

An Approach for Engineering Tuning of PI-Controller with Dynamic Process from Second Order

Georgi Petrov Terziyski*

*(Department of Electrical engineering, electronics and automation, University of Food Technologies, Plovdiv, Bulgaria
Email: g_terziyski@uft-plovdiv.bg)

Abstract:

An approach is proposed for engineering adjustment of the PI-controller with a dynamic second order process. There is a proposal to solve the problem by solving the characteristic equation. As a result of the third row dynamic system analysis, the adjustment parameters of the PI-controller are calculated. The transitional processes of the closed system (process-controller) are dealt with by assignment and disturbance. For the transitional process by assignment, overshoot $\sigma = 20\%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 0% inaccuracy is observed in theory. Therefore, the proposed approach for engineering adjustment for a PI-controller with a second-order dynamic process is suitable for use in third-order dynamic systems analysis.

Keywords — PI-controller, Dynamic process from second order

I. INTRODUCTION

Third order dynamic systems can be obtained in the following cases:

- process two aperiodic links operating with I-controller;
- process two aperiodic links working with PI-controller;
- process two aperiodic links, working with PID-controller with perfect differentiation;
- process three aperiodic links operating with a P-controller;
- process three aperiodic links, working with a PD-controller with perfect differentiation;
- process oscillating link operating with PI-controller;
- process oscillating link operating with PID-controller with perfect differentiation and
- process oscillating link operating with perfect differentiation first-order PD-controller.

Problems with the adjustment of controllers in third order systems

Third order dynamic systems are often used in industrial automation systems for a variety of production processes, but due to their complexity, few authors have attempted to do theoretical

research on them. The complexity is that the roots of the characteristic equation of the closed ACS (automatic control system) is three, and it is not clear how the third real root influences the stability of the system, and hence the indicators of quality of the transitional processes.

II. POSSIBLE OPTIONS FOR SOLUTION OF THE ASSIGNED TASK

In analyzing third order dynamic systems, the determination of dependencies between quality indicators and system parameters is considerably more complicated. One of the possible options for solving the task is through the use of Prof. Vishnegradski's diagram (Naplatanov, 1976). The diagram he suggests allows to judge not only sustainability but also some key quality indicators. In the study of dynamic systems of third order, he concluded that the nature of the transitional process can be determined without solving the characteristic equation of the system. For this purpose, it is sufficient for hyperbola built according to its parameters - X and Y to be supplemented with three auxiliary curves (Naplatanov, 1976). He has given an original word formulation of his criterion, which states: To be a dynamic third-order system

sustainable, it is both necessary and sufficient to fulfill the following two conditions: 1. All the coefficients of the characteristic equation must be positive; 2. The average output minus the output of the final coefficients of the characteristic equation of the system must be positive. Failure to comply with these conditions will make the third order dynamic system unstable or at the limit of resistance.

Other possible options for solving this task are by using Ziegler & Nicols first method, Koppelovich's nomograms and nomograms given in (Hinov et al., 1978). These are methods for determining the parameters for adjusting the controllers by known data for the transitional characteristic of the control plant (Dragotinov et al., 2008), (Terziyski et al., 2015).

The purpose of this article is to offer an engineering adjustment for a proportional-integral PI-controller with a dynamic second order process.

III. PROPOSAL FOR SOLVING THE PROBLEM BY SOLVING THE CHARACTERISTIC EQUATION

Figure 1 shows the structural diagram of a ACS comprising a second order process (two aperiodic units) and a PI-controller.

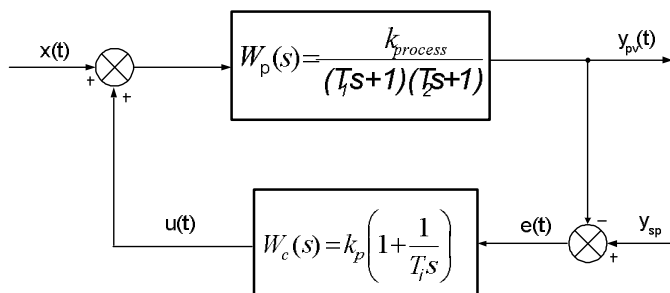


Fig.1. A system with a second order process and a PI-controller

The transfer function of the closed system (fig.1) regarding the assignment is the type

$$W_{sp}(s) = \frac{Y_{pv}(s)}{Y_{sp}(s)} = \frac{W_p(s) \cdot W_c(s)}{1 + W_p(s) \cdot W_c(s)} = \frac{\frac{k_{process}}{(T_1s+1)(T_2s+1)} \cdot k_p \left(\frac{T_i s + 1}{T_i s} \right)}{1 + \frac{k_{process}}{(T_1s+1)(T_2s+1)} \cdot k_p \left(\frac{T_i s + 1}{T_i s} \right)} = \frac{T_i s + 1}{\frac{T_1 T_2 T_i}{k_{process} k_p} s^3 + \frac{(T_1 + T_2) T_i}{k_{process} k_p} s^2 + \frac{(1 + k_{process} k_p) T_i}{k_{process} k_p} s + 1} \quad (1)$$

, where: $W_p(s)$ – is the transfer function of the control plant; $W_c(s)$ – is the transfer function of the controller; $k_{process}$ – is the coefficient of the control process; T_1 and T_2 are the time constants of the control process; k_p – is the proportionality factor of the controller; T_i – is the integration time constant of the controller; $y_{pv}(t)$ – is the process variable; y_{sp} – is the set point; $e(t)$ – is the adjustment error; $u(t)$ – is the control variable; $x(t)$ – is the disturbance variable.

The transfer function of the closed system (fig. 1) regarding the disturbance is the type

$$W_x(s) = \frac{Y_{pv}(s)}{X(s)} = \frac{W_p(s)}{1 + W_p(s) \cdot W_c(s)} = \frac{\frac{k_{process}}{(T_1s+1)(T_2s+1)}}{1 + \frac{k_{process}}{(T_1s+1)(T_2s+1)} \cdot k_p \left(\frac{T_i s + 1}{T_i s} \right)} = \frac{T_i}{k_p} \cdot \frac{s}{\frac{T_1 T_2 T_i}{k_{process} k_p} s^3 + \frac{(T_1 + T_2) T_i}{k_{process} k_p} s^2 + \frac{(1 + k_{process} k_p) T_i}{k_{process} k_p} s + 1} \quad (2)$$

propose that the analysis of the third-order dynamic system be carried out with a successively connected oscillating and aperiodic link, i.

$$W(s) = \frac{1}{T_o^2 s^2 + 2\xi T_o s + 1} \cdot \frac{1}{T s + 1} \quad (3)$$

Assuming that the time constant of the aperiodic link (first order low pass filter) is equal to the time constant of the oscillating link, i. $T = T_o$ is obtained

$$W(s) = \frac{1}{T_o^2 s^2 + 2\xi T_o s + 1} \cdot \frac{1}{T_o s + 1}. \quad (4)$$

For the polynomial in the denominator of expression (4) the characteristic equation is obtained

$$(T_o^2 s^2 + 2\xi T_o s + 1)(T_o s + 1) = T_o^3 s^3 + (2\xi + 1)T_o^2 s^2 + (2\xi + 1)T_o s + 1. \quad (5)$$

If we equal the corresponding coefficients in front of s^3 , s^2 etc. from the characteristic equation (5) to the coefficients of s^3 , s^2 etc. of the polynomial in the denominator of expression (1), the transfer function of the closed system regarding the assignment will have the final appearance

$$W_{sp}(s) = k_{sp} \cdot \frac{T_i s + 1}{T_o^3 s^3 + (2\xi + 1)T_o^2 s^2 + (2\xi + 1)T_o s + 1}, \quad (6)$$

where $k_{sp} = 1$ is called a coefficient of the system assignment.

The transfer function of the closed disturbance system will have the final appearance

$$W_x(s) = k_x \cdot \frac{T_o s}{T_o^3 s^3 + (2\xi + 1)T_o^2 s^2 + (2\xi + 1)T_o s + 1}, \quad (7)$$

$$\text{where } k_x = \frac{T_i}{k_p} \cdot \frac{1}{T_o} = \frac{T_i}{k_p} \sqrt[3]{\frac{k_{process} k_p}{T_1 T_2 T_i}} = \sqrt[3]{\frac{T_i k_{process}}{k_p T_1 T_2}}$$

is called the system disturbance factor.

By comparing the coefficients in front of the corresponding degrees of s in the polynomials of expressions (1) and (6), dependencies between the parameters of the transition process and the parameters of the system can be determined. Equivalent time constant is

$$T_o = \sqrt[3]{\frac{T_1 T_2 T_i}{k_{process} k_p}}. \quad (8)$$

Similarly, the attenuation coefficient ξ is determined. For it two expressions of s^2 and s of (6) are obtained, ie.

The first expression that can be determined ξ is

$$(2\xi + 1)T_o^2 = \frac{(T_1 + T_2)T_i}{k_{process} k_p}. \quad (9)$$

If we only express ξ we obtained

$$\xi = \frac{(T_1 + T_2)T_i - T_o^2 k_{process} k_p}{2T_o^2 k_{process} k_p}. \quad (10)$$

The second expression from which can be determined ξ is

$$(2\xi + 1)T_o = \frac{(1 + k_{process} k_p)T_i}{k_{process} k_p}. \quad (11)$$

If we express only ξ it is obtained

$$\xi = \frac{(1 + k_{plant} k_p)T_i - T_o k_{process} k_p}{2T_o k_{process} k_p}. \quad (12)$$

If the expressions (9) and (11) are divided into one another, it is obtained

$$T_o = \frac{T_1 + T_2}{1 + k_{process} k_p}. \quad (13)$$

If the expressions (10) and (12) are equal to one another, i.

$$\frac{(T_1 + T_2)T_i - T_o^2 k_{process} k_p}{2T_o^2 k_{process} k_p} = \frac{(1 + k_{plant} k_p)T_i - T_o k_{process} k_p}{2T_o k_{process} k_p} \quad (14)$$

and then simplified, an expression of the type (13) is obtained. This confirms that the expressions (8) and (13) are equal, i

$$T_o = \sqrt[3]{\frac{T_1 T_2 T_i}{k_{process} k_p}} = \frac{T_1 + T_2}{1 + k_{process} k_p}. \quad (15)$$

If an expression (15) is solved regarding the time constant of integration T_i , it is obtained

$$T_i = \frac{(T_1 + T_2)^3 k_{process} k_p}{(1 + k_{process} k_p)^3 T_1 T_2}. \quad (16)$$

The proportionality coefficient of the controller k_p can be determined by substituting expressions (16) and (13) into expression (11), ie.

$$k_p = \left[\frac{(T_1 + T_2)^2}{(2\xi + 1)T_1 T_2} - 1 \right] / k_{process} \quad (17)$$

Example: Transitional process of process is given with two aperiodic links. The following algorithm performs the following:

1. Take the transitional process of the process that is smooth and normalizing.

2. Since the process model is of second order (two consecutively connected aperiodic links with equal time constants) - the transitional characteristic is monotone with a transient delay, it is chosen to

approximate the method of Ormans (Badev, 2013). After the approximation, it is determined: $k_{plant} = 1$, $T_1 = T_2 = 20$ sec.

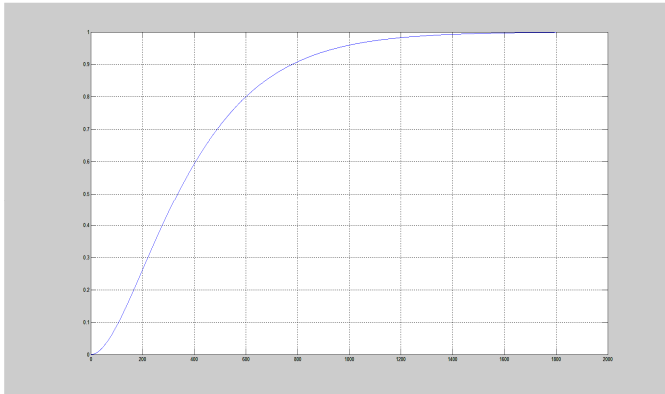


Fig.2. Transitional process of control process

3. By the expressions (16) and (17), the adjustment parameters of the PI-controller are calculated.

$$k_p = 1.092$$

$$T_i = 19.083 \text{ sec}$$

4. By the expression (12) the damping factor ξ is calculated and approximately what is the value of the overshoot σ from [3]

$$\xi = \frac{(1 + k_{plant}k_p)T_i - T_0k_{process}k_p}{2T_0k_{process}k_p} = 0.456$$

$$\sigma^2 = \exp\left(-\frac{\xi}{\sqrt{1-\xi^2}} \cdot 2\pi\right) = 0.04 \quad \text{or} \quad \text{only}$$

$$\sigma = 20 \%$$

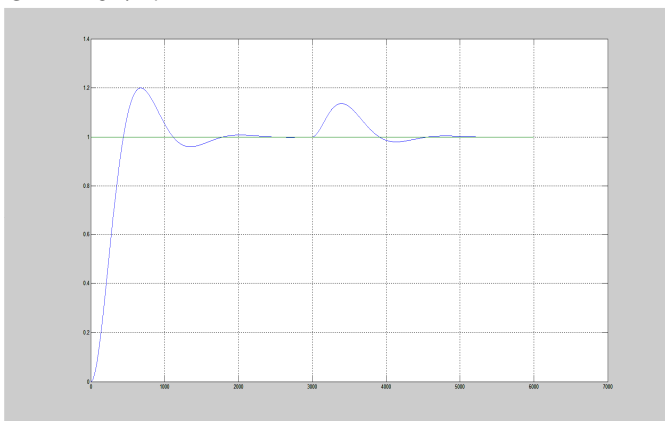


Fig.3. Transitional processes by assignment and by disturbance

5. Determine the maximum dynamic deviation y_1 in the expression given in [3]

$$y_1 = \exp\left(-\frac{\xi}{\sqrt{1-\xi^2}}\right) = 0.6.$$

6. If any of the above two parameters does not meet the prerequisites for quality, adjust the controller.

The transitional processes of the closed system (fig. 1) by assignment and by disturbance are shown in fig. 3. For the transitional process by assignment, overshoot $\sigma = 20 \%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 0 % inaccuracy is observed in theory. Therefore, the proposed sub-process for engineering adjustment of a PI-controller with a dynamic process with two aperiodic links is suitable for use in the analysis of third-order dynamic systems.

Conclusion

An approach is proposed for engineering adjustment of the PI-controller with a dynamic second-order process (two aperiodic links with equal time constants). There is a proposal to solve the problem by solving the characteristic equation. As a result of the analysis of the third order dynamic system, the adjustment parameters of the PI-controller are calculated.

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