

# A Comprehensive Comparative Analysis of Traffic Stream Models for Highway Volume and Capacity Optimization: A Case Study of the Pune Ring Road Corridor

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## Abstract:

Traffic volume studies and capacity forecasting are critical elements of urban transport infrastructure planning. This paper presents a comparative research analysis using empirical traffic projection data from the proposed Pune Ring Road (PRR) corridor project. We systematically apply five classic macroscopic traffic stream models: the Greenshields Linear Model, Greenberg Logarithmic Model, Underwood Exponential Model, Northwestern Exponential Model, and Drake Bell-Shaped Curve Model. Mathematical derivations for maximum flow ( $q_{max}$ ) and optimum velocity ( $v_0$ ) are established for each framework.

By grounding our equations in the project's empirical vehicle statistics alongside standard calibrated engineering boundaries, we evaluate how each model characterizes the transition from free-flow to congested traffic flow. A non-linear regression using the Levenberg-Marquardt algorithm is executed to rank the models based on Coefficient of Determination ( $R^2$ ) and Root Mean Squared Error (RMSE). This study establishes a definitive theoretical framework to assist civil engineers in selecting appropriate macroscopic models for structural highway designs, toll plaza geometry, and intersection grade-separations.

**Keywords — Traffic, Pune Ring Road, Greenshields Linear Model, Greenberg Logarithmic Model, Underwood Exponential Model, Northwestern Exponential Model, Drake Bell-Shaped Curve Model, Non-Linear Regression.**

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## I. INTRODUCTION

Modern arterial highway networks suffer from non-linear congestion spikes driven by rapid regional

urbanization. Traffic flow theory relies on macroscopic mathematical descriptions to evaluate the structural integrity of these networks. These

models analyze the continuous stream properties of three core variables:

- Flow ( $q$ ): Vehicles passing a point per unit time (PCU/hr)
- Speed ( $v$ ): Space-mean speed of the vehicle stream (km/hr)
- Density ( $k$ ): Number of vehicles occupying a unit length of roadway (PCU/km)

The basic equation governing all traffic stream operations is the fundamental identity:

$$q = v \cdot k$$

This research employs traffic field volume metrics from the Detailed Project Report (DPR) for the Pune Ring Road (PRR) bypass system. Empirical datasets from major project gateways—including the Talegaon Toll Plaza, Sutarwadi on Paud Road (SH-60), Mutha on Temghar–Lavasa Road (SH-57), Malkhed-Panshet Road, Khanapur-Velhe Road, and Khed Shivapur Toll Plaza—indicate that peak-hour traffic volumes are fast approaching or exceeding nominal road design thresholds.

To optimize corridor performance and prevent bottleneck failures, this paper analyzes five distinct macroscopic mathematical models. We provide algebraic proofs for each model's capacity parameters and run a quantitative comparative stress test using a combination of the project's long-term vehicle projections and standard design parameters.

## II. THEORETICAL FRAMEWORK & MODEL DERIVATIONS

### A. Greenshields Linear Model

Proposed by Bruce Greenshields in 1935, this foundational model assumes a linear relationship between speed and density.

*Mathematical Formulation:*

$$v = v_f \left( 1 - \frac{k}{k_j} \right)$$

*Derivation of Capacity ( $q_{max}$ ):*

*Using the fundamental identity:*

$$q = v \cdot k = v_f \left( k - \frac{k^2}{k_j} \right)$$

To determine the maximum traffic volume, take the first derivative with respect to density ( $k$ ) and set it to zero:

$$\frac{dq}{dk} = v_f \left( 1 - \frac{2k}{k_j} \right) = 0 \Rightarrow k_o = \frac{k_j}{2}$$

Substituting the optimum density ( $k_o$ ) back into the speed relationship gives the optimum speed ( $v_o$ ):

$$v_o = v_f \left( 1 - \frac{k_j/2}{k_j} \right) = \frac{v_f}{2}$$

The maximum capacity  $q_{max}$  is computed as:

$$q_{max} = v_o \cdot k_o = \left( \frac{v_f}{2} \right) \left( \frac{k_j}{2} \right) = \frac{v_f \cdot k_j}{4}$$

### B. Greenberg Logarithmic Model

Greenberg (1959) developed a logarithmic formulation based on fluid-dynamics analogy. It works well for high-density, congested traffic streams but struggles at low densities because velocity approaches infinity as density goes to zero.

**Mathematical Formulation:**

$$v = v_o \ln\left(\frac{k_j}{k}\right)$$

Where  $v_o$  is the speed at maximum flow.

**Derivation of Capacity ( $q_{max}$ ):**

$$\begin{aligned} q &= v \cdot k = v_o \cdot k \\ &= v_o \cdot k(\ln k_j - \ln k) \end{aligned}$$

Differentiating with respect to  $k$ ,

$$\begin{aligned} \frac{dq}{dk} &= v_o \left[ (\ln k_j - \ln k) \cdot (1) + k \cdot \left(-\frac{1}{k}\right) \right] = 0 \\ \ln\left(\frac{k_j}{k}\right) - 1 &= 0 \Rightarrow \ln\left(\frac{k_j}{k}\right) = 1 \Rightarrow k_o = \frac{k_j}{e} \end{aligned}$$

Substituting  $k_o$  back into the speed equation proves that the parameter  $v_o$  is the optimum speed,

$$v(k_o) = v_o \ln(e) = v_o$$

The maximum capacity  $q_{max}$  is:

$$q_{max} = v_o \cdot k_o = v_o \cdot \frac{k_j}{e}$$

**C. Underwood Exponential Model**

Underwood (1961) proposed an exponential model to address the infinite-speed limitation of Greenberg's model at low densities. However, this model has a different trade-off: speed never drops to zero, meaning it cannot mathematically represent a complete jam density ( $k_j = \infty$ ).

**Mathematical Formulation:**

$$v = v_f \cdot e^{-\frac{k}{k_o}}$$

Where  $k_o$  represents the optimum density at maximum flow.

**Derivation of Capacity  $q_{max}$ :**

$$q = v \cdot k = v_f \cdot k \cdot e^{-\frac{k}{k_o}}$$

Differentiating with respect to  $k$ ,

$$\begin{aligned} \frac{dq}{dk} &= v_f \left[ e^{-\frac{k}{k_o}} + k \cdot \left(-\frac{1}{k_o}\right) e^{-\frac{k}{k_o}} \right] = 0 \\ v_f \cdot e^{-\frac{k}{k_o}} \left(1 - \frac{k}{k_o}\right) &= 0 \Rightarrow k = k_o \end{aligned}$$

Substituting  $k_o$  into the velocity equation,

$$v_o = v_f \cdot e^{-1} = \frac{v_f}{e}$$

The maximum capacity  $q_{max}$  is:

$$q_{max} = v_o \cdot k_o = \frac{v_f \cdot k_o}{e}$$

**D. Northwestern Exponential Model**

Developed at Northwestern University, this model introduces an exponential polynomial structure to better capture speed drops as densities move past free-flow levels.

**Mathematical Formulation:**

$$v = v_f \cdot e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^2}$$

**Derivation of Capacity  $q_{max}$ :**

$$q = v \cdot k = v_f \cdot k \cdot e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^2}$$

Differentiating with respect to  $k$  using the product rule:

$$\begin{aligned} \frac{dq}{dk} &= v_f \left[ 1 \cdot e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^2} + k \cdot \left(-\frac{k}{k_o^2}\right) e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^2} \right] \\ &= 0 \\ v_f \cdot e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^2} \left(1 - \frac{k^2}{k_o^2}\right) &= 0 \Rightarrow k = k_o \end{aligned}$$

Substituting  $k_o$  into the velocity framework,

$$v_o = v_f \cdot e^{-0.5} = \frac{v_f}{\sqrt{e}}$$

The maximum capacity  $q_{max}$  is:

$$q_{max} = v_o \cdot k_o = \frac{v_f \cdot k_o}{\sqrt{e}}$$

**E. Drake Bell-Shaped Curve Model**

Drake et al. (1967) introduced a modified bell-shaped curve function. It uses a steeper exponential drop to better mirror how traffic transitions from stable flow to severe congestion.

**Mathematical Formulation :**

$$v = v_f \cdot e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^4}$$

**Derivation of Capacity  $q_{max}$  :**

$$q = v \cdot k = v_f \cdot k \cdot e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^4}$$

Differentiating with respect to k,

$$\frac{dq}{dk} = v_f \left[ 1 \cdot e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^4} + k \cdot \left( -\frac{2k^3}{k_o^4} \right) e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^4} \right] = 0$$

$$v_f \cdot e^{-\frac{1}{2}\left(\frac{k}{k_o}\right)^4} \left( 1 - 2\left(\frac{k}{k_o}\right)^4 \right) = 0$$

$$2\left(\frac{k}{k_o}\right)^4 = 1 \Rightarrow k = \frac{k_o}{\sqrt[4]{2}} \approx 0.8409 k_o$$

Substituting this peak-flow density parameter back into the speed equation yields,

$$v_{opt} = v_f \cdot e^{-\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^4} = v_f \cdot e^{-\frac{1}{4}} = \frac{v_f}{\sqrt[4]{e}}$$

The maximum capacity  $q_{max}$  is:

$$q_{max} = v_{opt} \cdot k_{opt} = \left(\frac{v_f}{\sqrt[4]{e}}\right) \left(\frac{k_o}{\sqrt[4]{2}}\right)$$

**III. REFERENCE DATASETS (SOURCED FROM DETAILED PROJECT REPORT)**

The analysis relies directly on empirical field metrics compiled from traffic reconnaissance, destination surveys, and cross-road counts.

TABLE I

BASE YEAR (2020) AND FORECASTED TRAFFIC VOLUME PROJECTIONS (VEHICLES/DAY)

This dataset maps out long-term demand trends across distinct vehicle classifications.

Year	Car / Jeep / Van	Mini Buses	Buses	Mini LC V	LC V	2-Axle Truck	3-Axle Truck	MAV (4-6 Axle)	Total Vehicles
2020	5,104	40	1,639	1,337	2,553	3,487	1,873	5,474	21,507
2026	8,098	55	2,238	1,971	3,764	5,139	2,762	8,067	32,093
2032	12,286	74	3,027	2,754	5,257	7,179	3,858	11,270	45,704
2038	17,811	100	4,062	3,698	7,059	9,640	5,180	15,134	62,683
2042	22,276	121	4,938	4,495	8,580	11,717	6,296	18,395	76,817

TABLE II

PROJECTED MAINLINE CORRIDOR TRAFFIC LOADINGS (PASSENGER CAR UNITS - PCU)

To standardise calculations, the conversion matrix maps traffic into equivalent PCU factors as per IRC Guidelines.

Year	Car / Jeep / Van	Mini Buses	Buses	Mini LC V	LC V	2-Axle Truck	3-Axle Truck	MAV (4-6 Axle)	Total PCU
2020	5,104	60	4,916	2,006	3,829	10,460	5,620	24,632	56,628
2026	8,098	82	6,715	2,957	5,645	15,418	8,285	36,301	83,503
2032	12,286	112	9,080	4,131	7,885	21,536	11,573	50,714	1,17,316
2038	17,811	150	12,186	5,546	10,588	28,920	15,540	68,101	1,58,843

2	22,27	181	14	6,7	12	35,1	18,8	82,77	1,93
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TABLE III

CROSS-ROAD INTERSECTION FLOW BASELINES (AADT PCU - 2020)

This dataset tracks intersection nodes crossing different roadway classes before adding separation flyovers.

Vehicle Mode / Type	Paud Road (SH-60)	Mutha Road (SH-57)	Malkhed Road (MDR)	Khanapur Road (MDR)
Two-Wheelers (Sc/Mc)	6,259	2,143	1,746	1,052
Auto Three-Wheelers	872	136	164	94
Car / Jeep / Utility	6,125	3,176	1,457	621
Buses (Govt. & Private)	854	92	160	15
Light Commercial Vehicles	1,326	180	259	130
Trucks & Heavy Machinery	1,029	112	186	52
Tractors & Trailers	402	104	64	48
Non-Motorised (Cycles etc.)	31	3	11	5
<b>Total Base Volume</b>	<b>17,220</b>	<b>5,981</b>	<b>4,073</b>	<b>2,020</b>

#### IV. ANALYTICAL ASSUMPTIONS

##### ENLISTMENT :

To construct macro performance models from daily field totals, specific transport engineering constants were defined.

##### F. Boundary Value Assumptions

- **Design Free-Flow Speed  $v_f$**  : Set at 100 km/hr based on standard design criteria for an access-controlled multi-lane regional expressway bypass in India.

- **Corridor Jam Density  $k_j$** : Defined as 150 PCU/km/lane representing the theoretical ceiling where vehicle streams reach a bumper-to-bumper standstill under mixed-traffic footprints.
- **Optimal Reference Density Boundary  $k_o$**  : Set at 65 PCU/km/lane to act as a shared calibration link to standardize exponential and bell-shaped curves fairly.

##### G. Traffic Flow and Distribution Assumptions

- **Peak-Hour Flow Factor**: Assumed at 10% of daily cumulative volumes (AADT x 0.10) to convert total daily PCU profiles into peak hourly design metrics (q).
- **Directional Splits**: Modelled at a (50/50) directional allocation, splitting standard flow counts equally across opposing lanes along the bypass mainline.
- **Homogeneous Passenger Car Conversions**: Assumed that converting vehicles into PCUs via IRC guidelines adequately accounts for speed variations in mixed traffic.
- **Environmental Adjustments**: Assumed constant clear daylight weather conditions, excluding temporary lane capacity drops from seasonal monsoons.

#### V. STATISTICAL REGRESSION & MODEL VALIDATION

##### H. Optimization Methodology:

To objectively assess how well each traffic stream model represents real-world conditions, a non-linear optimization routine was executed using the Levenberg-Marquardt least-squares algorithm. The validation objective functions minimize the residual sum of squares (RSS) between observed space-mean speed  $V_{obs}$  and predicted model velocity  $V_{pred}$  :

$$RSS = \sum_{i=1}^n (v_{obs,i} - v_{pred,i})^2$$

Model accuracy is quantified using Coefficient of Determination ( $R^2$ ) and Root Mean Squared Error (RMSE):

$$R^2 = 1 - \frac{\sum (v_{obs} - v_{pred})^2}{\sum (v_{obs} - \bar{v}_{obs})^2}$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (v_{obs,i} - v_{pred,i})^2}$$

### I. Goodness-of-Fit Performance Matrix

The following below summarizes the statistical regression outputs, showing the performance variations across the five models.

#### 1) Northwestern Exponential

- Optimized Parameter 1:  $v_f = 101.4$  km/hr
- Optimized Parameter 2:  $k_o = 63.8$  PCU/km
- Coefficient of Determination ( $R^2$ ): 0.9704
- Root Mean Squared Error (RMSE): 4.96 km/hr
- Optimization Status / Rank: Rank 1: Best fit across full density spectrum.

#### 2) Greenshields Linear

- Optimized Parameter 1:  $v_f = 98.6$  km/hr
- Optimized Parameter 2:  $k_o = 154.2$  PCU/km
- density passes the optimal threshold ( $k_o$ ). This structure prevents strict linear constraints while keeping low-density errors to a minimum.
- Coefficient of Determination ( $R^2$ ): 0.9695
- Root Mean Squared Error (RMSE): 5.03 km/hr
- Optimization Status / Rank: Rank 2: Excellent balance; stable boundaries.

#### 3) Drake Bell-Shaped

- Optimized Parameter 1:  $v_f = 99.1$  km/hr
- Optimized Parameter 2:  $k_o = 66.1$  PCU/km

- Coefficient of Determination ( $R^2$ ): 0.9423
- Root Mean Squared Error (RMSE): 6.92 km/hr
- Optimization Status / Rank: Rank 3: Strong fit; clear congestion drop-off.

#### 4) Underwood Exponential

- Optimized Parameter 1:  $v_f = 104.7$  km/hr
- Optimized Parameter 2:  $k_o = 61.3$  PCU/km
- Coefficient of Determination ( $R^2$ ): 0.8981
- Root Mean Squared Error (RMSE): 9.2 km/hr
- Optimization Status / Rank: Rank 4: Good at low density; safe free-flow speed.

#### 5) Greenberg Logarithmic

- Optimized Parameter 1:  $v_f = 41.2$  km/hr
- Optimized Parameter 2:  $k_o = 162.5$  PCU/km
- Coefficient of Determination ( $R^2$ ): 0.6842
- Root Mean Squared Error (RMSE): 16.19 km/hr
- Optimization Status / Rank: Rank 5: High errors; fits congested zones only.

### J. Comparative Regression Breakdown:

#### 1. Northwestern Exponential Model ( $R^2 = 0.9704$ , $RMSE = 4.96$ km/hr)

The Northwestern model achieved the highest statistical accuracy. Its exponential-quadratic form effectively mirrors how speeds gradually decline in light traffic, followed by an accelerated drop-off as density passes the optimal threshold ( $k_o$ ). This structure prevents strict linear constraints while keeping low-density errors to a minimum.

#### 2. Greenshields Linear Model ( $R^2 = 0.9695$ , $RMSE = 5.03$ km/hr)

Despite its mathematical simplicity, the linear model performs remarkably well. It captures standard traffic flow trends effectively and maintains reliable boundary constraints, making it a highly practical option for general roadway design.

**3. Drake Bell-Shaped Model ( $R^2 = 0.9423$ ,  $RMSE = 6.92$  km/hr)**

The Drake model performs well by maintaining a flat free-flow speed curve at low densities, which closely aligns with typical expressway conditions. However, its steeper fourth-power exponential drop yields slightly higher residual errors right at the capacity tipping point compared to the Northwestern model.

**4. Underwood Exponential Model ( $R^2 = 0.8981$ ,  $RMSE = 9.20$  km/hr)**

The Underwood model accurately represents low-density traffic conditions, capturing free-flow operations without inflating initial speeds. However, because its exponential tail never reaches absolute zero velocity, residual errors rise significantly in high-density, stop-and-go conditions ( $k > 110$  PCU/km).

**5. Greenberg Logarithmic Model ( $R^2 = 0.6842$ ,  $RMSE = 16.19$  m/hr)**

The logarithmic model produced the highest prediction error. Because velocity approaches infinity as density nears zero, it cannot accurately model low-density conditions ( $k < 15$  PCU/km). It remains highly accurate for forced-flow conditions and heavy queuing near major bottlenecks, but struggles when applied across the full traffic spectrum.

## VI. Infrastructure Application and Discussion

The intersection traffic database (Table 3) shows that cross-road intersections are quickly approaching critical volume conditions.

### K. Capacity Limits and Structural Upgrades

- Evaluating 4-Lane Configurations: Under a traditional linear approach (Greenshields capacity of 3,750 PCU/hr/lane), a standard 4-lane setup risks complete saturation during peak surges by the 2032 forecast horizon (117,316 PCU/day).
- The Non-Linear Reality: Advanced models (Drake/Northwestern) yield higher ideal lane capacities ( $>3,900$  PCU/hr/lane). However, they also reveal a sharp performance drop once mixed-traffic levels pass the optimum density point ( $k_0$ ).

This sudden breakdown mimics actual breakdown points observed on busy urban highway lanes.

### L. Structural Justification under IRC Regulations

The long-term projections confirm the findings in the project report. The peak volume across all four main intersections exceeds the critical (10,000 PCU/hr) limit defined by IRC:92-1985. To maintain steady travel times, preserve design speeds, and ensure safety, the mainline must expand to six access-controlled lanes and feature grade-separated interchanges.

## VII. Conclusion

This paper compared five macroscopic traffic stream frameworks using empirical datasets and calibrated assumptions from the Pune Ring Road project.

1. Greenshields remains a highly practical option for basic highway design due to its clear, manageable linear structure.
2. Greenberg effectively models high-density queuing near toll gates but struggles with low-density traffic streams.
3. Underwood and Northwestern are well-suited for open, high-speed highway segments but lose accuracy during severe congestion.
4. Drake offers a strong overall option for modern expressways by accurately mapping the rapid transition from stable flow to severe gridlock.

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