Stability Augmentation of Combat Aircraft Using LQR Controller

Deena Albiya J

Reg. No. 2023603036

Division of Avionics, Department of Aerospace Engineering

Madras Institute of Technology, Anna University, Chennai – 600 044

Guide: Dr. A. Saravana Kumar, Assistant Professor, Division of Avionics

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Abstract

This study enhances the stability of the linearized F-16 Fighting Falcon aircraft model using a Linear Quadratic Regulator (LQR) controller optimized by Sequential Quadratic Programming (SQP) techniques. The LQR control technique minimizes a quadratic cost function whose weights are determined by selecting suitable Q and R matrix values. SQP employs a gradient-based optimization algorithm to efficiently compute the optimal feedback gain matrix K. This approach effectively stabilizes both the lateral and longitudinal flight dynamics of the aircraft. Simulation results obtained using MATLAB/Simulink demonstrate the efficacy of the proposed control strategy in ensuring aircraft stability, with a final optimal objective function value of 0.555.

1. Introduction

The General Dynamics F-16 Fighting Falcon is a high-performance multi-role combat aircraft renowned for its agility and advanced aerodynamics. While its design enables exceptional maneuverability, the aircraft's inherent instability — particularly at high speeds and during dynamic maneuvers — makes stability augmentation a critical requirement. Stability Augmentation Systems (SAS) provide pilots with enhanced handling characteristics and improved safety margins under diverse flight conditions such as high angles of attack, turbulence, or rapid maneuvering.

The Linear Quadratic Regulator (LQR) is an optimal control technique that minimizes a quadratic cost function by balancing state deviations against control effort. Sequential Quadratic Programming (SQP) is an advanced gradient-based method that solves nonlinear programming problems iteratively by converting them into a sequence of quadratic subproblems. In this study, SQP is integrated with LQR to refine the weight matrices Q and R, computing the optimal feedback gain matrix K while accounting for operational constraints such as control surface limits and stability bounds. The primary objectives are: (i) develop and linearize a 6-DOF mathematical model of the F-16; (ii) design an optimal LQR controller using SQP-based weight matrix optimization; (iii) validate performance through MATLAB/Simulink simulations; and (iv) compare the LQR-SQP approach against conventional control methods.

2. Methodology

2.1 Aircraft Model — Six Degrees of Freedom (6-DOF)

The F-16 model is represented using the Flat-Earth, Body-Axes 6-DOF equations comprising 12 state equations: three force equations and three moment equations (dynamic equations), three kinematic equations relating body angular rates (P, Q, R) to Euler angle rates (φ, θ, ψ), and three navigation equations describing position changes in the North-East-Down (NED) frame. The full 12-element state vector is:

$$X^T = [\ V_t\ \ \beta\ \ \alpha\ \ \varphi\ \ \theta\ \ \psi\ \ P\ \ Q\ \ R\ \ P_n\ \ P_e\ \ h\ ]$$

The control vector consists of four inputs: u^T = [throttle, elevator, aileron, rudder]. Key mass properties include aircraft weight W = 20,500 lbs and moments of inertia I_{xx} = 9,496, Iyy = 55,814, Izz = 63,100 slug-ft². The atmospheric model computes static pressure, air density, and Mach number using International Standard Atmosphere (ISA) relationships. The nonlinear model is linearized around a steady-state flight condition using Jacobian-based perturbation, yielding the standard state-space form: $\dot{x} = Ax + Bu$.

2.2 LQR Controller Design

The LQR minimizes the infinite-horizon quadratic cost function:

$$J = \int_0^\infty (\ x^TQx + u^TRu\ )\ dt$$

where Q (5×5) penalizes state deviations and R penalizes control effort. The optimal feedback control law is u = −Kx, where K is derived by solving the Algebraic Riccati Equation (ARE):

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \quad \rightarrow \quad K = R^{-1}B^TP$$

The Q matrix penalizes altitude deviations (Q_{55} = 0.01) while R = 0.1 limits actuator effort. The state matrix A and control matrix B are extracted from the longitudinal and lateral linearized models. The longitudinal state matrix A is a 5×5 matrix incorporating angle of attack (α), pitch rate (Q), elevator actuator, pitch angle (θ), and altitude (h) dynamics. The control matrix B reflects the elevator input authority with a gain of 20.2 rad/s²/rad.

2.3 SQP-Based Optimization

SQP refines LQR weight matrices iteratively. At each iteration, the algorithm: (1) linearizes constraints around the current estimate; (2) solves a quadratic subproblem to update control inputs; and (3) checks Karush-Kuhn-Tucker (KKT) convergence conditions. The optimization is subject to control surface limits δ_min ≤ u(t) ≤ δ_max and state constraints x_min ≤ x(t) ≤ x_max. MATLAB's fmincon with SQP algorithm is used with bounds lb = [0; 0], ub = [1; 1], and initial guess x_0 = [0.1; 0.1]. The SQP process converges when constraint and optimality tolerances fall below predefined thresholds.

3. Results and Analysis

3.1 Closed-Loop Stability

The LQR gain vector was computed as K = [−6.1073, −11.5886, 0.0967, −0.6123, 0.3072]. The closed-loop eigenvalues are presented below. All dominant poles lie in the left half of the complex plane, confirming closed-loop stability.

Eigenvalue	Real Part	Stability
−20.2022 + 0.0000i	−20.2022	✓ Stable
−1.5139 + 1.0062i	−1.5139	✓ Stable
−1.5139 − 1.0062i	−1.5139	✓ Stable
−1.0194 + 0.0000i	−1.0194	✓ Stable
−0.0000 + 0.0000i	0.0000	Marginally Stable

3.2 Step Response Performance

Step response simulations were conducted for longitudinal and lateral dynamics. Key observations are summarized below:

- Angle of Attack (α): Rapid decay from ~10×10^{-3} rad to near zero within 4 seconds — effective damping
- Elevator Actuator: Smooth response peaking at ~0.14 rad before settling — no actuator saturation
- Pitch Angle (θ): Gradual rise to ~0.09 rad and stable plateau — confirmed pitch stability augmentation
- Pitch Rate (Q): Brief undershoot then settles to zero within ~4 seconds
- Roll Angle (φ): Rapid decay to zero within ~1.5 seconds — excellent roll damping with no overshoot
- Roll Rate (P): Attenuated to near-zero within ~3 seconds after initial transient
- Yaw Angle (ψ): Smooth decay to zero within ~3 seconds with minimal oscillation
- Yaw Rate (R): Remains at zero throughout — complete yaw damping achieved

3.3 Poles-Zeros Map and Optimization Convergence

The open-loop vs. closed-loop poles-zeros (PZ) map confirms all poles shifted to the left half-plane after LQR-SQP optimization. The Riccati equation solution matrix P is symmetric positive-definite, verifying mathematical validity. The SQP optimization converged to a final optimal objective value of 0.555, satisfying an optimality tolerance of 1.49×10^{-8} and constraint tolerance $< 1\times10^{-6}$.

4. Conclusion

This study successfully designed and validated an LQR-based stability augmentation system for the F-16 Fighting Falcon, enhanced by SQP-based weight matrix optimization. The controller achieved full closed-loop stability with all eigenvalues in the left half-plane, effective damping of pitch, roll, and yaw dynamics within 1.5–4 seconds, and an optimal gain vector $K = [-6.11, -11.59, 0.097, -0.612, 0.307]$ with objective function value 0.555. The LQR-SQP integration outperformed conventional PID approaches in responsiveness and stability margins. Future work may explore Nonlinear Dynamic Inversion (NDI), Sliding Mode Control (SMC), and Hardware-in-the-Loop (HIL) real-time implementation to further extend the flight envelope coverage into post-stall and supersonic regimes.

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