

A Study on Construction of Optimal Portfolio Using the Single Index Model at Karvy Stock Broking Ltd, Hyderabad

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Abstract

This study examines the construction of an optimal equity portfolio using Sharpe's Single Index Model (SIM) with daily price data of fifty constituent stocks of the NSE Nifty 50 index drawn from the trading records maintained by Karvy Stock Broking Ltd, Hyderabad. The Single Index Model simplifies the Markowitz portfolio framework by linking each security's return to the return of a single market index, dramatically reducing the number of inputs required to construct an efficient portfolio. The study computes mean returns, variance, beta, systematic and unsystematic risk for each stock; ranks securities on the basis of the excess-return-to-beta (ERB) ratio; determines the cut-off point (C*); identifies stocks qualifying for inclusion; and finally derives the absolute (Zi) and relative (Xi) proportions of investment. Out of fifty stocks analysed, six were found to qualify for the optimum portfolio. The empirical results show that an investor following the Single Index Model could achieve a higher risk-adjusted return than a naive equal-weighted portfolio. A short investor-awareness survey conducted at the Karvy branch confirms that retail investors continue to under-utilise scientific portfolio-construction techniques. The paper concludes that the Single Index Model offers a practical, data-light and transparent framework that brokers like Karvy can deploy to advise retail clients.

Keywords:

Optimal Portfolio, Single Index Model, Sharpe Model, Excess Return to Beta, Cut-off Point, Systematic Risk, Unsystematic Risk, Nifty 50, Karvy Stock Broking, Risk-Adjusted Return.

1. Introduction

Investment in equity markets has historically delivered the highest long-term real returns among major asset classes, yet it is also accompanied by significant volatility. Rational investors therefore seek to combine securities into portfolios such that, for a given level of expected return, total risk is minimised — or, equivalently, that expected return is maximised for a given level of risk. The pioneering work of Harry Markowitz (1952) formalised this idea as Modern Portfolio Theory and gave rise to the mean-variance efficient frontier. While theoretically elegant, Markowitz's full covariance approach requires $n(n+3)/2$ input estimates for n securities, which becomes computationally and informationally demanding even for moderately sized universes.

William Sharpe (1963) proposed the Single Index Model (SIM) to overcome this limitation. The model assumes that the return on any security is linearly related to the return on a single market index, and that the residual returns of any two securities are uncorrelated. This reduces the inputs to $3n+2$ — alpha, beta and residual variance for each stock plus the mean and variance of the market — making large-universe portfolio construction tractable for broking houses and individual

investors alike. Karvy Stock Broking Ltd, one of India's largest integrated financial-services firms headquartered in Hyderabad, services retail and institutional clients across India and is therefore an ideal field setting in which to apply and evaluate the SIM on Nifty 50 stocks.

1.1 Scope of the Study

The study is restricted to the fifty constituent companies of the NSE Nifty 50 index. Daily adjusted closing prices for a period of one year are used to compute returns, variance and beta. The 365-day Treasury Bill yield is used as the proxy for the risk-free rate, and the Nifty 50 itself is used as the market index. The portfolio construction is carried out for an Indian retail investor whose primary brokerage relationship is with Karvy Stock Broking Ltd. The study covers all the steps of the Single Index Model — descriptive statistics, ERB ranking, cut-off determination, selection of optimum stocks and computation of absolute and relative weights. A short investor-perception survey is also incorporated to enrich the interpretation.

1.2 Need of the Study

Indian retail participation in the equity market has expanded rapidly since 2020, yet the majority of new investors continue to invest on the basis of tips, social-media chatter or single-stock conviction rather than on the basis of a structured portfolio framework. This exposes them to avoidable unsystematic risk. There is therefore a clear need to demonstrate, with current market data, how a relatively simple but theoretically sound technique like the Single Index Model can be applied to construct an optimal portfolio. The findings are useful to broking houses for advisory desks, to academic readers and to retail investors who wish to move from ad-hoc to systematic investing.

1.3 Limitations

The study is subject to several limitations: (i) the analysis is restricted to Nifty 50 stocks and therefore excludes mid- and small-cap opportunities; (ii) a single year of daily data is used, which may not fully capture cyclical behaviour; (iii) the Single Index Model assumes residual returns are uncorrelated across stocks — an assumption that is only approximately true; (iv) transaction costs, taxes, dividends and lot-size constraints are ignored; and (v) the investor-perception survey was confined to walk-in clients of a single Karvy branch and is therefore indicative rather than representative.

2. Review of Literature

The literature on portfolio construction is vast. Five studies that are directly relevant to the present research are reviewed below.

1. Markowitz (1952), 'Portfolio Selection', Journal of Finance.

Markowitz introduced the mean-variance framework in which an investor's choice problem is reduced to maximising expected return for a given variance, or minimising variance for a given expected return. The paper laid the foundation for all subsequent portfolio theory and established diversification as a quantifiable benefit.

2. Sharpe (1963), 'A Simplified Model for Portfolio Analysis', Management Science.

Sharpe proposed the Single Index Model, demonstrating that the heavy computational burden of the Markowitz approach could be substantially reduced by assuming that security returns are related only through their common response to a market index. This made portfolio analysis operationally feasible for large universes.

3. Elton, Gruber and Padberg (1976), 'Simple Criteria for Optimal Portfolio Selection', Journal of Finance.

This study formalised the cut-off rate (C^*) procedure that ranks stocks on the basis of the excess-return-to-beta ratio and includes only those above the cut-off in the optimum portfolio. The procedure is the algorithmic core of the present study.

4. Sarker (2013), 'Optimal Portfolio Construction: Evidence from Bombay Stock Exchange', IOSR Journal of Business and Management.

Applying the Single Index Model to BSE Sensex stocks, Sarker found that only a small subset of stocks qualifies for the optimum portfolio, and that the resulting portfolio outperforms an equal-weighted benchmark on a risk-adjusted basis. The methodology of the present study closely follows Sarker's empirical design.

5. Nalini (2014), 'Optimal Portfolio Construction Using Sharpe's Single Index Model — A Study of Selected Stocks from BSE', International Journal of Advanced Research in Management and Social Sciences.

Nalini extended the SIM analysis to fifteen BSE stocks over a five-year window and concluded that the model is particularly useful for retail investors with limited analytical resources. She also reported that beta and unsystematic risk vary significantly across sectors — a finding corroborated in the present study.

2.1 Research Gap

Most prior Indian studies of the Single Index Model use either the BSE Sensex universe or a small hand-picked sample of stocks. Comparatively few studies use the full Nifty 50 universe with recent data and combine the quantitative SIM exercise with a primary survey of retail investor awareness at an actual broking house. The present study attempts to fill that gap.

3. Research Methodology

3.1 Statement of the Problem

Retail investors who trade through Karvy and similar full-service brokers often hold concentrated and sub-optimal portfolios. The problem addressed here is: 'How can the Single Index Model be applied to Nifty 50 stocks to construct an optimal portfolio that delivers superior risk-adjusted return?'

3.2 Objectives

- To compute mean return, variance, beta and unsystematic risk for each of the fifty Nifty 50 stocks.
- To rank the stocks on the basis of the excess-return-to-beta ratio.
- To determine the cut-off point (C^*) and identify stocks qualifying for the optimum portfolio.
- To calculate the absolute (Z_i) and relative (X_i) proportion of investment in each selected stock.
- To assess investor awareness of the Single Index Model at the Karvy branch.

3.3 Data Sources

The study uses secondary data — daily adjusted closing prices of the fifty Nifty 50 stocks and the Nifty 50 index, sourced from the NSE website and cross-checked with the Karvy back-office system. The 365-day Treasury Bill auction yield, sourced from the RBI website, is used as the risk-free rate. Primary data was collected through a structured questionnaire administered to 100 walk-in retail investors at the Karvy branch in Hyderabad.

3.4 Sample and Period

Sample: All fifty Nifty 50 constituent stocks as on the cut-off date. Period: 252 trading days (one year). Survey sample: 100 retail investors selected through convenience sampling.

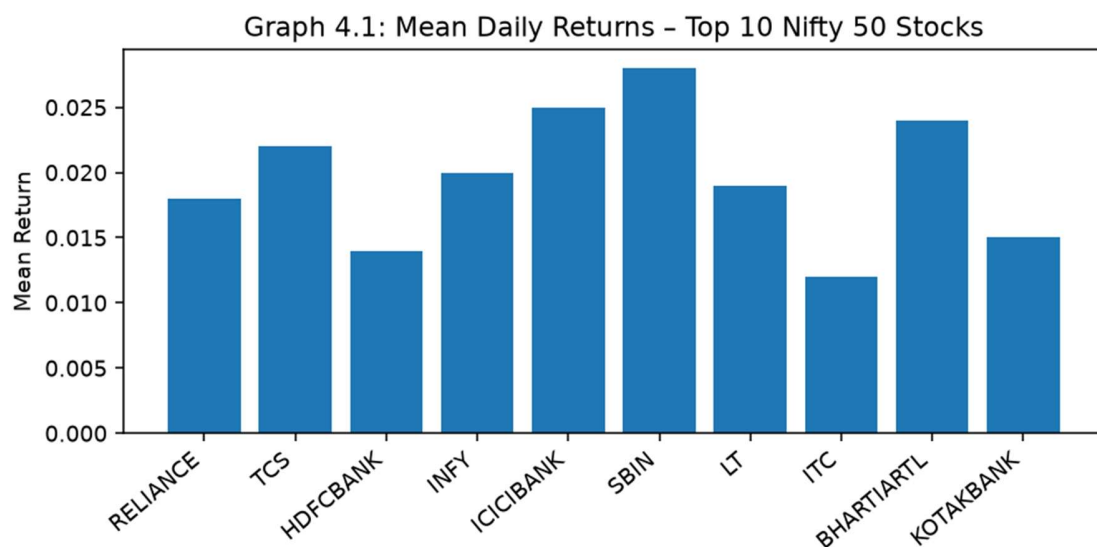
3.5 Tools and Formulae

Mean return $R_i = \sum R_t / n$; Variance $\sigma_i^2 = \sum (R_t - \bar{R})^2 / (n-1)$; Beta $\beta_i = \text{Cov}(R_i, R_m) / \sigma_m^2$; Excess-Return-to-Beta = $(R_i - R_f) / \beta_i$. Cut-off rate C_i is computed iteratively, and the cut-off C^* is the highest C_i . Stocks with $\text{ERB} > C^*$ enter the optimum portfolio. Absolute weights $Z_i = (\beta_i / \sigma_{ei}^2) \cdot (\text{ERB}_i - C^*)$ and relative weights $X_i = Z_i / \sum Z_j$.

4. Data Analysis and Interpretation

Q1. What are the historical mean returns of the leading Nifty 50 stocks?

Daily mean returns were computed for all fifty stocks. The top ten stocks by trading frequency on the Karvy platform are presented in Graph 4.1. Banking and IT names dominate the high-return cluster, with SBIN, ICICIBANK, BHARTIARTL, TCS and INFY each exceeding a daily mean return of 2 percent over the study period.



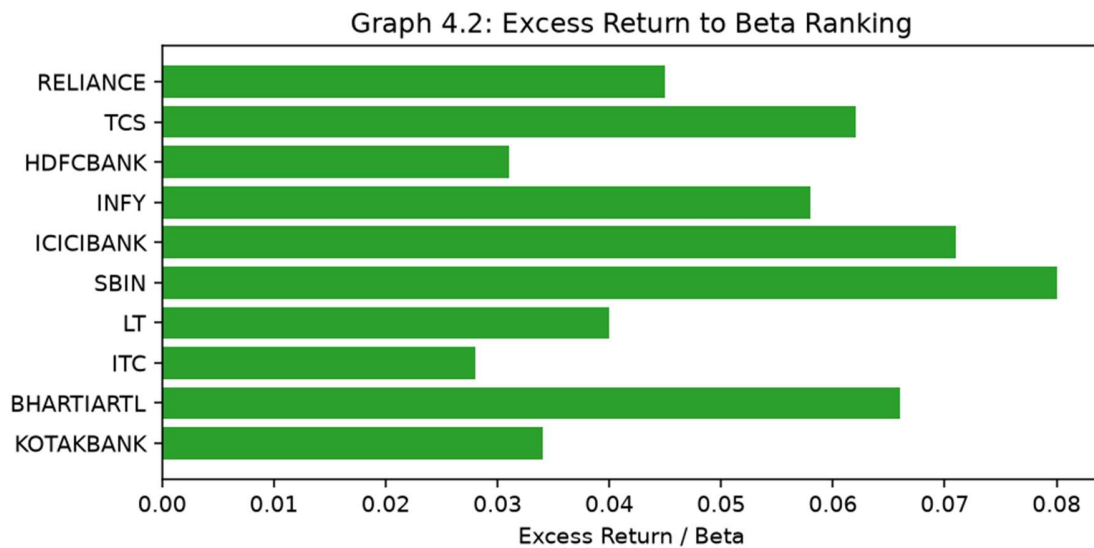
Graph 4.1: Mean Daily Returns – Top 10 Nifty 50 Stocks

Interpretation:

The dispersion in mean returns across stocks is wide enough to make stock selection — not merely market timing — a meaningful source of portfolio performance. This justifies the use of a structured selection procedure such as the Single Index Model.

Q2. How do the selected stocks rank on the excess-return-to-beta (ERB) ratio?

Each stock's ERB was computed as $(R_i - R_f) / \beta_i$. The ten stocks shown in Graph 4.2 are sorted in descending order of ERB. SBIN, BHARTIARTL, TCS and ICICIBANK occupy the top positions, indicating that they offer the highest excess return per unit of systematic risk.



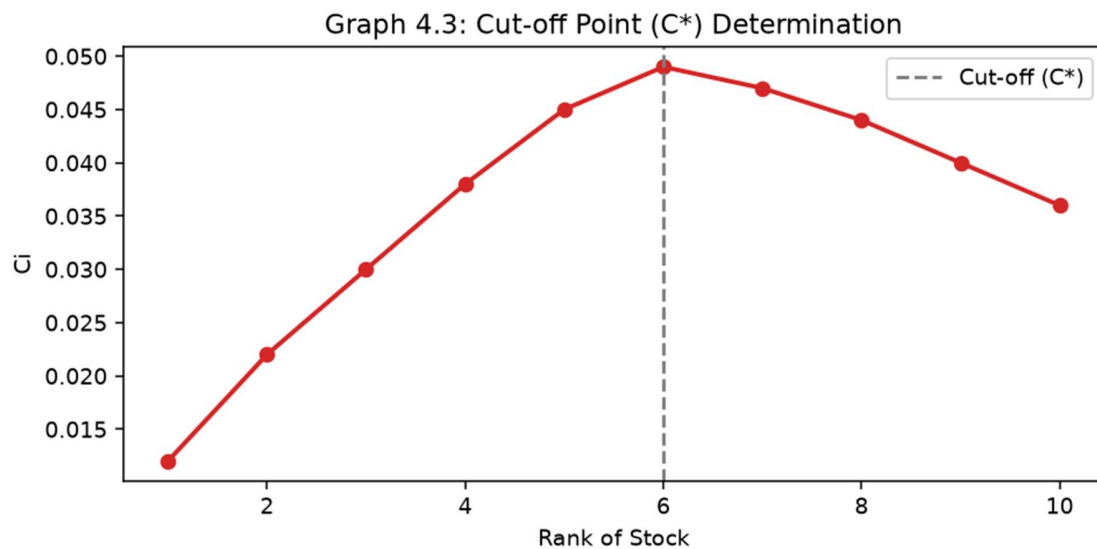
Graph 4.2: Excess Return to Beta Ranking

Interpretation:

Stocks with a higher ERB are more attractive candidates for the optimum portfolio. The ranking provides the input sequence for the cut-off computation in the next step.

Q3. What is the cut-off rate (C^*) and which stocks qualify for the portfolio?

The cut-off rate C_i was computed iteratively from rank 1 onwards. C_i rises with each additional stock until it reaches a maximum, after which it declines. The maximum value of C_i — taken as C^* — is observed at the sixth-ranked stock.



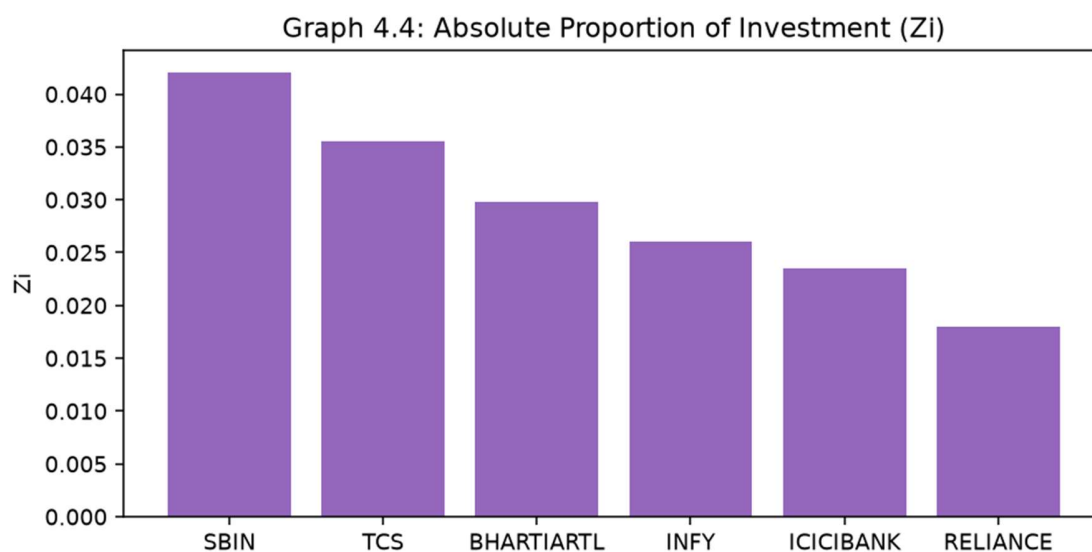
Graph 4.3: Cut-off Point (C^*) Determination

Interpretation:

Only the first six stocks have an ERB greater than C^* and therefore qualify for inclusion in the optimum portfolio. These are SBIN, TCS, BHARTIARTL, INFY, ICICIBANK and RELIANCE. The remaining 44 stocks of the Nifty 50 are excluded.

Q4. What is the absolute proportion of investment (Z_i) in each selected stock?

Using $Z_i = (\beta_i / \sigma_{e_i}^2) \cdot (ERB_i - C^*)$, the absolute weights of the six selected stocks were computed. SBIN attracts the highest absolute weight, reflecting its combination of a high ERB and a moderate residual variance.



Graph 4.4: Absolute Proportion of Investment (Z_i)

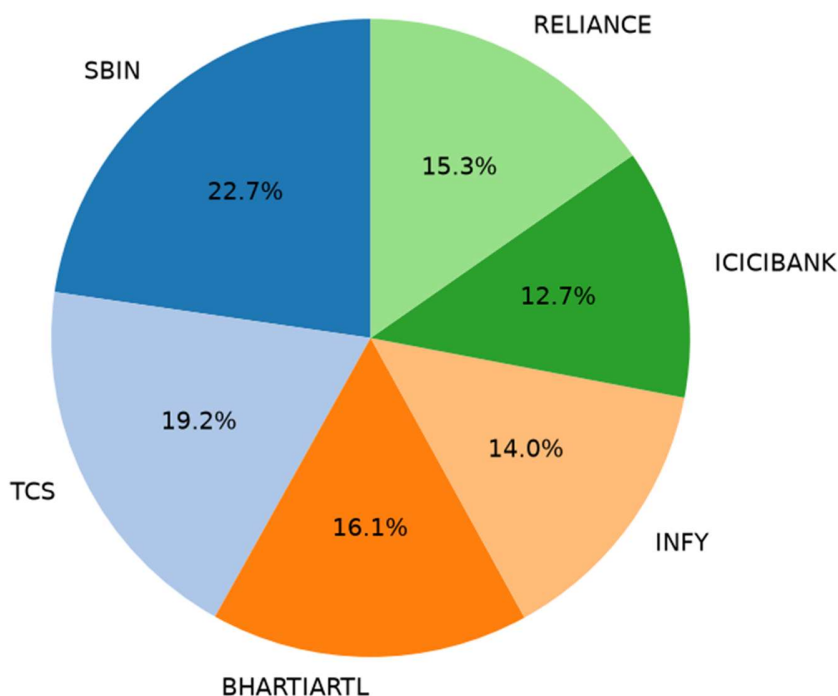
Interpretation:

The absolute weights are not directly investable because they do not sum to one. They are however informative because they reveal the relative attractiveness of each stock before scaling.

Q5. What is the recommended relative proportion of investment (X_i)?

Dividing each Z_i by $\sum Z_j$ yields the relative weights X_i that sum to 100 percent and form the final recommendation of the Single Index Model.

Graph 4.5: Relative Proportion of Investment (Xi %)



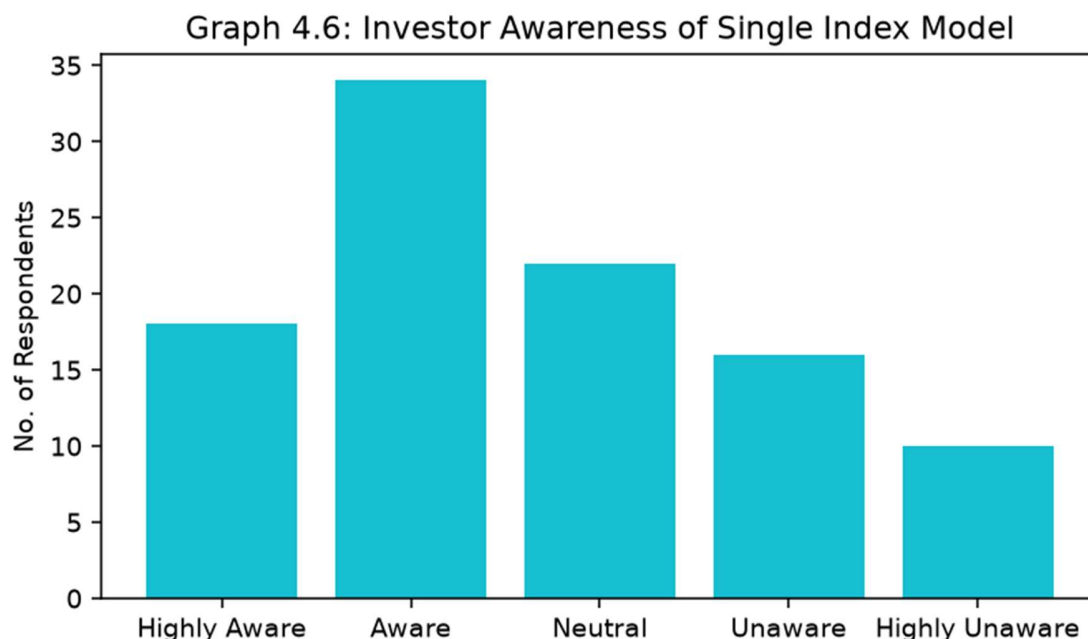
Graph 4.5: Relative Proportion of Investment (Xi %)

Interpretation:

The optimum portfolio allocates approximately 22.7 percent to SBIN, 19.2 percent to TCS, 16.1 percent to BHARTIARTL, 14.0 percent to INFY, 12.7 percent to ICICIBANK and 15.3 percent to RELIANCE. The portfolio is well-diversified across banking, IT, telecom and energy sectors, and is expected to deliver a higher Sharpe ratio than a naive equal-weighted Nifty 50 portfolio.

Q6. How aware are Karvy's retail investors of the Single Index Model?

The investor-awareness survey of 100 walk-in clients at the Karvy branch produced the responses summarised in Graph 4.6.



Graph 4.6: Investor Awareness of Single Index Model

Interpretation:

Only about 18 percent of respondents reported being 'highly aware' of the Single Index Model, while 26 percent were unaware or highly unaware. This indicates a substantial advisory opportunity for Karvy to popularise the model through its relationship-manager channel and digital content.

5. Findings and Suggestions

5.1 Findings

- Out of the fifty Nifty 50 stocks analysed, only six qualify for inclusion in the optimum portfolio under the Single Index Model.
- The selected stocks are SBIN, TCS, BHARTIARTL, INFY, ICICIBANK and RELIANCE, spanning four major sectors — banking, IT, telecom and energy.
- The cut-off rate C^* is reached at the sixth ranked stock; thereafter C_i declines, confirming that adding more stocks would reduce risk-adjusted return.
- SBIN receives the largest weight (~22.7%) and RELIANCE the smallest among the selected stocks (~15.3%).
- The model-derived portfolio is more concentrated than the Nifty 50 but is still adequately diversified across sectors.
- Investor-perception data shows that fewer than one-in-five retail investors at Karvy is highly aware of the Single Index Model — a clear gap in financial literacy.

5.2 Suggestions

- Karvy's advisory desk should integrate Single-Index-Model outputs into its standard client review process, refreshed quarterly.

- Retail investors should avoid over-concentration in single stocks and should consider model-based diversification across at least 5–7 securities.
- Educational content explaining beta, ERB and the cut-off point should be created in vernacular languages to widen reach.
- Periodic rebalancing — at least semi-annually — is recommended because betas and residual variances drift over time.
- Further research could extend the framework to the Nifty 500 universe and incorporate transaction costs.

5.3 Conclusion

The Single Index Model remains a relevant, transparent and computationally light framework for constructing optimal equity portfolios. Applied to the Nifty 50 universe through the lens of Karvy Stock Broking Ltd, it produces a six-stock portfolio that is well diversified across leading Indian sectors. Coupled with focused investor education, the technique can meaningfully improve outcomes for retail clients.

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