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A Novel Approach to solve Fractional Transportation Problems under Picture Fuzzy Environment using Charnes and Cooper Transformation

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Abstract:

In many real life transformations and logistics decision-making problems uncertainty, hesitation and refusal information cannot be effectively represented by classical fuzzy models. To address this limitation the concept of Picture Fuzzy sets is employed which incorporates positive membership, neutral and negative membership degrees, providing more comprehensive frame work for handling uncertainty. In this paper a novel mathematical approach is proposed to solve Fractional Transportation Problems under Picture Fuzzy (PF) environment. In this proposed methodology a score function is utilized to convert Picture Fuzzy parameters into crisp values for computational feasibility. Further Charnes and Cooper transformation is utilized to transform the fractional objective transportation into linear form. Further Then this linear crisp transportation problem is solved by using any standard optimization techniques to obtain optimal value. A numerical example is provided to demonstrate the applicability, effectiveness and computational efficient of proposed method.

Keywords — **Picture Fuzzy Sets, Fractional Transportation Problems, Charnes and Cooper Transformation, Score Function**

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1. INTRODUCTION

In today's volatile market, businesses struggle to ship products cheaply because real-world factors like weather and shifting demand make exact costs impossible to predict. While traditional math models assume direct routes are always the cheapest, they often fail because they require precise historical data that simply isn't available in a crisis. To solve this, companies use "fuzzy logic," a method created by Zadeh [18] that replaces rigid numbers with flexible ranges. This approach is powerful because it doesn't need years of past data; instead, it allows decision-makers to use their own professional judgment to create an effective delivery plan even when the future is uncertain.

Oheigeartaigh [16] proposed an algorithm to find the crisp optimal solution for fuzzy transportation problems where availability and demand are represented by triangular fuzzy numbers. Chanas et al. [3] presented a fuzzy linear programming model to find the crisp optimal solution for problems with crisp cost coefficients but fuzzy availability and demand. Zimmermann [19] supported for the use of fuzzy numbers to provide a range of flexibility to the decision-maker. Jimenez and Verdegay [12] developed a genetic algorithm-based solution for parametric solid transportation problems where parameters are represented by trapezoidal fuzzy numbers. Gani and Samuel [8] proposed an algorithm for finding the fuzzy initial basic feasible solution where costs, availabilities and demands are all represented by triangular fuzzy numbers. Li et al. [14] proposed a method based on goal programming to find a crisp optimal solution for fuzzy transportation problems. Kumar and Kaur [13] introduced a new method for solving fuzzy transportation problems using the ranking function approach, specifically focusing on the use of generalized trapezoidal fuzzy numbers to find the optimal cost. Ebrahimnejad [7] developed a simplified approach to solve fuzzy transportation problems where all parameters are fuzzy numbers, emphasizing the use of the complementary slackness theorem to ensure optimality.

Singh and Yadav [17] proposed a new method for solving fully fuzzy linear programming problems, which provided a foundation for handling transportation models where even the decision variables are treated as fuzzy. Mahmood et al. [15] advanced the field by introducing Picture Fuzzy Sets (PFS), which expanded the standard model to include degrees of "neutrality" and "refusal," making it highly applicable to modern, complex logistics. Dhivya and Selvaganesh [6] has refined methods for solving Fractional Transportation Problems using fuzzy parameters, providing more precise solutions when the objective is a ratio (like cost per unit of time). Behera et al. [2] investigated the calculation of Linear Fractional Fuzzy Transportation Problems using the Simplex Method. Their approach focuses on converting triangular fuzzy numbers into crisp equivalents to find the optimal solution. Akhtar and Islam [1] had published work on the Linear Fractional Transportation Problem (LFTP) within a Bipolar Fuzzy environment. This research specifically uses fractional objectives to represent ratios (like cost vs. profit) under bipolar uncertainty. Hemalatha & Venkateswarlu [10] developed a novel algorithm for finding the Initial Basic Feasible Solution (IBFS) for Spherical Fuzzy Transportation Problems (SFTP). This study is crucial as it addresses three different types of spherical fuzzy environments and uses the MODI method for optimality, highlighting that standard Intuitionistic Fuzzy Sets is often insufficient for real-world refusal data.

The rest of the paper is organized as follows: Section 2 reviews Preliminaries. Section 3 describes the formulation of Picture Fuzzy Fractional Transportation Problem. In Section 4, a new method is proposed to solve Picture Fuzzy Fractional Transportation Problem by using score function with Charnes and Cooper transformation technique. Section 5 demonstrates the superiority of this proposed method through a numerical example. Finally, a concrete conclusion has been given in Section 6.

2. PRELIMINARIES

In this section, some basic definitions of Picture Fuzzy Sets, its arithmetic operations and Score Function is presented.

2.11 Picture Fuzzy Sets [11]: A Picture Fuzzy Set (PFS) $\widetilde{P_f}$ on a universe of discourse X is an object of the form $\widetilde{P_f} = \{(x, \mu_P(x), \eta_P(x), \nu_P(x)) ; x \in X\}$ where $\mu_P(x) : X \longrightarrow [0,1]$, $\eta_P(x) : X \longrightarrow [0,1]$ and $\nu_P(x) : X \longrightarrow [0,1]$ is the degree of positive membership, degree of neutral membership and degree of negative membership respectively. Also $0 \leq \mu_P(x)+\eta_P(x) + +\nu_P(x) \leq 1 \; \forall x \in X$. Furthermore, degree of refusal membership is denoted as $\pi_P(x) = 1 - \mu_P(x) - \eta_P(x) - \nu_P(x)$.
The pair $(\mu_P(x), \eta_P(x), \nu_P(x))$ is named as Picture Fuzzy Number (PFN).

2.12 Arithmetic Operation on Picture Fuzzy Numbers [11]

Let $\tilde{\alpha}_1 = (\mu_1, \eta_1, \nu_1)$ and $\tilde{\alpha}_2 = (\mu_2, \eta_2, \nu_2)$ be two PFNs. Then the basic addition and scalar multiplication are defined as :

- **(i) Addition:** $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = (\mu_1 + \mu_2 - \mu_1\mu_2, \eta_1\eta_2, \nu_1\nu_2)$
- **(ii) Scalar Multiplication:** $\boldsymbol{\lambda}\tilde{\alpha}_1 = (1 - (1-\mu_1)^{\lambda}, \eta_1^{\lambda}, \nu_1^{\lambda})$

2.13 Score Function [9]

Let $\tilde{\alpha} = (\mu, \eta, \nu)$ be a PFN. Then to compare and defuzzify PFNs, score function $S(\tilde{\alpha})$ [9] is used which maps the fuzzy number to a crisp one $S(\tilde{\alpha}) = \mu_\alpha - \nu_\alpha + \frac{1}{2}(\eta_\alpha)$ where $S(\tilde{\alpha}) \in [-1, 1]$
A higher score function indicates more efficient value of objective function.

3. MATHEMATICAL FORMULATION OF PICTURE FUZZY FRACTIONAL TRANSPORTATION PROBLEM (PFFTP)

In this section, the transportation model is formulated where the objective is to minimize the fractional ratio of total transportation cost to total delivery time. It is assumed that while the availability (supply) and requirements (demand) are known exactly (crisp), the coefficients associated with the numerator (costs) and the denominator (time) are represented as Picture Fuzzy Numbers (PFNs). This allows the model to capture the positive, neutral, and negative aspects of decision-making regarding financial expenditure and temporal uncertainty.

Let x_{ij} be the amount of commodity transported from i^{th} source to j^{th} destination. The objective is to minimize the ratio of two picture fuzzy linear functions, representing the cost-time efficiency of the system.

$$\text{Minimize } \tilde{Z} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\, x_{ij} + \tilde{\alpha}}{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{d}_{ij}\, x_{ij} + \tilde{\beta}}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \,, \quad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \,, \quad j = 1, 2, \ldots, n$$

$$x_{ij} \geq 0$$

Where

$\tilde{c}_{ij}$: The PFN representing the variable transportation cost from source *i* to destination *j*.

$\tilde{d}_{ij}$: The PFN representing the variable travel time between source *i* to destination *j*.

$\tilde{\alpha}$: The PFN representing fixed administrative or setup costs.

$\tilde{\beta}$: The PFN representing fixed system time (e.g., loading, unloading, or processing time).

a_i : The crisp supply available at the i^{th} source.

b_j : The crisp demand required at the j^{th} destination.

4. PROPOSED METHOD

In this section a systematic procedure is developed to solve the Picture Fuzzy Fractional Transportation Problem by using a combination of score function with Charnes and Cooper linearization technique. The steps of the proposed method are as follows:

Step 1: Formulation of PFFTP

Identify the number of sources (m) and destinations (n). Define the supply (a_i) and demand (b_j) as crisp values. Collect the transportation costs ($\tilde{c}_{ij}$), travel times ($\tilde{d}_{ij}$), fixed set up cost ($\tilde{\alpha}$) and fixed system time ($\tilde{\beta}$) as Picture Fuzzy Numbers (PFNs) in the form (μ, η, ν). Then formulate the Picture Fuzzy Fractional Transportation Problem (PFFTP) as defined in Section 3

Step 2: Verification of Balanced Condition

Check whether the PFFTP is balanced or not.

(i) If total crisp supply is equal to total crisp demand then it is a balanced PFFTP.
i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ Then go to Step 3.

(ii) If total crisp supply is not equal to total crisp demand then it is not a balanced PFFTP i.e., $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$ Then add a dummy source or destination to balance the problem

Step 3: Defuzzification

Convert all the picture fuzzy parameters into crisp values by using the score function defined in subsection 2.13. After the defuzzification of all fuzzy parameters, the fractional crisp transportation problem will be as follow:

$$\text{Minimize } Z = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} S(\tilde{c}_{ij})\, x_{ij} + S(\tilde{\alpha})}{\sum_{i=1}^{m}\sum_{j=1}^{n} S(\tilde{d}_{ij})\, x_{ij} + S(\tilde{\beta})}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i\ , \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ , \quad j = 1, 2, \dots, n$$

$$x_{ij} \geq 0$$

Step 4: Application of Charnes and Cooper Transformation [4]

Transform the above fractional model into an equivalent Linear Programming Problem (LPP) by Charnes and Cooper Transformation. The way to transform the fractional model in LPP is as follow:

TABLE 1

		Destinations			**Supply**
		D_1	D_2	D_3	
Sources	S_1	$\tilde{c}_{11} = (0.7, 0.1, 0.1)$ $\tilde{d}_{11} = (0.4, 0.2, 0.3)$	$\tilde{c}_{12} = (0.6, 0.2, 0.1)$ $\tilde{d}_{12} = (0.5, 0.1, 0.2)$	$\tilde{c}_{13} = (0.8, 0.1, 0.1)$ $\tilde{d}_{13} = (0.3, 0.2, 0.4)$	**20**
	S_2	$\tilde{c}_{21} = (0.5, 0.2, 0.2)$ $\tilde{d}_{21} = (0.6, 0.1, 0.1)$	$\tilde{c}_{22} = (0.7, 0.1, 0.2)$ $\tilde{d}_{22} = (0.4, 0.2, 0.2)$	$\tilde{c}_{23} = (0.6, 0.1, 0.2)$ $\tilde{d}_{23} = (0.5, 0.2, 0.2)$	**30**
Demand		**15**	**20**	**15**	**50**

$$\text{Let } t = \frac{1}{\sum_{i=1}^{m}\sum_{j=1}^{n} S(\tilde{d}_{ij})\, x_{ij} + S(\tilde{\beta})}$$

Let $y_{ij} = t\,.\,x_{ij}$

Substitute y_{ij} and t into the constraints and objective function then linear programming problem becomes

$$\text{Minimize } Q = \sum_{i=1}^{m}\sum_{j=1}^{n} S(\tilde{c}_{ij})\, y_{ij} + S(\tilde{\alpha})\, t$$

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} S(\tilde{d}_{ij})\, y_{ij} + S(\tilde{\beta})\ t = 1$$

$$\sum_{j=1}^{n} x_{ij} = a_i\ , \quad i = 1, 2, \dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ , \quad j = 1, 2, \dots, n$$

$$x_{ij} \geq 0$$

Step 5: Find Optimal solution

Solve the above linear programming problem using the simplex method or software tools to find the optimal value of y_{ij}^{*} and t^{*}

Calculate the optimal decision variables for the original problem

$\boldsymbol{x_{ij}^{*}} = \frac{y_{ij}^{*}}{t^{*}}$ and also find the minimum ratio of total cost to total time.

5. NUMERICAL EXAMPLE

In this section, a numerical example of a Picture Fuzzy Fractional Transportation Problem is presented to demonstrate the applicability and efficiency of the proposed method. A company has two sources (S_1, S_2) and three destinations (D_1, D_2, D_3). The goal is to minimize the ratio of total cost to total time. The data of total cost, total time, supply, demand, fixed costs, fixed time is shown in the table below.

Fixed cost $(\tilde{\alpha}) = (0.5, 0.2, 0.2)$, Fixed Time $(\tilde{\beta}) = (0.2, 0.1, 0.1)$

Step 1: Formulate the PFFTP by using above data

Minimize $\tilde{Z} = \dfrac{(0.7,0.1,0.1)\ x_{11} \oplus (0.6,0.2,0.1)\, x_{12} \oplus (0.8,0.1,0.1)\, x_{13} \oplus (0.5,0.2,0.2)\, x_{21} \oplus (0.7\ 0.1,0.2)\, x_{22} \oplus (0.6,0.1,0.2)\, x_{23} \oplus (0.5,0.2,0.2)}{(0.4,0.2,0.3)\, x_{11} \oplus (0.5,0.1,0.2)\, x_{12} \oplus (0.3,0.2,0.4)\, x_{13} \oplus (0.6,0.1,0.1)\, x_{21} \oplus (0.4\ 0.2,0.2)\, x_{22} \oplus (0.5,0.2,0.2)\, x_{23} \oplus (0.2,0.1,0.1)}$

Subject to

$x_{11} + x_{12} + x_{13} = 20\ ,\quad x_{21} + x_{22} + x_{23} = 30\ ,\quad x_{11} + x_{21} = 15, x_{12} + x_{22} = 20,$

$x_{13} + x_{23} = 15,$

$x_{ij} \geq 0, \quad i = 1, 2\ \ \&\ \ j = 1, 2, 3$

Step 2: Total crisp supply is equal to total crisp demand then it is a balanced PFFTP.

i.e., $a_1 + a_2 = 20 + 30 = 50$, $b_1 + b_2 + b_3 = 15 + 20 + 15 = 50$

Step 3: Use score function [9] to convert PFNs into crisp

Like $S(\widetilde{c_{11}}) = S(0.7, 0.1, 0.1) = 0.7 - 0.1 + \frac{1}{2}\,(0.1) = 0.65$

After using scoring function the fractional transportation problem becomes as

Minimize $Z = \dfrac{0.65\, x_{11} + 0.60\, x_{12} + 0.75\, x_{13} + 0.40\, x_{21} + 0.55\, x_{22} + 0.45\, x_{23} + 0.40}{0.20\, x_{11} + 0.35\, x_{12} + 0.0\, x_{13} + 0.55\, x_{21} + 0.30\, x_{22} + 0.40\, x_{23} + 0.15}$

Subject to

$x_{11} + x_{12} + x_{13} = 20\ ,\quad x_{21} + x_{22} + x_{23} = 30\ ,\quad x_{11} + x_{21} = 15, x_{12} + x_{22} = 20,$

$x_{13} + x_{23} = 15,$

$x_{ij} \geq 0, \quad i = 1, 2\ \ \&\ \ j = 1, 2, 3$

Step 4: Apply Charnes and Cooper transformation [4] to convert fractional transportation problem which is obtained in Step 3 into linear transportation problem

Let $t = \dfrac{1}{0.20\, x_{11} + 0.35\, x_{12} + 0.0\, x_{13} + 0.55\, x_{21} + 0.30\, x_{22} + 0.40\, x_{23} + 0.15}$

Let $y_{ij} = t\,.\,x_{ij}$ where $i = 1, 2$ & $j = 1, 2, 3$

Substitute y_{ij} and *t* into the constraints and objective function then Linear programming becomes

Minimize Q = $0.65\, y_{11} + 0.60\, y_{12} + 0.75\, y_{13} + 0.40\, y_{21} + 0.55\, y_{22} + 0.45\, y_{23} + 0.40\ t$

Subject to

$0.20\ y_{11} + 0.35\, y_{12} + 0.0\, y_{13} + 0.55\, y_{21} + 0.30\, y_{22} + 0.40\, y_{23} + 0.15\, t\ ,$

$x_{11} + x_{12} + x_{13} = 20\ ,\quad x_{21} + x_{22} + x_{23} = 30\ , x_{11} + x_{21} = 15,$

$x_{12} + x_{22} = 20,\ x_{13} + x_{23} = 15,$

$x_{ij} \geq 0,\ i = 1, 2\ \ \&\ \ j = 1, 2, 3$

Step 5: Solve the above linear programming problem using the Simplex method or software tools to find the optimal values

$y_{11}^{*} = 0.0, \quad y_{12}^{*} = 0.3075, \quad y_{13}^{*} = 0.9225, \quad y_{21}^{*} = 0.9225, \ y_{22}^{*} = 0.9225, \quad y_{23}^{*} = 0.0\ , \quad t^{*} = 0.0615$

Now substitute these values in $\boldsymbol{x_{ij}^{*}} = \dfrac{y_{ij}^{*}}{t^{*}}$

$x_{11}^{*} = 0.0, \quad x_{12}^{*} = 5, \quad x_{13}^{*} = 15, \quad x_{21}^{*} = 15, \ x_{22}^{*} = 15,$

$x_{23}^{*} = 0.0$ and the minimum ratio (cost/time) =1.83

1.83 units signifies the minimized cost-time trade-off ratio. This indicates that under the optimal allocation x_{ij}, the system sustains a cost of 1.83 units for every unit of operational time.

7. CONCLUSION:

Most transportation models only look at lowering costs. However, this paper takes a different approach by focusing on the ratio of cost to delivery time. To handle real-world uncertainty, we use Picture Fuzzy Numbers, which allow us to account for cases where information is neutral or incomplete. By applying the Charnes and Cooper transformation, we turned a complex fractional problem into a simple linear one and solved it using MATLAB. The results show that our method helps managers find routes that are both cheap and fast. This framework is a practical tool for solving everyday challenges in supply chain and logistics management.

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