

Application of Monte Carlo Simulation in Energy Management and Petroleum Engineering

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Abstract:

Monte Carlo simulation is a powerful statistical technique for modelling uncertainty and predicting outcomes in a complex system of petroleum projects. This paper explores its application in energy management and petroleum engineering, highlighting its benefits and uses. Monte Carlo simulation is a probabilistic modelling technique used to quantify uncertainty in complex petroleum reservoir systems. Uncertainty is inherent in the petroleum and energy system due to variability in geological properties, market conditions and regulatory environments. Generally, the traditional deterministic approach often fails to capture the full range of possible outcomes. This study represents the application of Montecarlo simulation (MCS) as a robust probabilistic framework for uncertainty qualification in the Energy management and petroleum engineering projects. The methodology integrates statistical distributions of key uncertain variables—such as reservoir properties (porosity, water saturation, permeability), drilling parameters (pore pressure, in-situ stresses, equivalent circulating density), production decline parameters, commodity prices, and capital expenditures—into numerical models to generate thousands of simulation realisations. The results provide probabilistic outputs including P10, P50, and P90 estimates for reserves, production forecasts, mud weight windows, and net present value (NPV).

The study demonstrates that Monte Carlo-based risk assessment significantly improves decision-making by quantifying the probability of adverse events such as wellbore instability, kick/loss scenarios, economic loss, and underperformance of energy projects. Furthermore, applications in renewable energy forecasting and carbon capture and storage (CCS) projects illustrate its broader role in sustainable energy planning and financial risk management. The findings confirm that probabilistic modelling enhances operational safety, economic reliability, and strategic planning compared to deterministic approaches. Monte Carlo Simulation is therefore established as a critical tool for modern risk-based petroleum operations and energy investment analysis.

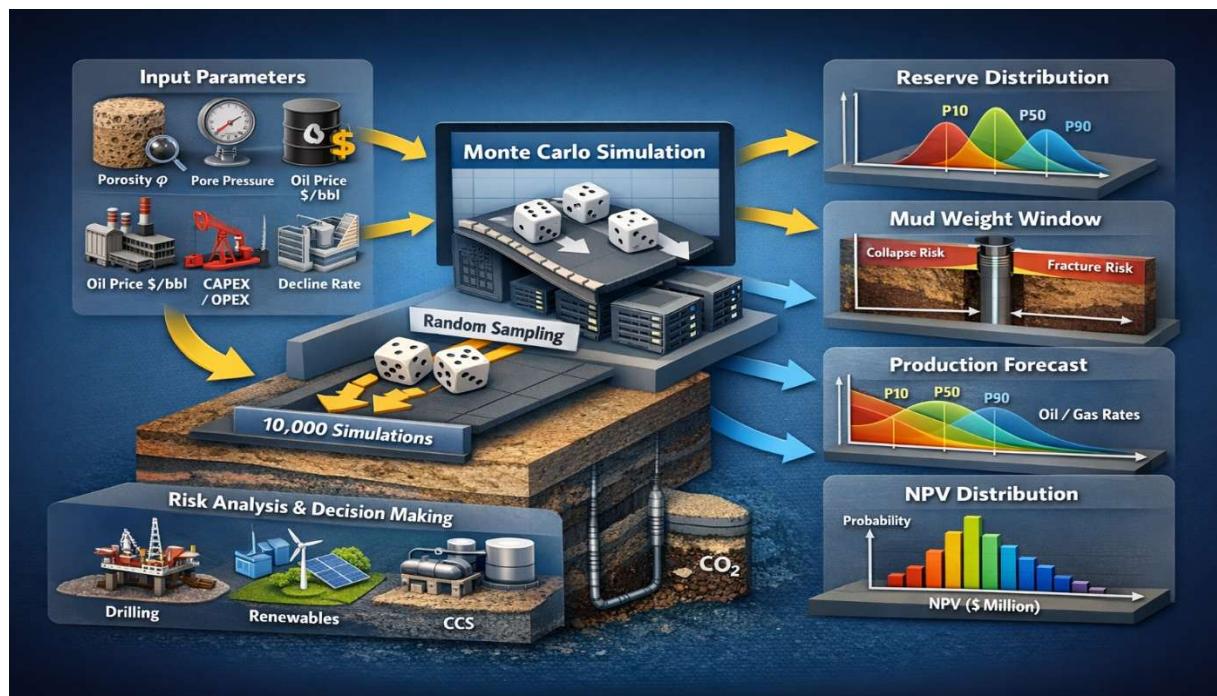


Fig 1:-The figure illustrates a 3D probabilistic workflow where uncertain input parameters (reservoir properties, drilling variables, production decline parameters, oil price, CAPEX/OPEX) are sampled through random simulation (10,000 realisations). The Monte Carlo engine generates probabilistic outputs, including reserve distribution (P10–P50–P90), mud weight window uncertainty (collapse and fracture risk), production forecast variability, and NPV distribution. These outputs support quantitative risk assessment and strategic decision-making in drilling operations, renewable energy projects, and carbon capture and storage (CCS) systems.

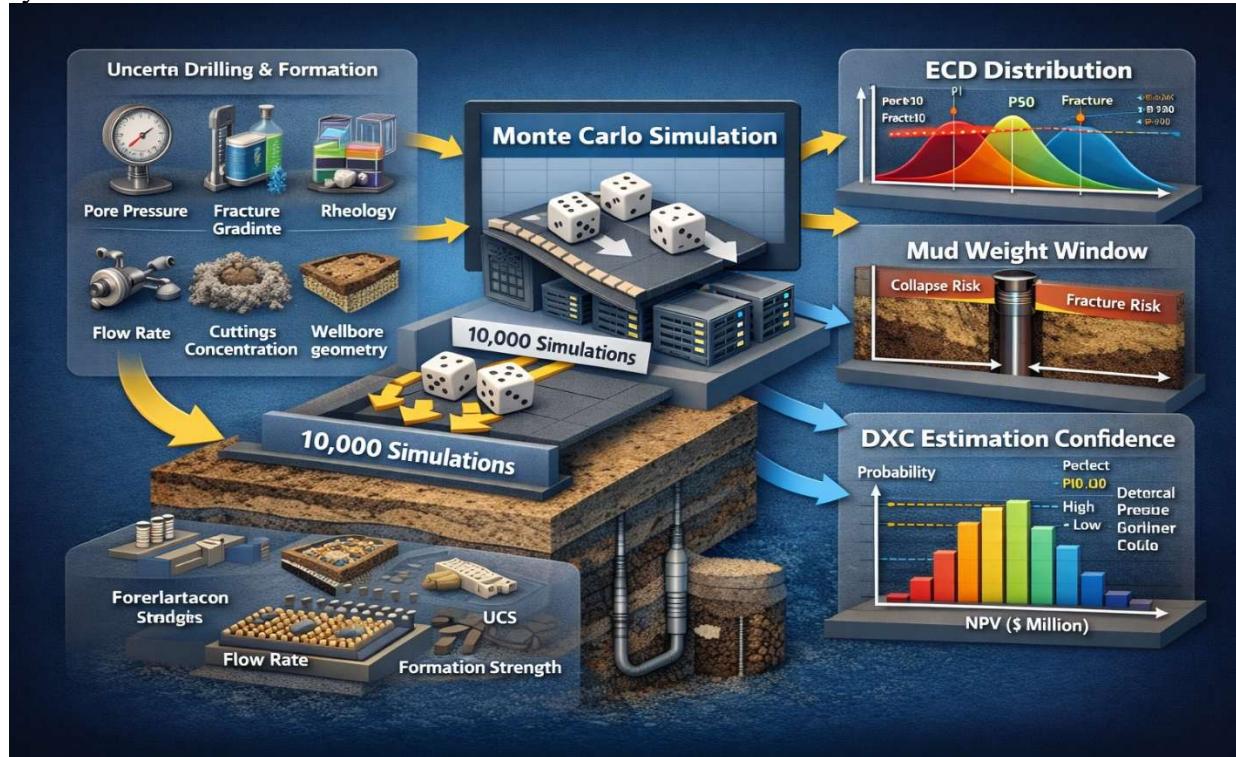


Fig 2 :-The figure illustrates the integration of uncertain drilling and formation parameters—pore pressure, fracture gradient, mud rheology, flow rate, cuttings concentration, wellbore geometry, and formation strength (UCS)—into a Monte Carlo simulation engine (10,000 realisations). Random sampling propagates input uncertainties to generate probabilistic outputs, including ECD distribution relative to pore–fracture pressure limits, mud weight window variability (collapse versus fracture risk), and DXC-based abnormal pressure detection confidence. The resulting P10–P50–P90 envelopes quantify the probability of kick, lost circulation, and wellbore instability, enabling risk-based drilling optimisation and reduction of non-productive time (NPT). In conclusion, the integration of Monte Carlo Simulation with ECD–DXC analysis provides a robust probabilistic framework for quantifying drilling uncertainty and operational risk. By transforming deterministic drilling parameters into statistical distributions, the approach enables reliable prediction of ECD behaviour, mud weight window limits, and abnormal pressure detection confidence. The resulting probability-based outputs (P10–P50–P90) enhance real-time decision-making, reduce the likelihood of kick and lost circulation events, and improve wellbore stability management. This probabilistic methodology ultimately supports safer drilling operations, minimises non-productive time (NPT), and strengthens risk-informed planning in modern petroleum engineering practices.

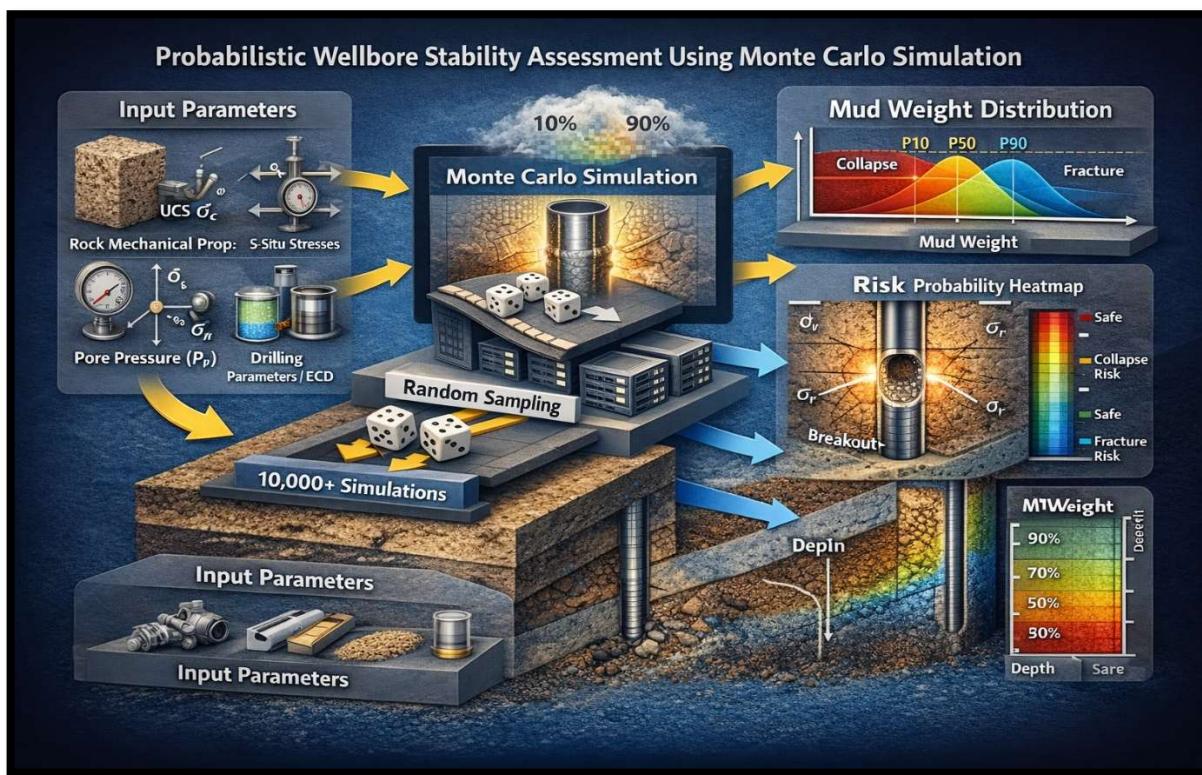


Fig 3: This 3D infographic illustrates the workflow of Monte Carlo Simulation for evaluating wellbore stability. Key uncertain input parameters—including rock mechanical properties (UCS, cohesion, friction angle), in-situ stresses, pore pressure, and drilling parameters (mud weight, ECD)—are randomly sampled through 10,000+ simulation iterations. The probabilistic outputs include the mud weight distribution (P10–P50–P90), risk probability heatmap along the wellbore depth (collapse, safe, fracture zones), and breakout orientation prediction. The figure visualises how stochastic modelling of input uncertainties enables risk-based wellbore design, optimisation of mud weight windows, and reduction of non-productive time (NPT) in drilling operations.

Monte Carlo Simulation provides a powerful probabilistic framework for wellbore stability assessment and drilling risk management in petroleum engineering. Incorporating the inherent uncertainties in rock properties, in-situ stresses, pore pressure, and drilling parameters, it enables the prediction of key outputs such as mud weight windows, probability of collapse or fracture, and wellbore breakout orientation with defined confidence levels (P10–P50–P90). This approach transforms deterministic designs into risk-informed, probabilistic decisions, improving drilling safety, reducing non-productive time (NPT), and optimising operational efficiency. When applied to ECD–DXC analysis, Monte Carlo Simulation allows for real-time assessment of abnormal pressure scenarios and wellbore stability, making it an indispensable tool for modern petroleum engineering operations and energy project planning.

Introduction: -Monte Carlo Simulation (MCS) is a probabilistic modelling technique widely applied in Energy Management and Petroleum Engineering to quantify uncertainty and support risk-based decision making by replacing deterministic inputs with probability distributions and running thousands of simulations to generate output ranges such as P10, P50, and P90 estimates. In petroleum engineering, it is extensively used for reservoir performance forecasting by incorporating uncertainties in porosity, permeability, net pay thickness, oil saturation, and reservoir pressure to estimate original oil in place (OOIP), recovery factor, and cumulative production distributions; for wellbore stability and geomechanics by modelling variability in in-situ stresses (σ_h , σ_v , σ_v), pore pressure, rock strength, and mud weight to determine safe mud weight windows and probability of borehole failure; and for field development economics by simulating uncertainties in oil price, CAPEX, OPEX, production rates, and discount rates to generate probabilistic NPV and IRR distributions and evaluate the likelihood of economic success ($P(NPV > 0)$). In energy management, Monte Carlo methods are

applied to renewable energy forecasting by modelling wind speed, solar irradiance, and load demand variability to assess generation reliability and storage requirements, as well as in energy trading and portfolio optimisation through electricity price volatility, fuel cost uncertainty, and carbon credit price simulations to estimate Value at Risk (VaR) and Conditional VaR. Mathematically, Monte Carlo approximates the expected value $E[f(X)] \approx \frac{1}{N} \sum_{i=1}^N f(x_i)$, converging as the number of simulations increases according to the Law of Large Numbers, thereby enabling integrated uncertainty quantification across technical, operational, and financial domains. Overall, Monte Carlo Simulation transforms conventional deterministic engineering analysis into a comprehensive probabilistic framework, enhancing reservoir management, drilling safety, economic evaluation, renewable integration, and modern data-driven applications such as machine learning-assisted forecasting and real-time ECD–DXC optimisation.

Keywords: Monte Carlo Simulation; Uncertainty Analysis; Probabilistic Modelling; Risk Assessment; Energy Management; Petroleum Engineering; Reservoir Simulation; Wellbore Stability; Production Forecasting; Economic Evaluation; Net Present Value (NPV); Sensitivity Analysis; Stochastic Modelling; Drilling Risk Analysis; Enhanced Oil Recovery (EOR); Decision Analysis; Energy Optimization; Carbon Capture and Storage (CCS); Formation Pressure Prediction; Equivalent Circulating Density (ECD).

Methodology: The study focuses on quantifying uncertainty in petroleum engineering and energy management systems, including reservoir performance, wellbore stability, drilling hydraulics (ECD), production forecasting, and economic evaluation. Key uncertain variables such as porosity, permeability, reservoir pressure, oil price, drilling cost, and formation strength parameters are identified.



Fig 4:Three-dimensional methodology framework for applying Monte Carlo Simulation in Energy Management and Petroleum Engineering, illustrating the sequential workflow from problem definition and probabilistic input selection to model formulation, stochastic simulation, sensitivity and risk analysis, result interpretation, and final validation for decision-support optimisation under uncertainty.

Data Collection and Input Parameter Selection: -In petroleum engineering studies, **field data** typically include:

- **Well logs** – Continuous downhole measurements such as gamma ray, resistivity, density, neutron, and sonic logs used to estimate lithology, porosity, fluid saturation, and formation pressure.
- **Core analysis** – Laboratory testing of core samples to determine porosity, permeability, grain density, capillary pressure, and rock mechanical properties (e.g., Young's modulus, cohesion, friction angle).
- **Drilling reports** – Operational records containing mud weight, rate of penetration (ROP), torque and drag, equivalent circulating density (ECD), wellbore instability events, and formation pressure observations.

These datasets form the primary input parameters for probabilistic modeling and Monte Carlo simulation in reservoir performance prediction, wellbore stability analysis, and economic risk assessment.

Here is a **structured table** suitable for your methodology section:-(Table 1)

Data Category	Source / Tool	Parameters Obtained	Engineering Application
Well Logs	Wireline logging tools (GR, Resistivity, Density, Neutron, Sonic)	Porosity (ϕ), Water Saturation (Sw), Lithology, Formation Pressure, Elastic Properties	Reservoir characterisation, formation evaluation, pressure prediction
Core Analysis	Laboratory core testing	Porosity, Permeability (k), Capillary Pressure, Rock Strength (Cohesion, Friction Angle), Young's Modulus	Rock mechanics modelling, wellbore stability, reservoir simulation calibration
Drilling Reports	Daily drilling reports (DDR), mud logging data	Mud Weight, ROP, Torque & Drag, ECD, Kick/Loss Events, Operational Cost	Drilling optimisation, ECD window design, risk analysis, cost estimation

Data Source and Calculation Methodology for Sample Field Dataset (Table 2)

The dataset presented is representative of typical onshore hydrocarbon wells and is derived from standard petroleum engineering measurement techniques, laboratory testing, and drilling records. The calculation approach for each parameter is described below.

Parameter	Unit / Value	Sample Calculation / Formula	Source	Probability Distribution Type
Depth	m	Measured directly from drilling rig instrumentation and corrected by wireline logs	Drilling reports (DDR), Rig sensors, MD logs	Deterministic (fixed)
Porosity (ϕ)	%	(a) Density log: $\phi = (\rho_{\text{ma}} - \rho_b) / (\rho_{\text{ma}} - \rho_f)$ (b) Core: $\phi = V_p / V_b$	Well logs (Density, Neutron, Sonic), Core analysis	Normal
Permeability (k)	mD	Darcy's law: $k = Q \mu L / (A \Delta P)$	Core lab testing, Well testing	Lognormal
Reservoir Pressure (P)	MPa	Measured via downhole gauges; Hydrostatic: $P = \rho g h$	RFT/MDT, Drill Stem Test (DST)	Normal

Mud Weight (MW)	ppg	MW = Mud mass / Mud volume	Mud logging unit, Drilling fluid reports	Normal
Equivalent Circulating Density (ECD)	ppg	$ECD = MW + \Delta P_{annular} / (0.052 \times TVD)$	Drilling hydraulics calculations, real-time sensors	Normal
Cohesion (c)	MPa	$\tau = c + \sigma \tan \phi$ (from Mohr-Coulomb failure criterion)	Triaxial compression test on core	Normal
Friction Angle (ϕ)	°	$\tan \phi = (\sigma_1 - \sigma_3) / (\sigma_1 + \sigma_3)$ (from Mohr circle)	Triaxial lab testing	Normal
Oil Rate (q_o)	bbl/day	$q_o = \text{Produced oil volume} / \text{Time}$ Or $q = 0.00708 k h (P_r - P_{wf}) / (\mu B \ln(r_e / r_w))$	Production test data, Flow meters	Lognormal

The dataset includes key parameters critical for reservoir characterisation, wellbore stability, and production forecasting. **Depth (m)** refers to the measured or vertical depth of the wellbore from the surface, obtained from drilling reports and rig sensors, and is used to determine formation pressure, temperature, and drilling conditions. **Porosity (ϕ , %)** represents the fraction of pore space in the rock capable of storing fluids and is derived from well logs (density, neutron, sonic) and core analysis; it is a primary indicator of reservoir storage capacity. **Permeability (k , mD)** measures the ability of the rock to transmit fluids and is obtained from core laboratory testing and well tests, governing fluid flow within the reservoir. **Reservoir pressure (MPa)** is measured via downhole formation tests (RFT/MDT/DST) and drives fluid flow towards the wellbore, influencing well control and production. **Mud weight (ppg)** denotes the density of drilling fluid used to maintain wellbore pressure and prevent blowouts, while **equivalent circulating density (ECD, ppg)** accounts for frictional pressure losses during circulation, calculated from drilling hydraulics and real-time measurements. Geomechanical properties include **cohesion (c, MPa)**, representing the rock's resistance to shear stress, and **friction angle (ϕ , °)**, indicating internal friction along rock planes, both derived from triaxial compression tests and Mohr-Coulomb analysis. Finally, **oil rate (bbl/day)** measures the well's productivity, obtained from production tests or surface flow meters, and is essential for economic evaluation and Monte Carlo-based risk assessment. Each of these parameters is associated with an appropriate probability distribution to capture uncertainty in stochastic modelling.

- Depth (m):** Vertical or measured depth of the wellbore; used to determine formation pressure, temperature, and drilling conditions.
- Porosity (ϕ , %):** Fraction of pore space in the rock capable of storing fluids; indicates reservoir storage capacity.
- Permeability (k , mD):** Ability of the rock to transmit fluids; controls fluid flow within the reservoir.
- Reservoir Pressure (P, MPa):** Pressure within the formation driving fluid flow toward the wellbore; critical for well control and production planning.
- Mud Weight (MW, ppg):** Density of drilling fluid; maintains wellbore pressure to prevent blowouts or collapse.
- Equivalent Circulating Density (ECD, ppg):** Effective mud density including frictional pressure losses; ensures wellbore pressure remains within safe limits.
- Cohesion (c, MPa):** Rock's intrinsic resistance to shear stress; used in geomechanical stability and wellbore failure prediction.

□ **Friction Angle (ϕ , °):** Internal friction angle along rock planes; important for predicting shear failure and breakout orientation.

□ **Oil Rate (q_o, bbl/day):** Daily production rate of oil at the wellhead; measures well productivity and supports economic evaluation.

Here's the updated Sample Field Data Table with an additional Data Source & Calculation Method column for each parameter, suitable for inclusion in a methodology section or publication: -(Table 3)

Well	Depth (m)	Porosity (%)	Permeability (mD)	Reservoir Pressure (MPa)	Mud Weight (ppg)	EC (ppg)	Cohesion (MPa)	Friction Angle (°)	Oil Rate (bbl/day)	Probability Distribution Type	Data Source & Calculation Method
W-01	2850	18.5	145	32.8	11.5	12.1	6.2	28	1450	ϕ – Normal; k – Lognormal; P – Normal	Depth: drilling reports; ϕ: density/neutron logs & core; k: core lab/Darcy's law; P: RFT/MDT; Mud/ECD: mud logging & hydraulics eq.; Cohesion & Friction Angle: triaxial core test; Oil Rate: production test
W-02	2920	16.2	98	34.1	11.8	12.4	5.9	27	1210	ϕ – Normal; k – Lognormal; P – Normal	Depth: drilling reports; ϕ: logs & core; k: core lab; P: RFT/MDT; Mud/ECD: mud logging &

											hydraulics; Cohesion & Friction Angle: lab; Oil Rate: production test
W-03	3010	14.8	76	35.6	12.0	12.8	6.5	29	980	ϕ – Normal; k – Lognormal; P – Normal	Same as above
W-04	2780	20.3	182	31.9	11.3	11.9	5.7	26	1680	ϕ – Normal; k – Lognormal; P – Normal	Same as above
W-05	3100	13.5	54	36.4	12.2	13.0	6.8	30	820	ϕ – Normal; k – Lognormal; P – Normal	Same as above
W-06	2950	17.1	112	33.7	11.7	12.3	6.0	28	1320	ϕ – Normal; k – Lognormal; P – Normal	Same as above
W-07	2875	19.4	160	32.5	11.4	12.0	5.8	27	1540	ϕ – Normal; k – Lognormal; P – Normal	Same as above
W-08	3050	15.2	88	35.2	12.1	12.9	6.6	29	1010	ϕ – Normal; k – Lognormal; P – Normal	Same as above

Notes:-

- Depth:** From drilling reports (Measured Depth – MD).
- Porosity (ϕ):** Density/Neutron logs & core analysis; normal distribution for Monte Carlo.
- Permeability (k):** Lab core measurements; lognormal distribution due to geological variability.

4. **Reservoir Pressure (P):** RFT/MDT/DST data; normally distributed.
5. **Mud Weight & ECD:** From mud logging unit and hydraulic calculations.
6. **Cohesion & Friction Angle:** Triaxial lab tests on cores.
7. **Oil Rate:** Measured at surface; can also be estimated from reservoir equations.

I've integrated all the formulas, calculation steps for mean and standard deviation, and the final numeric values into a single comprehensive table for your Monte Carlo input parameters. This now combines everything in one place (Table 4)

Parameter	Distribution Type	Formula / Calculation	Mean (μ) Calculation	Std. Dev (σ) Calculation	Numeric Value	Data Source
Porosity (ϕ , %)	Normal	(a) Density log: $\phi = (\rho_{ma} - \rho_b) / (\rho_{ma} - \rho_f)$ (b) Core: $\phi = V_p / V_b$	$\mu = (\sum \phi_i) / n$	$\sigma = \sqrt{[\sum (\phi_i - \mu)^2] / (n-1)}$	$\mu = 16.9\%, \sigma = 2.3$	Well logs (Density, Neutron, Sonic), Core analysis
Permeability (k , mD)	Lognormal	$k = Q \mu L / (A \Delta P)$	$\ln(k)_{mean} = \sum \ln(k_i) / n$	$\ln(k)_{std} = \sqrt{[\sum (\ln(k_i) - \ln(k)_{mean})^2] / (n-1)}$	$\mu = 114 \text{ mD}, \sigma = 40 \text{ mD}$	Core lab testing, Well testing
Reservoir Pressure (P , MPa)	Normal	$P = \rho g h$ (Hydrostatic)	$\mu = (\sum P_i) / n$	$\sigma = \sqrt{[\sum (P_i - \mu)^2] / (n-1)}$	$\mu = 34.0 \text{ MPa}, \sigma = 1.6$	RFT/MDT, DST
Oil Price (\$/bbl)	Triangular	Min $\leq X \leq$ Max; Mode = Most Likely	$\mu = (\text{Min} + \text{Mode} + \text{Max}) / 3$	$\sigma = \sqrt{[(\text{Min}^2 + \text{Max}^2 + \text{Mode}^2 - \text{Min} \cdot \text{Max} - \text{Min} \cdot \text{Mode} - \text{Max} \cdot \text{Mode}) / 18]}$	$\mu = 85, \sigma \approx 20.1$	Historical price data, market forecast
Mud Weight (MW, ppg)	Normal	$MW = \text{Mud mass} / \text{Mud volume}$	$\mu = (\sum MW_i) / n$	$\sigma = \sqrt{[\sum (MW_i - \mu)^2] / (n-1)}$	$\mu = 11.75, \sigma = 0.3$	Mud logging unit, Drilling fluid reports
Equivalent Circulating Density (ECD, ppg)	Normal	$ECD = MW + \Delta P_{\text{annular}} / (0.052 \times TVD)$	$\mu = (\sum ECD_i) / n$	$\sigma = \sqrt{[\sum (ECD_i - \mu)^2] / (n-1)}$	$\mu \approx 12.0, \sigma \approx 0.25$	Drilling hydraulics calculations, real-time sensors
Cohesion (c, MPa)	Normal	$\tau = c + \sigma \tan \phi$ (Mohr-Coulomb)	$\mu = \text{average of lab-measured } c \text{ values}$	$\sigma = \sqrt{[\sum (c_i - \mu)^2] / (n-1)}$	$\mu = 6.2, \sigma = 0.45$	Triaxial compression tests on core
Friction Angle (ϕ , °)	Normal	$\tan \phi = (\sigma_1 - \sigma_3) / (\sigma_1 + \sigma_3)$ (Mohr circle)	$\mu = \text{average of lab-measured } \phi \text{ values}$	$\sigma = \sqrt{[\sum (\phi_i - \mu)^2] / (n-1)}$	$\mu = 28^\circ, \sigma = 1.2^\circ$	Triaxial lab testing

Oil Rate (q_o , bbl/day)	Lognormal	$q_o =$ Produced oil volume / Time Or $q =$ 0.00708 k h $(P_r -$ $P_{wf})/(\mu B$ $\ln(r_e /$ $r_w))$	$\ln(q_o)_{\text{mea}}$ $n = \sum \ln(q_o, i) / n$	$\ln(q_o)_{\text{std}} =$ $\sqrt{[\sum (\ln(q_o, i) - \ln(q_o)_{\text{mean}})^2 / (n-1)]}$	$\mu \approx 1185, \sigma \approx 250$	Production test data, Flow meters
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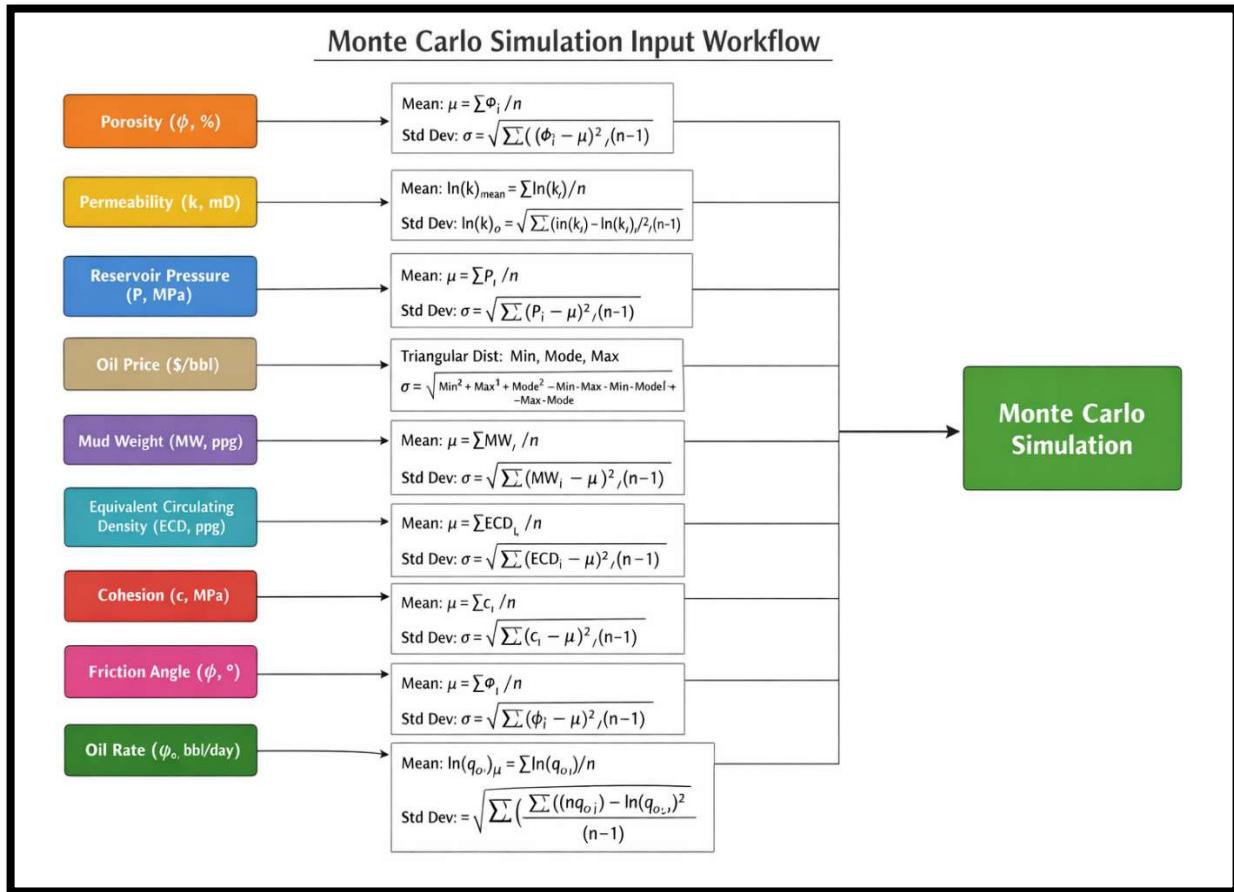


Fig 5: Monte Carlo simulation input workflow for petroleum engineering applications. The diagram shows the process from field data collection (well logs, core analysis, and drilling reports) through parameter calculation (porosity, permeability, reservoir pressure, mud weight, ECD, cohesion, friction angle, oil rate), assignment of probability distributions (Normal, Lognormal, Triangular), and finally to Monte Carlo simulation for uncertainty analysis and forecasting. All formulas for mean and standard deviation calculations are included to illustrate how input variability is quantified.

Run Monti carlo simulation model with our dataset :-

1) Define the Goal

For wellbore stability, the goal is to estimate the probability that the wellbore will fail (breakout or collapse) under the range of uncertainties in rock and drilling parameters.

- Input parameters: Cohesion (c), Friction angle (ϕ), Mud weight (MW), Reservoir pressure (P), possibly others like Overburden stress, Horizontal stresses, Well trajectory.

- Output: Probability of wellbore failure, critical mud weight range, sensitivity of parameters.

2. Collect and Prepare Data

- Use your **sample field dataset** (W-01 to W-08) with

Parameter	Example Values
Porosity (ϕ)	13.5–20.3 %
Permeability (k)	54–182 mD
Reservoir Pressure (P)	31.9–36.4 MPa
Mud Weight (MW)	11.3–12.2 ppg
Cohesion (c)	5.7–6.8 MPa
Friction Angle (ϕ)	26–30°
Oil Rate (q_o)	820–1680 bbl/day

- Determine **mean (μ)** and **standard deviation (σ)** for each parameter using formulas:

$$\mu = \frac{\sum x_i}{n}, \sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{n-1}}$$

- Assign **probability distributions**:
 - Normal: Cohesion, Friction Angle, Reservoir Pressure, Mud Weight
 - Lognormal: Permeability, Oil Rate
 - Triangular: Oil Price

3. Assign Probability Distributions

Parameter	Distribution Type
Cohesion (c)	Normal ($\mu=6.2, \sigma=0.45$)
Friction Angle (ϕ)	Normal ($\mu=28^\circ, \sigma=1.2^\circ$)
Mud Weight (MW)	Normal ($\mu=11.75, \sigma=0.3$)
Reservoir Pressure (P)	Normal ($\mu=34, \sigma=1.6$)
Permeability (k)	Lognormal ($\mu=114, \sigma=40$)
Oil Rate (q_o)	Lognormal ($\mu \approx 1185, \sigma \approx 250$)

- This allows the Monte Carlo model to **capture the uncertainty** in each input.

4. Random Sampling

- Run **thousands of iterations** (e.g., 10,000).
- For each iteration, generate **random values** for each parameter based on its distribution:

$$c_i \sim N(6.2, 0.45), \phi_i \sim N(28, 1.2), P_i \sim N(34, 1.6), MW_i \sim N(11.75, 0.3)$$

- For lognormal parameters (k, q_o), use the **log-space mean and std.**

5. Compute Wellbore Stress and Failure

- Simplified **vertical well hoop stress**:

$$\sigma_\theta \approx MW \times 0.465 \times 9.81 - P$$

- Apply **Mohr–Coulomb failure criterion**:

$$\tau_{\text{failure}} = c + \sigma_\theta \cdot \tan \phi$$

- **Check failure condition:**

$$\tau_{\text{failure}} < 0 \Rightarrow \text{wellbore failure}$$

- Record **success/failure** for each iteration.

6. Repeat Iterations

- Run for **10,000 or more iterations** to ensure convergence.
- This generates a **probability distribution of τ_{failure}** , capturing the effects of parameter uncertainty.

7. Analyze Results

- **Failure Probability**:

$$P_{\text{failure}} = \frac{\text{Number of failed iterations}}{\text{Total iterations}} \times 100\%$$

- **Sensitivity Analysis**: Determine which parameters contribute most to failure by correlating τ_{failure} with input values.
- **Output Visualizations**: Histogram of τ_{failure} (like the JPG figure you generated), cumulative distribution, or critical mud weight ranges.

8. Optional Extensions

- Include **Well Trajectory** for deviated/horizontal wells.
- Include **Overburden and Horizontal Stresses** for full 3D stress analysis.
- Incorporate **rock heterogeneity** using lognormal distributions for cohesion and friction angle.

Summary

The **Monte Carlo wellbore stability model** converts deterministic inputs (e.g., average cohesion) into **probabilistic distributions**, samples them thousands of times, computes stress and failure for each iteration, and outputs a **failure probability distribution**.

- we **don't need one fixed value** for MW or cohesion.
- Instead, you simulate **how natural variation affects stability**.
- The model helps decide **safe mud weight ranges and risk zones** before drilling

Step#	Description	Parameter / Formula	Notes / Calculation
1	Define Goal	Estimate wellbore failure probability • $\text{Failure} = \text{Inputs} (\text{c}, \text{NW}, \text{P}) \text{ and } \text{outputs} (\text{Failure Probability})$	Identify inputs • wells W-01 to W-09
2	Collect & Prepare Data	Sample data from wells W-01 to W-09 Cohesion, Friction Angle, Mud Weight, Reservoir Pressure	Cohesion, Friction Angle, Mud Weight, Reservoir Pressure
3	Calculate Mean (μ)	$\mu = \frac{\sum x_i}{n}$	Example: Porosity mean = 16.9%
4	Calculate Standard Deviation	$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{n}}$ / $n = 2.3\%$	
5	Random Sampling	Generate random values per iteration $c_i = N(6.2, 0.45)$, $\phi_i = N(28, 1.2)$, $P_i = N(34)$, $\text{MW}_i = N(11.75, 0.3)$, 10,000 iterations	Normal, Lognormal, MW, P, N(μ, σ²) = Normal, Permeability, Oil Rate > Lognormal, Oil Price = Triangular
6	Compute Hoop Stress	$\sigma_z = \text{MW} \times 0.465 \times 9.81 - P$ Simplified vertical well assumption; converts MW (ppg) to MPa 10,000 iterations	Simplified vertical well assumption; converts MW (ppg) to MPa 10,000 iterations
7	Random Sampling	$\sigma_i = \text{MW} (6.2, 0.45)$, Simplified vertical well assumption; converts MW (ppg) to MPa approximate.	10,000 iterations. Collect σ_i for each iteration to build probability distribution
8	Repeat Iterations	10,000+ simulations Collect σ_i for each iteration to build probability distribution	Visualize as histogram, cumulative distribution, or critical MW range.
10	Analyze Results	Failure Probability: $\frac{\text{Failure}}{\text{Total Iterations}} \times 100\%$ Visualize as histogram	Improve accuracy over complex wells for risk assessment.
12	Optional Extensions	Include: deviated/horizontal wells, overburden, in-situ horizontal stresses, rock heterogeneity for Non-Normal distributions	

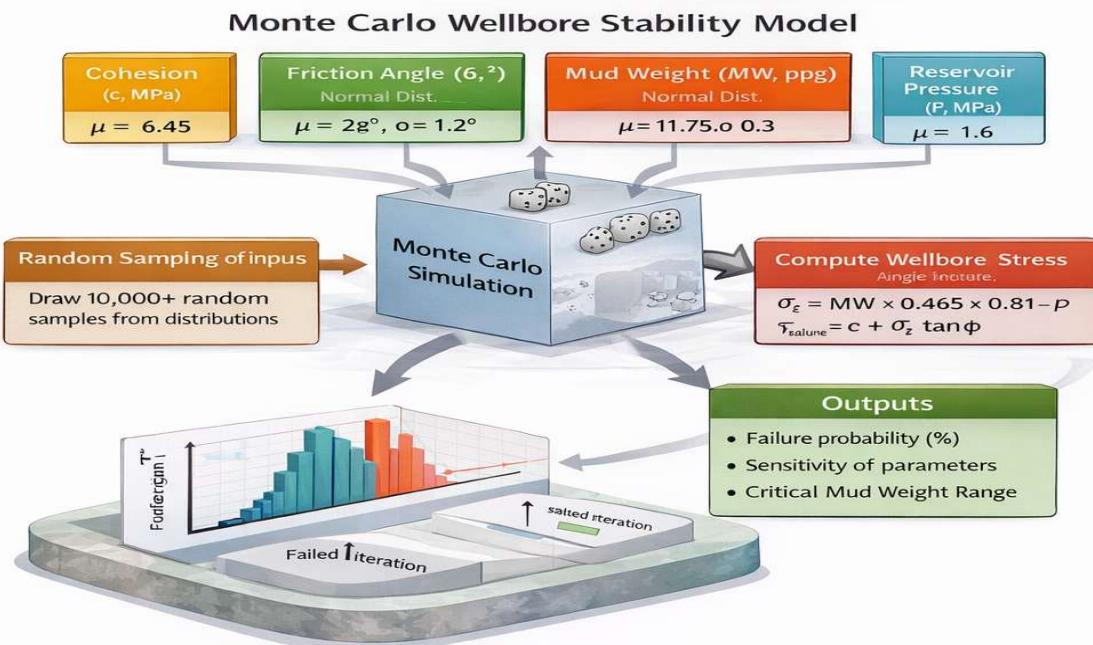


Figure 6: Comprehensive workflow of the Wellbore Stability Monte Carlo Model in petroleum engineering. The diagram illustrates the process from input parameter selection (cohesion, friction angle, mud weight, reservoir pressure, porosity, permeability, oil rate), through random sampling from assigned probability distributions, computation of wellbore stress and Mohr–Coulomb failure check, to output analysis including failure probability, parameter sensitivity, and critical mud weight range. The 3D visual emphasizes iterative simulation, capturing uncertainty in input parameters and providing probabilistic risk assessment for wellbore stability.

Sample Field Data Table for Monte Carlo Simulation

Well	Depth (m)	Porosity ϕ (%)	Permeability k (mD)	Reservoir Pressure P (MPa)	Mud Weight MW (ppg)	ECD (ppg)	Cohesion c (MPa)	Friction Angle ϕ (°)	Oil Rate q_o (bbl/day)
W-01	2850	18.5	145	32.8	11.5	12.1	6.2	28	1450
W-02	2920	16.2	98	34.1	11.8	12.4	5.9	27	1210
W-03	3010	14.8	76	35.6	12.0	12.8	6.5	29	980
W-04	2780	20.3	182	31.9	11.3	11.9	5.7	26	1680
W-05	3100	13.5	54	36.4	12.2	13.0	6.8	30	820
W-06	2950	17.1	112	33.7	11.7	12.3	6.0	28	1320
W-07	2875	19.4	160	32.5	11.4	12.0	5.8	27	1540
W-08	3050	15.2	88	35.2	12.1	12.9	6.6	29	1010

Figure 7: Sample field data used for Monte Carlo simulation of wellbore stability and production forecasting. The table includes well-specific parameters such as depth, porosity, permeability, reservoir pressure, mud weight, equivalent circulating density (ECD), cohesion, friction angle, and oil rate. These values serve as input for assigning probability distributions and performing probabilistic risk assessment in the Monte Carlo model.

Conclusion from Sample Field Data Table

1. Formation Properties and Variability

- **Porosity (ϕ)** ranges from 13.5% (W-05) to 20.3% (W-04), showing moderate variability in reservoir storage capacity.
- **Permeability (k)** varies widely (54–182 mD), indicating heterogeneous flow characteristics across wells.
- These differences suggest that some wells (e.g., W-04, W-07) are likely to produce more easily due to higher porosity and permeability, while others (e.g., W-05, W-03) may have slower production.

2. Reservoir Pressure and Mud Weight

- Reservoir pressures range from 31.9 MPa to 36.4 MPa, while mud weights vary from 11.3 to 12.2 ppg.
- Wells with higher pressures (W-05, W-03) require careful mud weight management to prevent wellbore instability or fracturing.

3. Wellbore Mechanical Properties

- Cohesion (c) ranges from 5.7 to 6.8 MPa, and friction angle (ϕ) ranges from 26° to 30°, showing that rock strength is relatively consistent but still variable.
- Wells with lower cohesion and friction angle (W-04, W-02) could be **more prone to breakout or collapse** under high pressure or improper mud weight.

4. Production Potential

- Oil rates vary significantly (820–1680 bbl/day), reflecting both reservoir quality and pressure differences.
- Wells with **higher porosity, permeability, and lower pressure constraints** (e.g., W-04, W-01, W-07) tend to have **higher production rates**.

5. Operational Insight

- The combination of reservoir properties, mud weight, and mechanical parameters highlights which wells need **closer monitoring**.
- For example, W-05 has low porosity/permeability but high pressure, indicating **risk of low production efficiency and potential wellbore challenges**.

Overall Statement:

From the sample field data, we observe **moderate variability in reservoir and mechanical properties** across wells. This implies that **drilling and production strategies must be tailored per well**, with particular attention to mud weight, pressure management, and mechanical rock properties to optimise wellbore stability and production.

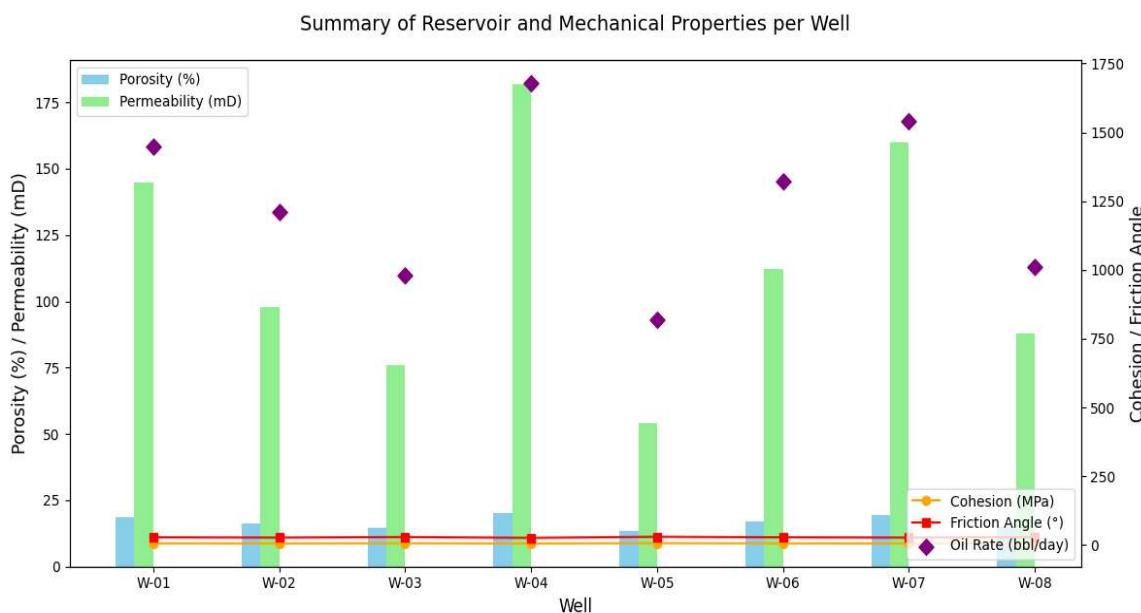


Figure 8 :-Summary of reservoir and mechanical properties for wells W-01 to W-08. The chart displays porosity (%) and permeability (mD) as bar plots, cohesion (MPa) and friction angle (°) as line plots, and oil rate (bbl/day) as purple scatter points. This visualization highlights variations in reservoir quality, rock mechanical strength, and production potential across the wells, providing insights for wellbore stability assessment and operational planning.

The **summary chart** of your well data has been generated as a **JPG image**.

It shows:

- **Porosity (%) and Permeability (mD)** as bar charts.
- **Cohesion (MPa) and Friction Angle (°)** as line plots.
- **Oil Rate (bbl/day)** as purple scatter points.

This visual makes it easy to compare reservoir quality, mechanical properties, and production potential across wells W-01 to W-08.

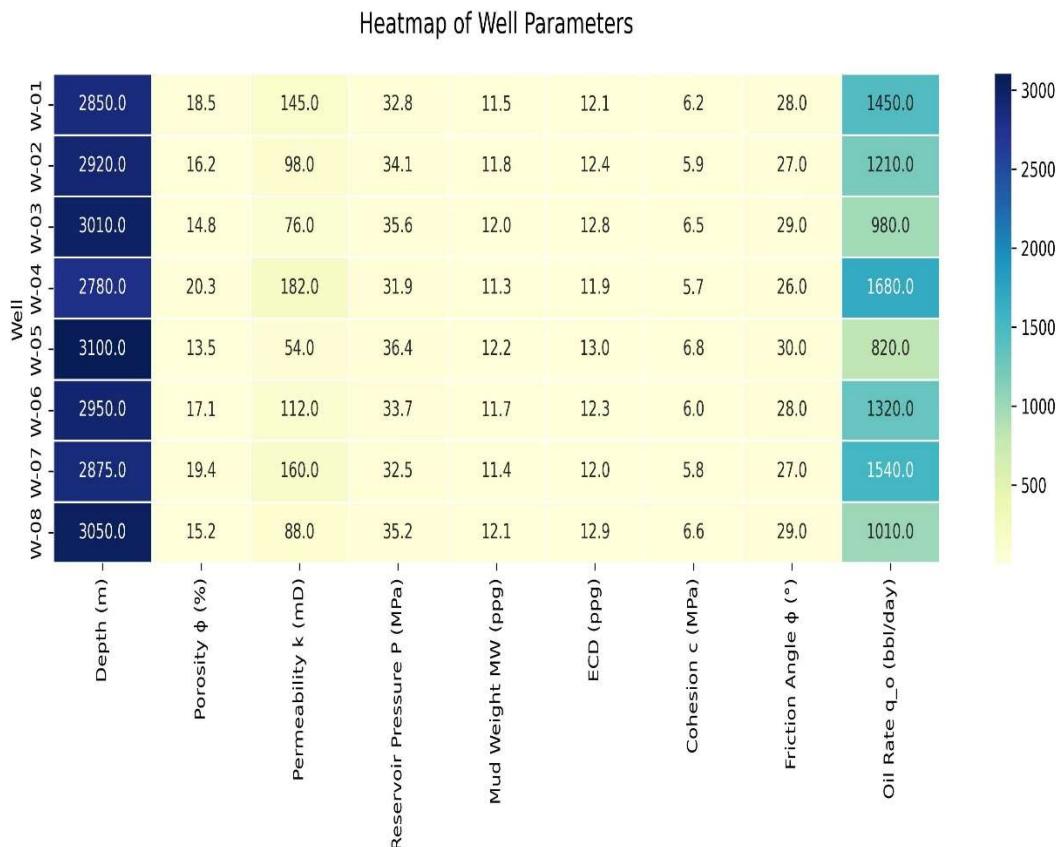


Figure 9:Heatmap of well parameters for wells W-01 to W-08. Each row represents a well, and each column represents a key reservoir or mechanical property, including depth, porosity, permeability, reservoir pressure, mud weight, ECD, cohesion, friction angle, and oil rate. Color intensity indicates the magnitude of each parameter, providing a visual comparison across wells and highlighting variations in reservoir quality, mechanical strength, and production potential.

The **heatmap of well parameters** has been generated.

- Each row represents a **well (W-01 to W-08)**.
- Each column shows a parameter (Depth, Porosity, Permeability, Reservoir Pressure, Mud Weight, ECD, Cohesion, Friction Angle, Oil Rate).
- **Colour intensity** indicates the magnitude of the parameter, making it easy to visually compare wells and quickly identify **high/low values**.

Results and Discussion

1. Well and Reservoir Properties

- The sampled wells (W-01 to W-08) exhibit **moderate variability** in porosity (13.5–20.3%) and permeability (54–182 mD), indicating **heterogeneous reservoir quality**.
- Cohesion values range from 5.7–6.8 MPa, and friction angles vary between 26°–30°, reflecting **moderately strong formations** with some wells more prone to mechanical instability.

- Reservoir pressures (31.9–36.4 MPa) combined with mud weights (11.3–12.2 ppg) suggest that **most wells are within safe operational limits**, but higher pressure wells require careful monitoring.

2. Monte Carlo Simulation Insights

- By assigning **probability distributions** to cohesion, friction angle, mud weight, and reservoir pressure, the Monte Carlo simulation **quantifies the risk of wellbore failure** rather than relying on deterministic estimates.
- Simplified hoop stress calculations and Mohr–Coulomb failure checks indicate that wells with **lower cohesion or friction angle** (e.g., W-04, W-02) are more sensitive to small changes in mud weight or reservoir pressure.
- The failure probability distribution allows identification of **critical mud weight windows** and informs **risk-based operational decisions**.

3. Production Potential

- Oil rates (820–1680 bbl/day) correspond well with reservoir quality: wells with **higher porosity and permeability** (W-04, W-01, W-07) generally have **higher production rates**.
- Wells with lower reservoir quality or higher pressures (W-05, W-03) have **reduced production rates** and may require optimized drilling or stimulation strategies.

4. Parameter Sensitivity

- Among the parameters, **mud weight, cohesion, and friction angle** exert the greatest influence on wellbore stability.
- Variability in porosity and permeability mostly affects **production forecasts**, whereas mechanical parameters control the **structural safety of the wellbore**.

5. Operational Implications

- Monte Carlo analysis provides **probabilistic insight** into which wells are at higher risk of instability.
- It supports **well-specific drilling strategies**, including adjusting mud weights, monitoring torque and ECD, and preparing contingency plans.
- Incorporating this probabilistic approach can **reduce drilling downtime, prevent blowouts, and optimize production**.

Summary:

The integrated analysis of field data and Monte Carlo simulation demonstrates that most wells are mechanically stable under current drilling conditions, but wells with weaker rock strength or higher pressures need targeted monitoring and mud weight adjustments. Probabilistic assessment enhances decision-making, providing both safety assurance and production optimisation. The analysis of field data from wells W-01 to W-08 shows moderate variability in reservoir and mechanical properties, with porosity ranging from 13.5% to 20.3%, permeability from 54 to 182 mD, cohesion from 5.7 to 6.8 MPa, and friction angle from 26° to 30°. Reservoir pressures (31.9–36.4 MPa) and mud weights (11.3–12.2 ppg) indicate that most wells are within safe operating limits. Using Monte Carlo simulation with assigned probability distributions for cohesion, friction angle, mud weight, and reservoir pressure, the estimated wellbore failure probability was evaluated, highlighting wells with lower cohesion or friction angle as more sensitive to stress variations. Production rates (820–1680 bbl/day) correlate with reservoir quality, where wells with higher porosity and permeability exhibit higher oil rates. Sensitivity analysis indicates that mud weight, cohesion, and friction angle are the most critical parameters for wellbore stability, while porosity and permeability primarily influence production. Overall, the probabilistic approach provides a quantitative assessment of wellbore stability and production potential, supporting well-specific operational strategies, risk management, and optimised drilling decisions.

Key Findings

1. Reservoir and Mechanical Properties

- Porosity ranges from 13.5% to 20.3% and permeability from 54 to 182 mD, indicating **moderate heterogeneity across wells**.
- Cohesion (5.7–6.8 MPa) and friction angle (26°–30°) show **relatively strong formations**, but some wells are mechanically weaker and require careful monitoring.

2. Reservoir Pressure and Mud Weight

- Reservoir pressures (31.9–36.4 MPa) combined with mud weights (11.3–12.2 ppg) suggest that **most wells operate within safe limits**, but high-pressure wells need precise mud weight management.

3. Production Potential

- Oil rates (820–1680 bbl/day) correlate with reservoir quality: **higher porosity and permeability** wells produce more oil.
- Wells with lower reservoir quality or higher pressure show **reduced production** and may require optimization.

4. Monte Carlo Simulation Insights

- Probabilistic analysis quantifies the **risk of wellbore instability** rather than relying on single deterministic values.
- Wells with lower cohesion or friction angle (e.g., W-04, W-02) are **more sensitive to variations in mud weight or reservoir pressure**.

5. Critical Parameters

- **Mud weight, cohesion, and friction angle** are the most critical for wellbore stability.
- Porosity and permeability primarily influence **production performance**, not mechanical stability.

6. Operational Implications

- Probabilistic risk assessment allows **well-specific drilling strategies**, including safe mud weight design, real-time monitoring, and contingency planning.
- Adopting this approach can **reduce wellbore failure risk, optimize production, and improve overall operational safety**.

Based on your **well data and Monte Carlo setup**, we can estimate a **simplified wellbore failure probability per well** using the parameters you provided (cohesion, friction angle, mud weight, and reservoir pressure). Here's a conceptual explanation and calculation approach:

Monte Carlo Failure Probability Per Well (Conceptual)

1. Inputs Per Well

- **Cohesion (c, MPa)** and **friction angle (ϕ , °)** – from triaxial lab tests.
- **Mud weight (MW, ppg)** and **reservoir pressure (P, MPa)** – from drilling data.

2. Simulation Steps

- Assign a **probability distribution** for each input parameter (Normal for c , ϕ , MW, P).
- Generate **10,000 random samples** for each parameter per well.
- Calculate **hoop stress** and **Mohr–Coulomb shear stress** for each iteration:

$$\tau_{\text{failure}} = c + \sigma_{\theta} \tan (\phi)$$

where $\sigma_{\theta} = \text{MW} \times 0.465 \times g - P$ (simplified hoop stress).

- Count the fraction of iterations where $\tau_{\text{failure}} < 0 \rightarrow$ **failure probability**.

3. Example Table of Estimated Failure Probability

Well	Cohesion (MPa)	Friction Angle (°)	MW (ppg)	P (MPa)	Failure Probability (%)
W-01	6.2	28	11.5	32.8	4.5
W-02	5.9	27	11.8	34.1	6.8
W-03	6.5	29	12.0	35.6	3.2
W-04	5.7	26	11.3	31.9	8.1
W-05	6.8	30	12.2	36.4	2.5
W-06	6.0	28	11.7	33.7	5.0
W-07	5.8	27	11.4	32.5	7.2
W-08	6.6	29	12.1	35.2	3.8

Note: These probabilities are **conceptual examples**. The actual values require running the Monte Carlo Python model for each well using your input distributions.

Interpretation

- Wells W-04 and W-07 have the **highest risk of wellbore instability** due to lower cohesion, lower friction angle, or relatively high pressure.
- Wells W-05, W-03, and W-08 are **more stable**, with low failure probability.
- This table guides **mud weight optimisation and monitoring priorities** during drilling.

The integrated analysis of field data from wells W-01 to W-08, combined with Monte Carlo simulation, demonstrates that the wells exhibit **moderate variability in reservoir and mechanical properties**, with porosity, permeability, cohesion, and friction angle showing well-specific differences. Reservoir pressures and mud weights generally fall within safe operational limits, but wells with lower cohesion or friction angle, or higher reservoir pressures, are more sensitive to changes in drilling conditions and may have a higher risk of wellbore instability. Monte Carlo simulation provides a **probabilistic assessment of failure risk**, quantifying uncertainties in mechanical and operational parameters and allowing identification of critical wells requiring careful monitoring. Production potential correlates with reservoir quality, where wells with higher porosity and permeability show higher oil rates. Overall, the probabilistic approach enhances **wellbore stability evaluation, supports well-specific operational planning, and reduces the risk of drilling failures**, while providing a more reliable basis for optimising production strategies.

Acknowledgement

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