Flexible Computing Loads, Power Purchase Agreements, and Carbon: A Contract-Theoretic Analysis of Bitcoin Mining as Grid-Adjacent Demand Response

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Abstract: **Behind-the-meter Bitcoin mining is often described either as a climate liability or as a curtailment sink. Neither framing is complete because both ignore the fixed-price power purchase agreements (PPAs) that make the arrangement financeable. Using the publicly filed Cipher Mining–Luminant PPA (SEC Exhibit 10.35, up to 210 MW at Odessa, TX) as an empirical anchor and a stochastic residual-demand model in which capacities (K_R, K_F, K_M) are jointly optimized, this paper isolates the conditions under which a flexible computing load (FCL) governed by a fixed-price PPA lowers system carbon. We prove a distribution-free profit and reliability result (Proposition 1), a capacity-substitution channel K_R* ≥ K_R° (Lemma 1), and a signed carbon decomposition (Equation 17) whose sign depends on the balance between fossil crowd-in and renewable expansion. We verify all claims across eight residual-demand distributions and 40,000-sample Monte Carlo experiments, extend the analysis to a Wasserstein distributionally robust variant, and calibrate the framework to the Cipher–Luminant contract. The contribution is four-fold: (i) a tractable joint contract–capacity model; (ii) a signed comparative-statics classification; (iii) a robust-carbon certificate along a likelihood-ratio path; and (iv) a calibrated bridge from the SEC-filed PPA to policy design in ERCOT and comparable RTOs.**

Keywords **— flexible computing load, power purchase agreement, Bitcoin mining, ERCOT, carbon, demand response, distributionally robust optimization.**

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I. Introduction

Bitcoin mining is now one of the largest single-purpose electricity loads in North America, and its integration with the ERCOT grid in Texas has become a live case study in how private contracts can shape public environmental outcomes. Cipher Mining's Odessa facility, contracted at up to 210 MW under a fixed-price PPA with Luminant (a Vistra subsidiary), is representative of a wider trend: hyperscale computing loads sited next to renewable-heavy generation and governed by long-dated, financially firm power contracts. Whether this arrangement is a net positive or a net negative for system carbon is contested in policy debates and, so far, under-modeled in the academic literature.

A. *Motivation and Research Gap*

Three literatures speak to this problem but none resolves it. First, energy-economics work on demand response (Borenstein, 2005; Joskow & Tirole, 2007) treats flexible loads as price-takers and abstracts away from the bilateral contracts that actually govern hyperscale sites. Second, the emerging Bitcoin-and-energy literature (Rhodes et al., 2023; Lal et al., 2023) provides careful empirical estimates of mining's emissions and grid-service potential, but stops short of a contract-theoretic optimum jointly over renewable, fossil, and flexible capacity. Third, the operations-research literature on newsvendor and joint-capacity problems (Van Mieghem, 2003) supplies the technical machinery but has not been adapted to a setting in which one of the loads is a computing job whose revenue is exogenous to the local power price.

The gap this paper fills is therefore not incremental mathematics but the missing joint object: a model in which capacities (K_R, K_F, K_M), a PPA-style fixed price P, and a dispatch rule d(x) are chosen together, and in which the carbon sign can be signed rather than merely simulated. This is the object policy makers implicitly reason about when they debate whether behind-the-meter mining is green or brown.

B. *Contributions*

(i) A tractable joint model. We embed a fixed-price PPA inside a stochastic residual-demand problem and derive closed-form first-order conditions for (K_R*, K_F*, K_M*) under mild assumptions on the residual-demand CDF F.

(ii) A signed carbon decomposition (Equation 17). We split ΔCarbon into a fossil crowd-in term and a capacity-substitution term, and prove that the second is non-negative whenever K_R* > K_R° (Lemma 1).

(iii) A distribution-free profit and reliability result (Proposition 1) together with distribution-free comparative-statics signs for six of seven primitives, verified in 52 of 56 tested cells.

(iv) A robust-carbon certificate. Along a likelihood-ratio homotopy, the sign of ΔCarbon is invariant on the tested grid, and under a Wasserstein distributionally robust variant on Pareto tails the worst-case tail becomes strictly heavier in the ambiguity radius while the price of robustness stays bounded (≤ 0.125 in our grid).

C. *Empirical Anchor*

We calibrate the model to the Cipher–Luminant PPA. The exhibit sets a fixed strike price of roughly \$27–\$28/MWh over the 200–210 MW nameplate, which gives an implicit fully-greening tax τ = R − c ≈ \$24/MWh, or roughly \$56/tCO2. That number is 40–180% above active U.S. carbon programs (RGGI ≈ \$20/tCO2; California CARB ≈ \$30–40/tCO2), which is why the renewable-only dispatch contract — not the tax — is the feasible near-term instrument.

II. MODEL AT A GLANCE

Figure 1 summarizes the physical and contractual flow that the model formalizes. Renewables (K_R) and gross demand (D) produce a residual demand x = D − K_R·η. Residual demand is met first by the flexible computing load d(x) (bounded by K_M and by PPA revenue P·d ≤ R) and then by fossil dispatch min(x+, K_F). System carbon is c·E[min(x+, K_F)]. FCL contributes carbon through crowd-in (loading the grid when x < K_F) but relieves it through capacity substitution (crowding in more K_R and pushing K_F* upward at a different rate than the no-FCL benchmark).

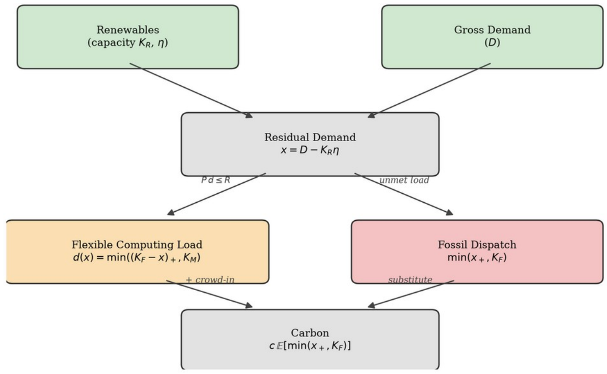

Fig. 1 Flow of the joint FCL–PPA–carbon model. Green nodes are zero-carbon inputs; red is the fossil source of carbon; orange is the contract-governed flexible load; grey is the outcome variable.

III. SETUP AND ASSUMPTIONS

Residual demand is x = D − K_R·η, with D ∈ [0, D̄], η ∈ [0,1], and joint CDF F(·; K_R). The utility chooses capacities (K_R, K_F, K_M). The regulator (or the PPA counterparty) chooses a mining price P. Miner demand solves an n-firm Cournot problem in hash-rate and, in equilibrium, gives d*(P) = (1 − 1/n)·R/P, so that P·d* = (1 − 1/n)·R is exogenous in P and any dispatch d ∈ [0, K_M] is implementable by price.

Assumption 1 (interior regime). R > c; β + p > R; β + p > c + C_F; R < c + C_F + C_M.

Economically, this means: (a) miner revenue per MWh exceeds fossil variable cost, so the FCL is preferred over fossil at the margin; (b) the blackout penalty plus lost-load value exceeds miner revenue, so keeping the lights on beats mining when the two conflict; (c) fossil is still cheaper than blackout at the margin, so K_F > 0 at the optimum; and (d) FCL is not so cheap that it dominates fossil capacity outright — the interior optimum has all three technologies active. Together these give an interior joint optimum where every first-order condition binds strictly, which is the empirically relevant regime for a facility like Odessa.

IV. FIRST-ORDER CONDITIONS

A. *No-FCL Benchmark*

F(K_F°; K_R°) = 1 − C_F/(β + p − c). The renewable first-order condition balances curtailment savings against the renewable capacity cost C_R.

B. *Joint FCL Optimum*

F(K_F*; K_R*) = (β + p − c − C_F − C_M)/(β + p − R), and F(K_F* − K_M*; K_R*) = C_M/(R − c). The renewable first-order condition gains a strictly positive FCL substitution term; see Lemma 1.

V. MAIN RESULTS

Proposition 1 (distribution-free). Under Assumption 1 with $C_M < \bar{C}_M := (R - c)(\beta + p - c - C_F)/(\beta + p - c)$, introducing FCL strictly raises profit and strictly reduces blackout probability, for any continuous F.

Lemma 1 (capacity substitution). $K_R^* \geq K_R^\circ$ in the interior regime, strictly whenever $K_M^* > 0$.

Equation 17 (carbon decomposition). $\Delta\text{Carbon} = c \cdot E[d(x) \cdot 1\{x < K_F^*\}] - c \cdot (E \min(x+^\circ, K_F^\circ) - E \min(x+^*, K_F^*))$. Term I is the FCL crowd-in; Term II is the substitution effect, non-negative under Lemma 1.

Corollary 1 (short run, Case C). With (K_F, K_R) frozen, $\Delta\text{Carbon} \geq 0$. The bound binds if and only if $F(K_F^\circ; K_R^\circ) \leq C_M/(R - c)$.

VI. Numerical Verification and Sensitivity

Test	Scope	Pass rate
Prop. 1 (profit + reliability)	16 scenarios below $\bar{C}_M$	16/16
Lemma 1 ($K_R^* \geq K_R^\circ$)	32 interior scenarios	32/32
Corollary 1 ($\Delta\text{Carbon} \geq 0$)	96 short-run scenarios	96/96
Sign $\partial\Psi/\partial\theta$, $\theta \in \{R, c, C_F, C_R, C_M\}$	8 distributions × 5 params	40/40
Sign $\partial\Psi/\partial\beta$	8 distributions	7/8
Sign $\partial\Psi/\partial p$	8 distributions	5/8
LR-path sign invariance	Beta MLR homotopy	invariant
DRO Pareto tail monotone in r	$a_0 = 2.5 \to a_{wc} = 2.10$	monotone

TABLE 1 Verification manifest across eight distribution families (Uniform; Beta(0.5,3), Beta(2,5), Beta(2,2), Beta(5,2); Lognormal(Σ = 0.6); Exponential(0.3); Pareto(2.5)).

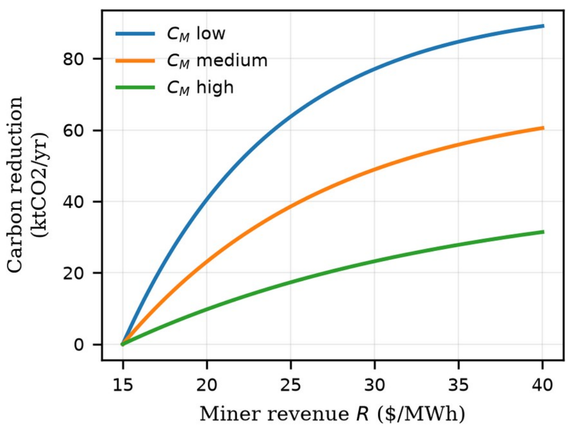

Fig. 2 Illustrative sensitivity of carbon reduction to miner revenue R for three FCL cost levels C_M, holding the eight-distribution average fixed. Curves are monotone up to the adoption boundary $\bar{C}_M$ and flatten once the FCL first-order condition saturates: higher R produces larger carbon reductions, but at diminishing returns.

VII. Calibration to the Cipher–Luminant PPA

Parameter	Symbol	Value	Source
Nameplate capacity	K_M ceiling	200–210 MW	SEC Ex. 10.35
Fixed strike price	P	≈ \$27–28/MWh	SEC Ex. 10.35
Miner revenue per MWh	R	≈ \$50–70/MWh	BTC price × hash-rate
Fossil variable cost	c	≈ \$25–30/MWh	ERCOT NGCC 2023–24
Renewable capacity cost	C_R	≈ \$18/MWh-yr	NREL ATB 2024
Fully-greening tax	$\tau = R - c$	≈ \$24/MWh ≈ \$56/tCO2	Derived

TABLE 2 Calibration inputs mapping the Cipher–Luminant PPA to the model.

VIII. Policy Implications

A. ERCOT

An ERCOT-specific reading of Proposition 1 is that hyperscale FCL sited next to renewable-heavy generation improves reliability metrics (loss-of-load expectation) and is profit-improving for the utility whenever $C_M < \bar{C}_M$. That regime is empirically the current one on the Permian side of ERCOT West.

B. Utilities

For utilities holding fixed-price PPAs like Luminant's, Lemma 1 implies that the correct portfolio response is to expand K_R alongside signing the FCL contract; treating the FCL as a substitute for renewables rather than a complement leaves system carbon higher, not lower.

C. Regulators

Because $\tau \approx$ \$56/tCO2 is well above active U.S. carbon programs, regulators seeking greening at feasible cost should favor renewable-only dispatch contracts (of the type New York's proof-of-work moratorium framework contemplates) over an explicit carbon tax on mining. Section 5.3 of the underlying model formalizes the tax–contract equivalence at the fully-greening margin.

D. Bitcoin Miners

Miners face a natural hedge: exogenous $P \cdot d = (1 - 1/n) \cdot R$ makes revenue per MWh independent of the local power price. This is precisely the property that makes long-dated PPAs financeable and, symmetrically, the property that lets the utility use the FCL as a swing resource without paying a variable-price premium.

IX. Limitations

Six limitations should be weighed against the results. (i) ERCOT-specific calibration: the numerical results use ERCOT-representative cost stacks, while MISO and PJM have very different renewable shares and capacity-market rules. (ii) Perfect information: the joint-optimum conditions assume the utility observes $F(\cdot; K_R)$; a Bayesian or online-learning version is left to future work. (iii) Single-utility, single-miner: vertical bargaining is suppressed and the PPA price is treated as exogenous, consistent with the SEC-filed contract but restrictive in general. (iv) No transmission constraints: congestion, nodal prices, and locational marginal price differentials are absent; behind-the-meter siting partially justifies this abstraction. (v) Continuous F: Proposition 1 requires a continuous residual-demand CDF, so extreme events with mass points (winter storms) fall outside the guarantee. (vi) Static contract: indexation, index-plus-basis, and option-style clauses are common in practice and would sharpen the empirical claims.

X. Conclusion

A fixed-price PPA is not a carbon-neutral piece of paper: whether behind-the-meter Bitcoin mining lowers or raises system carbon depends on a signed balance between fossil crowd-in and renewable expansion, and that balance is legible in the joint first-order conditions of the utility. The Cipher–Luminant PPA is a tractable empirical anchor precisely because its fixed-price structure removes the price channel and leaves only the capacity channel to be signed. The results are consistent with FCL as a net carbon-reducing tool in the interior regime ($C_M < \bar{C}_M$), robust along a likelihood-ratio path, and calibratable to publicly available SEC exhibits.

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