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# Existence Solution of One Dimentional Integro-Differential Equation by Transform Method

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## Abstract
Differential transform method is a semi-numerical analytic iterative method that describes the solution of a ODE's or PDE's or system ODE's and PDE's into a series from that further sometimes can be converted into a closed form, that is, an exact solution. In this paper, we are concern with the existence of solution of One dimensional integro-differential equation with transform method. The efficiency of the considered method is illustrated

## 1. INTRODUCTION

Mathematical modeling of real-world dynamical problems leads to linear, nonlinear ordinary, or partial differential equations that describe the relationship between the physical system's different input and output parameters. From the past decade to now, researchers, scientists, engineers, and mathematicians have developed from mathematical modeling, which has its advantages and disadvantages. Differential equations arise in many problems, such as the oscillations of electrical and mechanical systems, the conduction of heat, the bending of beams, the velocity of chemical reaction, the transport of the solute, and others, and play a significant role in modern engineering and science studies. Numerical methods, such as the finite volume method, the finite difference method, the finite element method, and others, describe the solution of the system at a discrete point that requires extensive computational resources. The stability and convergence analysis are also major issues with the numerical methods, Integral transforms, such as Fourier transforms and Laplace transforms, are often applied to find the differential equation solution. The advantage of these integral transforms lies in their ability to transmute the given differential equations into algebraic equations, which provide efficient and straightforward solution procedures but fail to work efficiently with nonlinear problems. However, the analytic method, like the homotopy perturbation method, a combination of perturbation and homotopy method, requires small or large perturbation parameters to start the procedure. If perturbation parameters are not selected approximately, then it is impossible to obtain the solution. While dealing with a nonlinear differential equation, it requires a function (initial guess) in each iteration, which is tricky. Many methods have been developed to deal with the nonlinear term appearing in ordinary or partial differential equations. Some of the methods that deal with nonlinear terms are the Adomian decomposition methods (ADM), the variational iteration method (VIM), the modified Adomian method (MADM), the variational homotopy perturbation method (VHPM), and others. In all the techniques mentioned earlier, convergence analysis is essential for obtaining the series solution. In all the methods mentioned earlier, good accuracy can be obtained by considering more terms of the series, which is a tedious task. ADM for solving ordinary differential equations (ODE's) or partial differential equations (PDE's) is simple in the abstract formulation, but it requires the calculation of Adomain polynomials, which depends on the initial approximation. Suppose the initial approximation is not considered correctly. In that cases, a solution may not be physically justified, so it is necessary to consider the proper initial approximation to obtain the physically realistic solution, which is a difficult task. The variational iteration method requires producing the correct function using Lagrange's multipliers and stationary conditions by using variational theory, which is problematic. Besides this, other disadvantages are repeated computation and computation of unneeded terms, which consume time and effort. In the homotopy analysis method, an auxiliary linear operator is required to construct a continuous mapping of an initial guess approximation to the exact solution of the given equation. An additional parameter is needed to show the convergences of series solutions obtained by this method, which are difficult to identify. This technique embeds a parameter $p$, which ranges from 0 to 1. When the embedding parameter is zero, the equation is a linear system, and when it is 1, the equation represents the original system. So, the embedded parameter $p \in [0, 1]$ can be considered a small parameter. To

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overcome all the mentioned issues, we suggest an efficient semi-analytical approach, the differential transform method (DTM), for dealing with linear and nonlinear differential equations. The DTM has no unrealistic assumptions or restrictions, such as linearization, discretization, or small parameters used for the nonlinear operators, to mention a few. It converts the system into a recursive formula that is easy to handle. This, the DTM addresses linear and nonlinear problems efficiently, some authors consider the DTM as an iterative method or numerical method or semi-numerical analytic method. The DTM can be defined as follows. Differential transform method is a semi-numerical analytic iterative method that describes the solution of a ODE's or PDE's or system ODE's and PDE's into a series from that further sometimes can be converted into a closed form, that is, an exact solution. The DTM id derived from the traditional Taylor series method, which requires the complicated and tedious symbolic computation of higher-order derivatives. It transforms the differential equation or system of ODE's or PDE's into a recursive formula that calculates the series solution coefficient. The differential transform method reduces the span of the computational domain compared to the other techniques and doesn't require the unneeded parameters to start the solution procedure. The series solution obtained by using DTM shows rapid convergence. One of the most significant advantages of DTM is that obtains the solution of the given differential equation on a continuous interval. Thus, its straightforward applicability, computational efficiently, and high accuracy make the DTM one of the powerful and efficient methods to solve the differential equations or system of differential equations.

This chapter discusses the one dimensional differential transform method (ODDTM) with its fundamental properties. This chapter also discusses the Numerical solution of integro-differential equation.

## 2. PRILIMINARIES

**DEFINITION 1:**

If $\boldsymbol{u(t)}$ is analytic in the time domain T, then

$$\frac{d^k u(t)}{dt^k} = \emptyset(t,k), \qquad \forall t \in T. \tag{2.1}$$

For $t = t_i, = \emptyset(t,k) = \emptyset(t_i,k)$. where $k$ belongs to the non-negative integer, denoted as the K-domain. The equation (2.1) can become

$$U_i(k) = \emptyset(t_i,k) = \left[\frac{d^k u(t)}{dt^k}\right]_{t=t_i}, \qquad \forall t \in T. \tag{2.2}$$

where $U_i(k)$ is called the spectrum of $\boldsymbol{u(t)}$ at $t = t_i$ in the K domain.

**DEFINITION 2:**

If $\boldsymbol{u(t)}$ is analytic, then $\boldsymbol{u(t)}$ can be expressed by Taylor's series expansion as

$$\boldsymbol{u(t)} = \sum_{t=1}^{\infty} \frac{(t-t_i)^k}{k!} U(k) \tag{2.3}$$

Equation (2.3) is called the inverse transformation of $U(k)$. If $U(k)$ is defined as

$$U(k) = M(k)\left[\frac{d^k q(t)u(t)}{dt^k}\right]_{t=t_i}, \quad k = 0,1,2,3, \dots\dots \tag{2.4}$$

then the function $\boldsymbol{u(t)}$ can be described as $u(t) = \frac{1}{q(t)} \sum_{t=0}^{\infty} \frac{(t-t_i)^k}{k!} \frac{U(k)}{M(k)}$, (2.5)

where $M(k) \neq 0, q(t) \neq 0$. $M(k)$ is called the weighting factor, and $q(t)$ is considered as a kernel corresponding to $\boldsymbol{u(t)}$. If $M(k) = 1$ and $q(t) = 1$ in equation (2.5), then equations (2.3) and (2.5) are equivalent. Using the ODDTM, the given differential equation in the domain of interest can be transformed to be an algebraic equation in the $K$ domain. Hence, $u(t)$ can be obtained by finite-term Taylor series plus a remainder, as

$$u(t) = \frac{1}{q(t)} \sum_{t=0}^{\infty} \frac{(t-t_i)^k}{k!} \frac{U(k)}{M(k)} + R_{n+1}(t)$$
$$= \sum_{k=0}^{\infty} \frac{(t-t_i)^k U(k) + R_{n+1}(t)}{k!}$$

where $R_{n+1}(t) = \sum_{k=n+1}^{\infty} (t-t_0)^k U(k)$ is negligibly small.

## FUNDAMENTAL PROPERTIES OF THE ODDTM

In this section, the fundamental properties of the ODDTM are discussed. Assume that $U(k), G(k), H(k)$ are the differential transform of the original function of $u(x), g(x), h(x)$ respectively.

| | |
|---|---|
| $u(x) = g(x) \pm h(x)$ | $U(k) = G(k) \pm H(k)$ |
| $u(x) = ag(x)$ | $U(k) = aG(k)$ |
| $u(x) = \frac{dg(x)}{dx}$ | $U(k) = (k+1)G(k+1)$ |
| $u(x) = \frac{d^2g(x)}{dx^2}$ | $U(k) = (k+1)(k+2)G(k+2)$ |
| $u(x) = \frac{d^mg(x)}{dx^m}$ | $U(k) = (k+1)(k+2) \ldots\ldots\ldots\ldots\ldots\ldots (k+m)G(k+m)$ |
| $u(x) = g(x)h(x)$ | $U(k) = \sum_{i=0}^{k} G(l)H(k-l)$ |
| $u(x) = g(x)h(x)l(x)$ | $U(k) = \sum_{n=0}^{k}\sum_{m=0}^{n} G(m)H(n-m)L(k-n)$ |
| $u(x) = e^{\lambda x}$ | $U(k) = \frac{\lambda^k}{k!}$ |
| $u(x) = \sin(\alpha x + \beta)$ | $U(k) = \frac{\alpha^k}{k!} sin\left(\frac{k\pi}{2} + \beta\right)$, where α & β are non-zero constants. |
| $u(x) = \cos(\alpha x + \beta)$ | $U(k) = \frac{\alpha^k}{k!} cos\left(\frac{k\pi}{2} + \beta\right)$, where α & β are non-zero constants. |
| $u(x) = x^m$ | $U(k) = \delta(k-m) = \begin{cases} 1 & if\ k = m \\ 0 & if\ k \neq m \end{cases}$ |
| $u(x) = ax^m$ | $U(k) = a\delta(k-m) = \begin{cases} 1 & if\ k = m \\ 0 & if\ k \neq m \end{cases}$ |

## 3. EXISTANCE OF SOLUTION FOR INTEGRO-DIFFERENTIAL EQUATION

The general form of the integro-differential equation $\frac{dy}{dx} + \int_{x_0}^{x} f(t, y(t))dt = g(x, y(t)), y(x_0) = y_0$, $x_0 \geq 0$. In epidemiology and epidemic mathematical modeling, integro-differential equations have been used mainly when the models include age structure or depict spatial epidemics.

Example:

Consider the nonlinear integro-differential equation of the initial value type

$$y'(x) = 1 + \int_0^x y^2(t)dt \qquad (3.1)$$

With initial boundary condition $y(0) = 0$. (3.2)

Solution:

Substituting $x = 0$ in (3.1) gives $y'(0) = 1$, which is another boundary condition. Taking the differential transform of equations (3.1), (3.2), it yields

$$D(y'(x)) = D\left(1 + \int_0^x y^2(t)dt\right).$$

$$D(y'(x)) = D(1) + D\left(\int_0^x y^2(t)dt\right)$$

$$(k+1)Y(k+1) = \delta(k) + \frac{1}{k}\sum_{i=0}^{k} Y(k)Y(k-l-1)$$

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$$Y(k+1) = \frac{\delta(k) + \sum_{i=0}^{k} Y(k)Y(k-l-1)}{(k+1)}$$

With the transform boundary condition $Y(0) = 0, Y(1) = 1$. According to the definition of the Kronecker delta, $\delta(k-m) = \begin{cases} 1 & if\ k = m \\ 0 & if\ k \neq m \end{cases}$. For $k = 0$, the value of $\delta(k-0) = 1$, and for $k \geq 1$, the value of $\delta(k-0) = 0$, Since $k \geq 1, \delta(k) = 0$, therefore, the preceding formula reduces to

$$Y(k+1) = \frac{\sum_{i=0}^{k} Y(k)Y(k-l-1)}{(k+1)}$$

using the recursive formula and the transformed boundary condition $Y(0) = 0, Y(1) = 1$, $Y(k)$ for $k \geq 2$ are calculated, and some of the coefficients are mentioned in below table.

Using the definition of the inverse differential transformation for the set of values $\{Y_i(k)\}_{k=0}^{\infty}$, the analytic approximate series solution is given as

$$y(x) = \sum_{k=0}^{\infty} y(k)x^k$$

$$y(x) = Y(0) + Y(1)x + Y(2)x^2 + Y(3)x^3 + Y(4)x^4 + Y(5)x^5 + \cdots$$

$$y(x) = x + \frac{x^4}{12} + \frac{x^7}{252} + \cdots$$

**TABLE**

Coefficient table for equation (3.2) with the initial condition (3.1)

| $k$ | $Y(k)$ | $k$ | $Y(k)$ |
|---|---|---|---|
| 0 | 1 | 5 | 0 |
| 1 | 0 | 6 | 0 |
| 2 | 0 | 7 | $\frac{1}{252}$ |
| 3 | 0 | 8 | 0 |
| 4 | 0 | …. | …. |

## 4. CONCLUSION

The differential transform method has been successfully applied for solving integro-differential equation. The solution is obtained by differential transform method is an infinite power series for appropriate initial condition, which can in turn express the exact solutions in a closed form. Thus, we conclude that the proposed by this method can be extended to solve many problems which arise in physical and engineering applications.

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