

# Friction-Based Vibration Damping in Gas Turbine Blades: Experimental Insights and Design Strategies for Under-Platform Dampers

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## Abstract:

Under-platform dampers (UPDs) remain the most widely deployed friction-damping device in gas turbine bladed disks, yet their nonlinear, history-dependent contact behaviour means that neither their performance nor their optimal geometry can be established analytically alone: both depend on experimental characterization. This review surveys the experimental infrastructure the field has built for this purpose — single- and dual-damper test rigs, contact-force and relative-displacement instrumentation, and the parameter-identification workflows that convert measured data into usable contact-model inputs — and the design-optimization literature that this infrastructure supports, spanning symmetric and asymmetric wedge geometries, reliability- and uncertainty-aware formulations, and, most recently, gradient-free topology optimization. Particular attention is given to a gap that recurs across this literature: essentially all published UPD test rigs and optimization studies operate at or near room temperature and a single nominal preload, so the extent to which a geometry optimized under laboratory conditions remains close to optimal once operating temperature and its attendant contact-parameter drift are taken into account is presently unresolved. The review closes by identifying this thermally-robust design question, together with the closely related uncertainty-quantification literature, as the most consequential open direction for UPD design methodology.

**Keywords —** *Under-Platform Damper; Test Rig; Parameter Identification; Damper Geometry Optimization; Asymmetric Damper; Uncertainty Quantification; Turbine Blade Vibration*

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## I. INTRODUCTION

An under-platform damper is, mechanically, one of the simplest components in a gas turbine: a loose metallic insert, held in place only by centrifugal force, wedged beneath the platforms of two adjacent blades. That simplicity is deceptive. Since Griffin's foundational treatment first brought dry-friction damping explicitly into turbomachinery blade design [1], the behaviour of such an unbonded interface has been governed entirely by the nonlinear, load- and history-dependent physics of dry friction contact, and neither the damping it delivers nor the geometry that would deliver the most of it can be predicted from first principles with the confidence a bonded structural component would allow. Every stage of UPD development —

constitutive contact modelling, forced-response prediction, and geometric design — therefore depends on dedicated experimental infrastructure in a way that few other turbomachinery components do.

This review surveys that infrastructure and the design-optimization literature built on top of it. It is a deliberately narrower and more current companion to the comprehensive review recently published by Yuan, Gastaldi, Denimal Goy and Chouvion [20], which spans the full breadth of friction-damper modelling, testing, and design across shrouds, ring dampers, and UPDs together; the present review instead concentrates specifically on the UPD experimental and optimization literature, at a level of case-by-case detail that a broader survey cannot accommodate, and updates

that coverage with several studies published since. Readers seeking the constitutive contact-modelling and solution-method side of this field — the physics that the rigs and optimization studies reviewed here exist to calibrate and validate — are referred to the first paper in this review series, which treats that literature specifically.

Section 2 characterizes the principal UPD geometric families and the design variables that distinguish them. Section 3 surveys experimental test rig architectures, instrumentation, and the parameter-identification workflow that connects measured data to usable contact-model inputs. Section 4 reviews the design-optimization literature built on this experimental foundation, from geometry and asymmetry optimization through reliability-aware and topology-optimization formulations. Section 5 synthesizes this literature comparatively, Section 6 identifies open challenges, and Section 7 concludes.

## II. UNDER-PLATFORM DAMPER CONFIGURATIONS AND DESIGN VARIABLES

Figure 1 illustrates the three geometric families that dominate the published literature. The

symmetric wedge, or cottage-roof, damper (Fig. 1a) is geometrically the simplest and historically the earliest studied configuration: a single contact angle and a single preload govern both platform interfaces identically by construction. Panning, Sextro and Popp showed computationally in 2002 that this enforced symmetry is not, in general, damping-optimal [2], motivating the asymmetric wedge configuration (Fig. 1b), in which the two contact angles  $\alpha_1$  and  $\alpha_2$  differ deliberately, redistributing normal load and slip amplitude unevenly between the two blade interfaces. Panning and colleagues subsequently validated this asymmetric concept experimentally and, ultimately, in-engine [3], and it now appears widely in the optimization literature reviewed in Section 4. The seal-wire or strip damper (Fig. 1c) trades the wedge's angled contact for a slender, largely flexible element running along the platform gap; it is mechanically distinct enough from the wedge families that its damping mechanism — combining local friction slip with the strip's own bending compliance — is typically treated as a separate design problem in the literature rather than a further point on the same geometric parameter space.

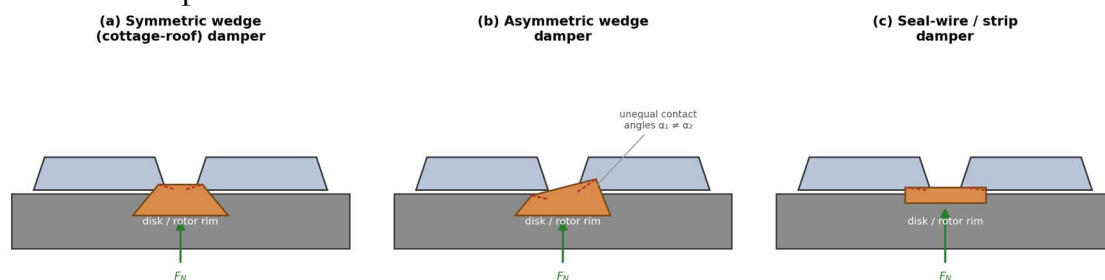


Fig. 1 Principal under-platform damper geometric families: (a) the symmetric wedge (cottage-roof) damper, in which both platform contact angles are equal by construction; (b) the asymmetric wedge damper, in which unequal contact angles  $\alpha_1 \neq \alpha_2$  redistribute load and slip between the two interfaces; (c) the seal-wire / strip damper, which relies additionally on the element's own bending compliance.

Across all three families, the design variables that recur throughout the literature reviewed in Section 4 are: the contact angle (or angles, for asymmetric geometries), the contact-patch radius of curvature or flatness, damper mass (which sets the achievable centrifugal preload at a given rotor speed and radial location), and, for strip dampers specifically, cross-sectional stiffness. It is worth noting explicitly that

every one of these variables interacts with the contact constitutive parameters reviewed in the authors' companion paper on contact modelling — friction coefficient, contact stiffness, and, where the coupled models discussed there are used, the normal-tangential coupling strength — so that a geometry optimized against one calibrated parameter set is not guaranteed to remain optimal

against another. This interaction is the central thread connecting Sections 3 and 4 of the present review.

### III. EXPERIMENTAL TEST RIGS AND PARAMETER IDENTIFICATION

#### A. Rig Architectures: From Single-Interface to Dual-Damper Configurations

Because a UPD's behaviour cannot be predicted analytically with confidence, essentially every contact model surveyed in the authors' companion review requires experimental calibration, and a

distinct sub-literature has developed purpose-built rigs for this purpose. Figure 2 sketches the architecture common to most of these rigs: a single blade specimen, rigidly clamped at its root, is fitted with one or two dampers pressed against machined platform ledges by a controllable, measurable preload standing in for the centrifugal load a rotating engine would supply; a shaker excites the blade, and the resulting response is measured optically or with accelerometers.

**Representative dual-damper under-platform-damper test rig architecture**

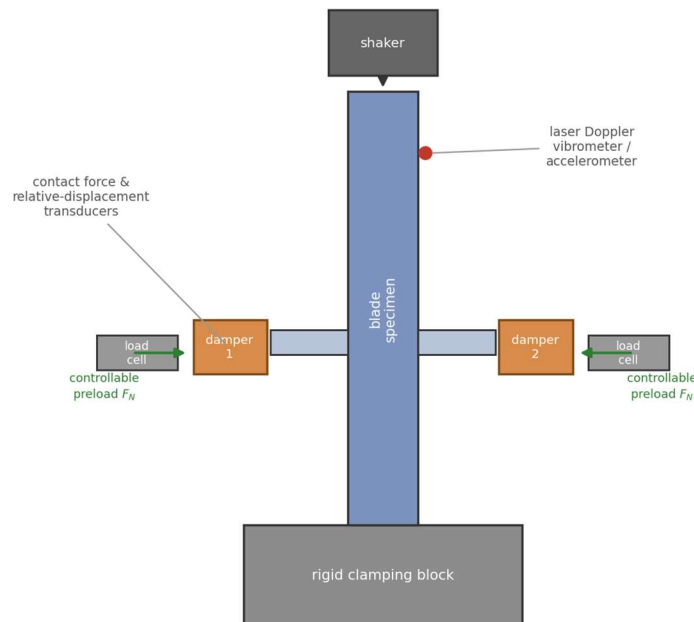


Fig. 2 Representative architecture of a dual-damper under-platform-damper test rig, showing the essential elements common to the majority of rigs surveyed in this review: rigidly clamped blade specimen, controllable preload with load-cell instrumentation at each damper, shaker excitation, and non-contact response measurement.

Firrone's 2009 rig [8] was among the first to instrument two dampers of differing geometry simultaneously on a single blade specimen, allowing direct comparison of damper kinematics against numerical prediction. Botto, Umer, Gastaldi and Gola extended this dual-damper architecture in 2017 to extract full hysteresis loops from the coupled two-damper configuration [5], and Botto and Umer's 2018 rig [6] represented a further step

change by measuring contact force directly at each damper — rather than inferring it indirectly from blade response — a distinction the authors argued was essential because blade-response measurements alone under-constrain the local nonlinearities the contact models of Section 2 attempt to represent. Umer and Botto's 2019 companion study [7] coupled this local force measurement explicitly back to global blade dynamics, closing the loop

between local contact behaviour and the system-level response a design engineer ultimately cares about. Gastaldi, Berruti and Gola's 2020 rig [4] took a complementary approach, decoupling the damper parameter measurement from blade-structural dynamics entirely so that contact parameters could be identified without the confounding influence of the specimen's own modal behaviour.

| Study (year)                            | Rig configuration                            | Loading method                   | Measured quantities                   | Key contribution                                       |
|-----------------------------------------|----------------------------------------------|----------------------------------|---------------------------------------|--------------------------------------------------------|
| Firrone (2009) [8]                      | Single blade, two UPDs, differing geometry   | Static preload rig               | Damper kinematics                     | Geometry-dependent kinematics vs. simulation           |
| Botto, Umer, Gastaldi & Gola (2017) [5] | Single blade, two UPDs                       | Static preload rig               | Blade FRF, hysteresis loops           | Dual-damper coupled dynamics                           |
| Botto & Umer (2018) [6]                 | Single blade, two force-instrumented dampers | Controllable, measurable preload | Contact force + relative displacement | First direct measurement of damper-local contact force |
| Umer & Botto (2019) [7]                 | As [6], coupled to blade dynamics            | Controllable preload             | Contact parameters vs. blade response | Links local contact measurement to global FRF          |
| Pesaresi et al. (2017) [9]              | Single blade, single UPD                     | Static preload rig               | FRF, validated nonlinear model        | End-to-end rig-model validation workflow               |
| Gastaldi, Berruti & Gola (2020) [4]     | Purpose-built UPD parameter rig              | Static, decoupled from blade     | Contact force, displacement           | Sensitivity analysis of contact parameters             |
| Usmanov et al. (2024) [12]              | 48-blade disk, multiple UPDs                 | Rotating (vacuum spin rig)       | Blade Tip-Timing response             | Mistuning + UPD effects at rotating, multi-blade scale |

TABLE I  
COMPARATIVE SUMMARY OF PRINCIPAL UNDER-PLATFORM-DAMPER EXPERIMENTAL TEST RIG STUDIES DISCUSSED IN THIS REVIEW.

### B. Parameter Identification Methodology

Whatever the rig architecture, the workflow that converts measured data into usable contact-model parameters follows a common pattern, sketched in Fig. 3: an initial parameter set is assumed for the chosen contact law, a forward nonlinear forced-response solve (typically MHB–AFT, as reviewed in the authors' companion paper) predicts the resulting frequency response, and the discrepancy between predicted and measured amplitude drives an iterative parameter update until convergence. Mohammadali and Ahmadian's efficient model-order-reduction approach [13] addressed a specific bottleneck in this workflow — the cost of repeatedly re-solving a full-order nonlinear model at each identification iteration — and the same general identification methodology underlies the room-temperature coupled-contact parameter identification later carried out by Mahmoodi and Ahmadian [14] for the normal–tangential coupling law reviewed in the authors' companion paper.

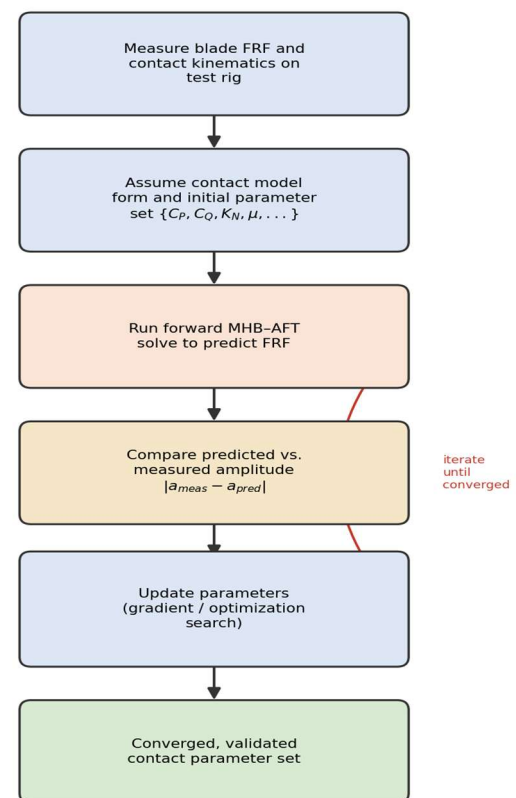


Fig. 3 General structure of the contact-parameter identification workflow common to the rig studies reviewed in this section: an assumed parameter set is refined iteratively by comparing forward nonlinear forced-response predictions against measured frequency-response data until convergence.

A recurring finding across this identification literature, consolidated into design guidance by Sanliturk, Gokdag and Akalin [15], is that identified contact parameters are sensitive not only to nominal geometry but to manufacturing tolerance, surface finish, and wear state — sources of variability a single deterministic identification run cannot capture. This observation is the direct motivation for the uncertainty-aware optimization literature reviewed in Section 4.2, and is revisited as an open challenge in Section 6.

#### ***C. High-Temperature and Rig-to-Engine Validation***

Two extensions of the room-temperature rig literature deserve separate mention. First, Schwingshackl and colleagues directly measured contact parameters of flat-on-flat interfaces representative of turbine friction joints as a function of temperature [10], demonstrating that the parameters the rigs of Section 3.1 identify at room temperature are not, in general, valid at engine operating temperature — a finding the authors' companion review on thermal and tribological effects treats at length. Second, Hartung and colleagues validated a nonlinear forced-response tool against both a rotating rig and full-engine measurements simultaneously [11], characterizing the operative friction coefficient as a function of temperature, preload, and wear state rather than as a single identified constant. Between them, these two studies mark the field's furthest published extension beyond the static, room-temperature rig architecture of Fig. 2, and, taken together with the rotating multi-blade rig of Usmanov et al. [12], indicate the direction rig development would need to continue in for the design-optimization literature reviewed next to be validated under conditions fully representative of engine service.

### **IV. DESIGN OPTIMIZATION OF UNDER-PLATFORM DAMPERS**

#### ***D. Geometry and Asymmetry Optimization***

The earliest and still most widely practised UPD optimization approach treats damper geometry as a small set of continuous parameters — contact angle, patch curvature, mass — swept or searched against a forced-response objective evaluated by the finite-

element and MHB-based methods reviewed in the authors' companion paper. Panning, Sextro and Popp's 2002 study [2] is the field's foundational result in this vein, showing computationally that a symmetric wedge geometry (Fig. 1a) is not, in general, damping-optimal, and that deliberately breaking contact-angle symmetry (Fig. 1b) can increase achievable damping. Because a full nonlinear forced-response evaluation is expensive to repeat at every candidate geometry, Gastaldi and Gola proposed in 2017 an analytical pre-optimization step [16] that identifies promising regions of the asymmetric design space cheaply, before committing to the more expensive full optimization — a screen-then-refine strategy that recurs, in more elaborate form, throughout the rest of this section.

#### ***E. Reliability- and Uncertainty-Aware Optimization***

A deterministic optimum — the single geometry that minimizes predicted resonance amplitude for one assumed, fixed set of contact parameters — is only as trustworthy as the identification workflow of Section 3.2 that produced those parameters, and Section 3.2 already noted that identified parameters vary with manufacturing tolerance, surface finish, and wear state. Krack, Tatzko, Panning-von Scheidt and Wallaschek reframed the design problem accordingly in 2014, posing friction-damper design as a reliability-optimization problem under explicitly uncertain friction and excitation parameters rather than a single-point search [17]. Yuan, Fantetti, Denimal, Bhatnagar, Pesaresi, Schwingshackl and Salles quantified the scale of the underlying uncertainty directly, propagating measured friction-parameter variability through a nonlinear forced-response model via polynomial chaos expansion and Sobol sensitivity analysis [22], while Yuan, Jones, Setchfield and Schwingshackl closed the loop by proposing a robust design-optimization formulation, combining detailed nonlinear contact modelling with a surrogate model to keep the resulting robust search computationally tractable [21]. More recently, Yang, She, Wu, Mi, Zang and Li extended this uncertainty treatment to a full multi-source formulation spanning manufacturing, excitation, and wear variability

simultaneously, evaluating sensitivity of blade amplitude to each source individually [23]. Figure 4 illustrates, schematically, the qualitative distinction this literature has established between a

deterministic (nominal) optimum and a robust (worst-case-across-conditions) optimum: the two need not — and, on the evidence of [21], typically do not — coincide.

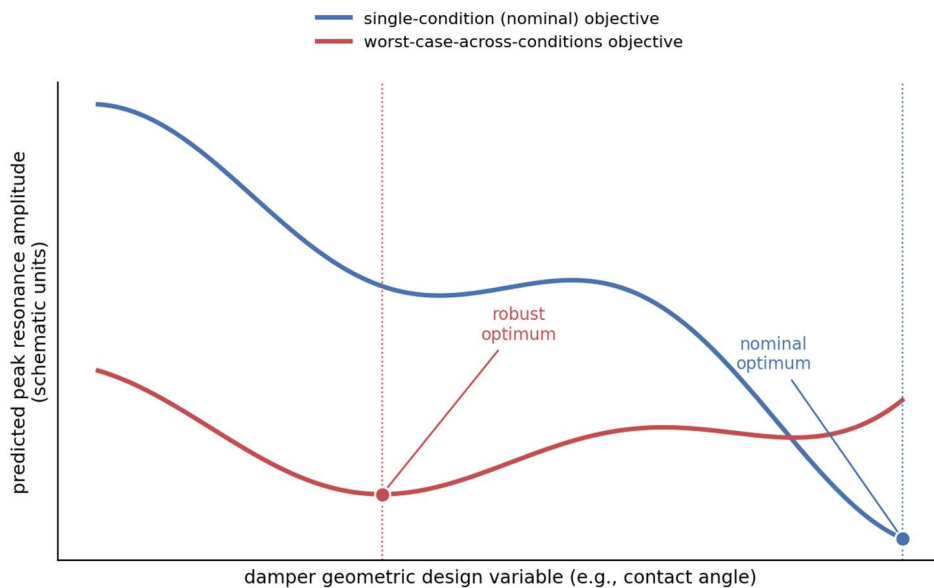


Fig. 4 Schematic illustration of the qualitative distinction between a deterministic (nominal) design optimum and a robust (worst-case-across-conditions) design optimum for a generic UPD geometric design variable. Curves are illustrative of a well-established qualitative finding in the reviewed literature [17], [21], [23], not simulation output from a specific study.

#### F. Topology Optimization

Where Sections 4.1–4.2 treat damper shape as a small number of hand-chosen parameters, a more recent thread instead lets the damper's shape itself emerge from an optimization algorithm with comparatively few prior assumptions. Denimal, Elhaddad, Wong and Salles combined a moving-morphable-components shape parametrization with a global optimization algorithm to minimize resonance amplitude directly [18], and Denimal, Renson, Wong and Salles extended this approach in 2022 with an efficient global optimization algorithm specifically suited to the expensive, noisy objective function a nonlinear friction-contact forced-response evaluation presents [19], reporting that the resulting search identified efficient damper geometries not obviously anticipated by the hand-parametrized families of Section 4.1 within a comparatively small number of iterations. This topology-optimization thread remains, at the time of writing, the least mature of the four reviewed in this section, and — in common with every other

optimization study reviewed here — has been demonstrated only against room-temperature, single-condition contact parameter sets.

#### G. Emerging Multi-Physics Damper Concepts

A final, smaller literature treats the damper itself as a more active or multi-physics device rather than a passive friction wedge. Wu, Fan, Li and Zhao examined hybrid dampers combining a wavy dry-friction profile with piezoelectric elements [24], and Brinkmann, Hoffmann, Panning-von Scheidt and Stürer examined the transient resonance passage — rather than steady-state forced response alone — of a mistuned bladed disk with and without UPDs [25], connecting the steady-state optimization literature reviewed above to the run-up and run-down transients more representative of actual engine operation. Neither line of work is yet mature enough to appear in Table 2 alongside the geometry- and topology-optimization studies, but both indicate directions in which the field's optimization scope is actively broadening.

| Study (year) | Design variables | Objective | Method | Key finding |
|--------------|------------------|-----------|--------|-------------|
|--------------|------------------|-----------|--------|-------------|

|                                                     |                                |                                          |                                               |                                                       |
|-----------------------------------------------------|--------------------------------|------------------------------------------|-----------------------------------------------|-------------------------------------------------------|
| Panning, Sextro & Popp (2002) [2]                   | Contact angle symmetry         | Minimize resonance amplitude             | Parametric FE forced-response sweep           | Breaking symmetry increases achievable damping        |
| Gastaldi & Gola (2017) [16]                         | Asymmetric wedge geometry      | Screen favourable geometric regions      | Analytical pre-optimization                   | Cheap screening before full optimization              |
| Krack et al. (2014) [17]                            | Generic friction-damper design | Reliability under parametric uncertainty | Nonlinear normal modes + reliability search   | Frames design as reliability, not point, optimization |
| Yuan, Jones, Setchfield & Schwingshackl (2021) [21] | Damper geometry                | Robust peak-amplitude minimization       | Surrogate-assisted optimization               | Robust optimum differs from deterministic optimum     |
| Denimal, El-haddad, Wong & Salles (2021) [18]       | Damper topology (parametrized) | Minimize resonance amplitude             | Moving morphable components + gradient search | Topology framing recovers non-intuitive geometries    |
| Denimal, Renson, Wong & Salles (2022) [19]          | Damper topology                | Minimize resonance amplitude             | MMC + efficient global optimization           | Novel efficient geometries found in few iterations    |

TABLE II  
COMPARATIVE SUMMARY OF PRINCIPAL UNDER-PLATFORM-DAMPER DESIGN-OPTIMIZATION STUDIES DISCUSSED IN THIS REVIEW.

## V. COMPARATIVE SYNTHESIS

Read together, Sections 3 and 4 show a field whose experimental and design-optimization capability has matured substantially: dual-damper rigs now measure local contact force directly rather than inferring it from global blade response [6], [7]; identification workflows are standardized enough to support explicit uncertainty quantification rather than single-point parameter estimates [21]–[23]; and optimization has broadened from hand-parametrized geometry sweeps [2] through reliability-aware formulations [17] to topology optimization with comparatively few prior shape assumptions [18], [19]. What has not broadened, on the evidence reviewed here, is the operating envelope under which any of this experimental or optimization work has been conducted: every rig in Table 1 and every optimization study in Table 2 operates at or near room temperature and a single representative preload, with the partial exceptions of the high-temperature contact-parameter measurements of Schwingshackl et al. [10] and the rig-and-engine validation of Hartung et al. [11], neither of which feeds into a published UPD geometry-optimization study to date.

## VI. OPEN CHALLENGES AND FUTURE DIRECTIONS

### H. Thermally-Robust Damper Design

The single most consequential gap this review identifies follows directly from Section 3.3: because

contact parameters measured at room temperature are known to differ from their values at engine operating temperature [10], [11], a damper geometry identified as optimal — whether by the deterministic methods of Section 4.1, the reliability-aware methods of Section 4.2, or the topology-optimization methods of Section 4.3 — against room-temperature contact parameters is not guaranteed to remain close to optimal once evaluated against contact parameters representative of hot-section conditions. Figure 4's schematic distinction between a nominal and a robust optimum, established in the reviewed literature for parametric manufacturing and wear uncertainty [17], [21], [23], has not yet been extended to temperature as the source of parametric variation, despite temperature being, if anything, a larger and more systematic source of contact-parameter drift than the manufacturing tolerances that literature already treats.

### I. Coupling to Constitutive Model Development

A second, related gap is that the optimization literature reviewed in Section 4 is built almost entirely on tangential-only or externally-driven normal-load-variation contact laws; none of the studies in Table 2 optimizes against a coupled normal–tangential contact model of the kind reviewed in the authors' companion paper on contact constitutive modelling, despite that coupling having been shown, at room temperature, to materially affect resonance-amplitude prediction.

Whether an optimum identified under a coupled contact model differs from one identified under an uncoupled model — in the same qualitative sense that Fig. 4 illustrates for nominal versus robust optima — is an open question this review's authors are pursuing directly in ongoing work.

#### ***J. Toward Rotating, Multi-Blade, and Transient Validation***

Most rigs surveyed in Table 1 test a single blade or blade pair under static preload; the rotating, 48-blade rig of Usmanov et al. [12] is a rare exception, and the transient run-up analysis of Brinkmann et al. [25] a rare departure from steady-state forced response as the sole design metric. Extending experimental validation, and eventually optimization itself, toward rotating multi-blade assemblies under transient operating conditions — where mistuning, thermal transients, and centrifugal preload all vary simultaneously — remains a substantially larger undertaking than any single rig or optimization study reviewed here has yet attempted, and represents a natural long-term direction for the field once the room-temperature and thermally-extended single-blade problems addressed above are better resolved.

## **VII. CONCLUSION**

Under-platform damper design has progressed from parametric geometry sweeps against a single deterministic contact-parameter set to reliability-aware and topology-optimization formulations capable of accounting for manufacturing and wear uncertainty explicitly, underpinned throughout by an experimental test-rig and parameter-identification literature that has itself progressed from inferring damper behaviour indirectly through blade response to measuring local contact force and displacement directly. What this progression has not yet done is extend beyond the room-temperature, fixed-preload envelope in which essentially every rig and optimization study reviewed here has been conducted, despite independent evidence that the contact parameters this literature identifies and optimizes against are themselves temperature-dependent. Closing that gap — establishing whether, and how much, thermally-robust UPD design differs from the room-temperature-optimal designs

this literature has so carefully developed — is the specific contribution the present authors are pursuing in the research programme this review series introduces.

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