

Exploring Golden Ratio, Geometric Structures and Infinite Processes in the Azerbaijan 20 Qəpik Coin: A Mathematical Interpretation

Dr. Dipayan Datta

Associate Professor

Department of Public Health Dentistry
Agartala Government Dental College and IGM Hospital,
Tripura, India

Abstract:

The Azerbaijan 20 qəpik coin exhibits a structured geometric composition incorporating a regular pentagon, radial constructions, and a spiral motif. This study examines the extent to which these elements can be interpreted within a coherent mathematical framework centered on the golden ratio ($\phi \approx 1.618$). While the pentagon intrinsically encodes ϕ through its diagonal-to-side ratio, the diagonal itself is not explicitly represented in the design. Instead, the presence of angular markers and symbolic elements allows for a geometric reconstruction of this relationship. The spiral, although not a strict representation of the golden spiral, introduces a complementary visual element suggestive of iterative and recursive processes associated with ϕ including continued fractions and Fibonacci convergence. Considered within the unified design system of the coin series, the selective integration of these features on the 20 qəpik coin may reflect a non-arbitrary and conceptually structured design approach. Without asserting authorial intent, the paper presents a mathematically consistent interpretation linking proportion, recursion, and geometric abstraction within a single visual framework.

Keywords — Azerbaijan, 20 qəpik coin; Pentagon; Golden ratio; Infinite processes.

INTRODUCTION

Geometric principles have long influenced coin design, from classical Greek symmetry to structured motifs in Islamic numismatics. In modern currency, geometric elements often serve both aesthetic and symbolic purposes.¹

The Azerbaijan 20 qəpik coin presents a structured composition featuring a regular pentagon, radial lines, and a spiral motif. This study examines whether these elements can be interpreted within a coherent mathematical framework centered on the golden ratio ($\phi \approx 1.618$), while maintaining a clear distinction between intrinsic geometric properties and interpretive analysis.

DESIGN FEATURES OF THE COIN



Figure 1. The Azerbaijan 20 qəpik coin (Source: Google images, Numista)

The observe design includes the following characteristics:

Map of Azerbaijan, AZƏRBAYCAN RESPUBLİKASI and 20 QƏPIK.

The reverse design includes the following characteristics:

A. The Pentagon and Linear Construction

The geometric background is centered on a regular pentagon in an upright orientation. Rather than using standard diagonals, the design employs a specific set of radial and tangential lines.

B. The Circumcircle

The most prominent circular element in the background is a partial circumcircle.

Visual Evidence: If you follow the curved arc that sweeps across the top and through the Greek letter β at the upper left, you can see it is part of a larger circle that would perfectly enclose the pentagon.

Geometric Function: In geometry, a circumcircle passes through all five vertices of a regular pentagon. On this coin, the arc aligns with the vertices, suggesting the pentagon is "inscribed" within the circle.

Three straight lines are originating from top three vertices and approaching to the centre which seems to be three radii of the circle. The top left one is bisecting the top left internal angle.

C. The Incircle

While the full incircle is not drawn with a continuous line, the internal arcs suggest its presence through tangency.

Visual Evidence: Look at the curved lines that dip toward the midpoints of the pentagon's sides.

Geometric Function: A true incircle (or inscribed circle) is the largest possible circle that can fit inside the pentagon. For it to be an incircle, it must be tangent to all five sides at their exact midpoints.

D. Mathematical Labels and Variables

The coin uses these symbols to represent the labels of a geometric proof:

α : This lowercase Greek letter is the most prominent. It appears in four specific locations:

Top Left: Above the major arc.

Top Right: To the right of the staircase's top steps.

Middle Left: Just above the "2" in the denomination.

Bottom Right: Near the base of the staircase.

β : Located near the upper half top left internal angle just above the bisecting radius of the circumcircle.

E. The Spiral Staircase

Structure: It features a central vertical pillar with distinct steps winding upward from the bottom right toward the top center.

F. Typography and Denomination

The Numeral "20": Rendered in a bold, clean font. The "2" has a flat base, and the "0" is slightly oval.

"QƏPIK": Located directly beneath the "20". The QƏPIK (pronounced [gæ'pic]) is a monetary unit of Azerbaijan.

DESIGNER CONTEXT

The coin was designed by Robert Kalina, known for incorporating structured visual systems and symbolic clarity in currency design. The "Science and Education" theme was a deliberate choice by the Central Bank of Azerbaijan to highlight the nation's commitment to intellectual and scientific advancement. As part of the 2006 currency redesign by Robert Kalina, each denomination was assigned a specific cultural or economic pillar, with the 20 QƏPIK serving as the ambassador for academic progress. The theme is visually expressed through two primary symbols on the coin's reverse: The Spiral Staircase: A numismatic metaphor for the endless ascent of knowledge. It represents the process of learning as a continuous, upward journey where each step builds upon the last. The Pentagon and Geometric Symbols: These represent the foundations of science. By including precise geometric shapes, the design emphasizes the importance of logic, mathematics, and structural integrity as the building blocks of a modern society, which may support the presence of conceptual geometric motifs.^{1,2}

GOLDEN RATIO IN PENTAGON GEOMETRY³⁻⁵

These relationships arise directly from pentagonal symmetry and require no additional assumptions.

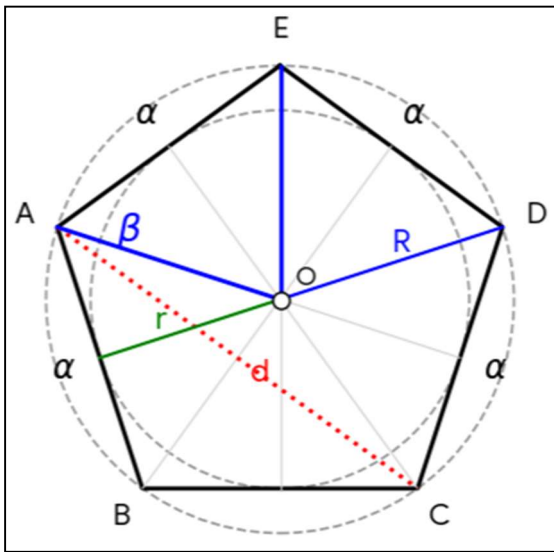


Figure 2. Basic properties of geometry of a regular pentagon

A regular pentagon intrinsically encodes the golden ratio. The fundamental relationships include:

Diagonal-to-Side Ratio: $d / s = \phi$

Angular Relation

The golden ratio emerges from the ratio of the chord lengths corresponding to the central angles.

Consider a regular pentagon inscribed in a circle of radius R (Figure 2). The side subtends a central angle of 72° , so its chord length is:

$$\text{Side} = 2R \sin 36^\circ$$

The diagonal spans two sides, subtending a central angle of 144° , so its length is:

$$\text{Diagonal} = 2R \sin 72^\circ$$

Ratio of diagonal to side:

$$\text{Diagonal} / \text{Side} = \frac{2R \sin 72^\circ}{2R \sin 36^\circ} = 2 \cos 36^\circ = \phi$$

This relationship is an intrinsic property of the regular pentagon's angular structure.

Radius Relationship

As r is the radius of the smaller incircle,

$$2r / R = \phi$$

ABSENCE OF THE DIAGONAL: STRUCTURAL INTERPRETATION

Although the coin shows or presents features like circumcircle, incircle, their radii along with the pentagon, a notable feature of the design is the absence of the pentagon's diagonal, despite its central role in defining ϕ .

Rather than interpreting this omission as intentional, it is more appropriate to observe that:

The diagonal is not explicitly depicted, while a spiral motif occupies a comparable visual space.

This allows a structural interpretation, in which:

- The diagonal represent ϕ as a static proportional relationship
- The spiral introduces a dynamic visual process

This interpretation remains conceptual and does not imply deliberate encoding.

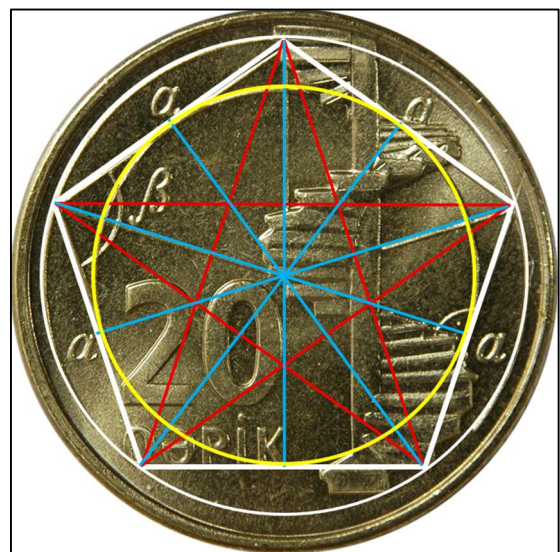


Figure 3. The hidden geometry inside the design

GEOMETRIC RECONSTRUCTION

Figure 2 shows, R is the radius of the circumcircle. The side length α of a regular pentagon inscribed in this circle is given by: $\alpha = 2R \sin 36^\circ$ and the diagonal d (connecting non-adjacent vertices) is given by: $d = 2R \sin 72^\circ$

The ratio of the diagonal to the side is therefore:

$$d / \alpha = 2 \cos 36^\circ = 2 \sin 54^\circ$$

As the half of the internal angle of the regular pentagon (54°) is denoted with β in the coin, we can establish:

$$d = 2\alpha \sin \beta$$

Thus, the ‘missing’ diagonal can be rediscovered using the symbols and geometric relationships present in the coin as hints:

- ❖ 2 is mentioned in the coin value.
- ❖ α is mentioned along the sides.
- ❖ β is mentioned along the bisector of one internal angle.
- ❖ The pentagon subtly embodies trigonometric relationship through the golden ratio.

The present paper proposes that the combination of a pentagon, two Greek letters, and a spiral staircase in the coin design does not appear to be arbitrary. This design framework could, in principle, have been applied to other denominations in the series (1, 3, 5, 10, or 50 qəpik which were developed within a unified design system). Its specific appearance on the 20 qəpik coin suggests a selective design choice. Within this context, the arrangement may be interpreted as a structured means of encoding the geometric relationship required to reconstruct the pentagon’s diagonal, which is not explicitly depicted. This reflects both the aesthetic elegance and the subtle concealment within the design, making it more intellectually engaging.

IMPORTANCE OF THE SPIRAL^{3,6}

The nature of a specific type of spiral is known to be directly associated with ϕ and called the golden spiral. The spiral depicted on the coin more closely resembles an Archimedean spiral, commonly associated with uniform radial increments. It is

important to distinguish between the spiral types (Table 1).

Table 1. Comparison of different types of spirals

Feature	Golden Spiral	Spiral in the coin
Type	Logarithmic	Archimedean
Growth	Exponential	Linear
Equation	$r = ae^{b\theta}$	$r = a + b\theta$
Relation to ϕ	The growth rate is $\frac{\ln(\phi)}{\pi/2}$	Angular increment between the successive points is $\frac{360^\circ}{\phi}$

Therefore, the spiral appearing on the coin can be said to have certain association with ϕ .

INFINITE PROCESSES AND THE GENERATIVE STRUCTURE OF ϕ ⁵⁻⁷

The golden ratio satisfies:

$$\phi^2 = \phi + 1$$

This identity serves as a generative mechanism underlying multiple infinite mathematical constructions.

Infinite Algebraic Forms

$$\phi = \sqrt{1 + \phi}$$

$$\text{or, } \phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \dots}}}}$$

Again,

$$\phi = 1 + \frac{1}{\phi}$$

$$\text{or, } \phi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}$$

These representations demonstrate ϕ as a product of recursive self-reference.

Geometric Self-Similarity

$$\text{As, } d / \alpha = \phi$$

The diagonals of a pentagon generate smaller pentagons indefinitely, forming a **geometric infinite sequence**.

Fibonacci Convergence

The Fibonacci sequence is defined by:

$$F_n = F_{n-1} + F_{n-2}$$

Or,

$$\frac{F_n}{F_{n-1}} = 1 + \frac{F_{n-2}}{F_{n-1}}$$

Finally it leads to:

$$\lim_{n \rightarrow \infty} \frac{F_n}{F_{n-1}} = \phi$$

GENERAL INTERPRETATION

A regular pentagon can be constructed such that, upon joining its non-adjacent vertices, a pentagram is formed, within which a smaller, inverted regular pentagon naturally emerges (Figure 3). This process may be iteratively continued, generating a theoretically infinite sequence of nested, self-similar pentagons. At each stage of this construction, the ratio of the length of a diagonal to that of a side remains invariant, and this constant ratio is identified as the Golden Ratio, denoted by ϕ .

Notably, the same constant arises independently in number theory as the limiting value of the ratio of consecutive terms in the Fibonacci sequence as the sequence progresses toward infinity. Consequently, the infinite hierarchy of nested pentagons may be interpreted as a geometric analogue of convergence, wherein each successive iteration preserves an identical proportional relationship, reflecting the recursive structure through which ϕ emerges analytically. In this sense, the construction provides a visual representation of the infinite continued fraction expansion of ϕ thereby establishing a direct conceptual link between geometric self-similarity and an inherently arithmetic process.

Across its various mathematical representations, a unifying characteristic of ϕ is its emergence as the

limit of iterative processes. Within this framework, the spiral motif may be interpreted as a symbolic analogue of such iteration, representing continuous progression and incremental growth without termination. Although the spiral does not explicitly encode ϕ in a strict mathematical sense, it aligns conceptually with the principles of recursion, convergence, and geometric scaling that underlie the constant.

The overall design may therefore be understood as integrating two complementary manifestations of the Golden Ratio: on one hand, ϕ appears as a fixed geometric proportion inherent in the structure of the pentagon and pentagram, and on the other, it arises as the limit of an infinite iterative process, conceptually reflected in the spiral form. This duality encapsulates both the static and dynamic aspects of ϕ unifying its role as a constant geometric ratio with its interpretation as the limit of recursive and convergent systems.

Alternate Interpretation

It is important to acknowledge:

- The spiral may be purely decorative
- No direct evidence confirms intentional encoding of ϕ
- The pentagon itself alone guarantees the presence of ϕ

Thus, the present work represents a mathematically consistent interpretation, not a claim of authorial intent.

CONCLUSION

The Azerbaijan 20 qəpik coin demonstrates a structured interplay between geometry and visual design. The pentagon inherently encodes the golden ratio, while the spiral introduces a dynamic element suggestive of iterative processes. Although the spiral does not mathematically represent ϕ , it provides a coherent conceptual counterpart to the pentagon's proportional structure. Together, these elements support an interpretation linking geometry, recursion, and symbolic growth within a unified visual framework.

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Author's Note: The author spent almost two decades in the field of Dental Surgery. Besides his academic career, he has a longstanding interest in numismatics and mathematics.

REFERENCES

- [1] Bank of the Republic of Azerbaijan. Coin designs and monetary circulation [Internet]. Baku: The Bank; Available from: cbar.az
- [2] Numista. 20 qəpik – Azerbaijan [Internet]: Numista; c2024. Available from: numista.com
- [3] Livio M. The golden ratio: the story of phi. New York: Broadway Books; 2002.
- [4] Dunlap RA. The golden ratio and Fibonacci numbers. Singapore: World Scientific; 1997.
- [5] Posamentier AS, Lehmann I. The fabulous Fibonacci numbers. Amherst (NY): Prometheus Books; 2007.
- [6] Weisstein EW. Golden ratio. MathWorld [Internet]: Wolfram Research; 2024. Available from: wolfram.com
- [7] Kappraff J. Connections: the geometric bridge between art and science. 2nd ed. Singapore: World Scientific; 2001.