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Experimental Arrangement for Performing Spatial Filtering of a Wavefront

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Abstract:

E In some technical applications, it is necessary to filter certain spatial frequency components of a wavefront. In this publication, the experimental setup for performing spatial filtering is visualized. It uses a converging lens, in whose focal plane an opaque obstacle is placed at the positions corresponding to the frequencies to be blocked, and transparent in the rest. Subsequently, a new lens with the same focal length as the first lens will be placed, so that the original wavefront with the filtered spatial frequencies is reproduced at its output. The pair of lenses is referred to as *afocal*, and the described operation is known as *spatial filtering*.

Keywords **— spatial filtering, pinhole, Fourier transform, collimator, Ronchi gratings**

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I. INTRODUCTION

Figure 1 shows a coherent optical computer, which allows the insertion of masks or filters in the transform plane, partially or completely blocking certain spatial frequencies, preventing them from reaching the image. *This process is called spatial filtering.*

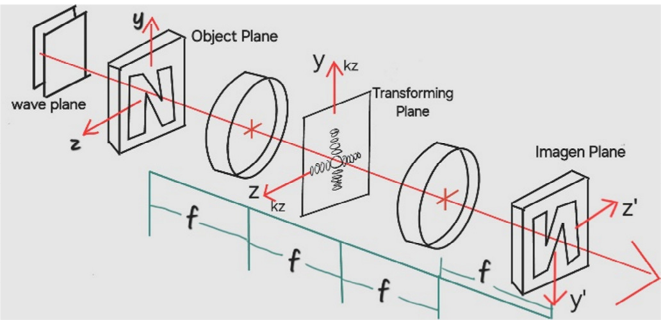


Figure 1. Obtaining the Fourier Transform of an Object f(x).

When an object is placed at the focal distance of a lens and illuminated with a collimated light beam, the image of the object in the back focal plane will be the Fourier transform of the object. Considering an object represented by the function $f(x)$ illuminated with a collimated beam, the propagation to the back focal plane is performed as shown in Figure 1. Calculating the propagation up to just before the first surface of the lens, that is in the y-plane, the function g^- is obtained.

$$g^-(y) = A\frac{e^{i\frac{2\pi}{\lambda}f}}{i\lambda f}\int_{-\infty}^{+\infty} f(x)e^{i\frac{\pi}{\lambda f}(x-y)^2}dx \qquad (1)$$

When the wave propagates within the lens, this is equivalent to a block multiplication, and the function g^- is multiplied by the transmittance of the lens, resulting in the function g^+ in the following way:

$$g^{+}(y) = A\frac{e^{i\frac{2\pi}{\lambda}f}}{i\lambda f}\int_{-\infty}^{+\infty} f(x)e^{i\frac{\pi}{\lambda f}(x-y)^2}dx\, e^{-i\frac{\pi}{\lambda f}y^2} \qquad (2)$$

And finally, propagating again to the back focal plane, the Fourier transform is obtained, given by:

$$\Psi(\xi) = A\frac{e^{i\frac{2\pi}{\lambda}f}}{i\lambda f}\int_{-\infty}^{+\infty}\left[A\frac{e^{i\frac{2\pi}{\lambda}f}}{i\lambda f}\int_{-\infty}^{+\infty} f(x)e^{i\frac{\pi}{\lambda f}(x-y)^2}dx * e^{-i\frac{\pi}{\lambda f}y^2}\right]e^{i\frac{\pi}{\lambda f}(y-\xi)^2}dy \qquad (3)$$

Simplifying the function:

$$\Psi(\xi) = -A^2\frac{e^{i\frac{4\pi}{\lambda}f}}{\lambda^2 f^2}\, e^{i\frac{\pi}{\lambda f}\xi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x)\, e^{i\frac{\pi}{\lambda f}x^2}\, e^{i\frac{\pi}{\lambda f}y^2} * e^{-i*2\pi\frac{(x+\xi)}{\lambda f}y}\, dx\, dy \qquad (4)$$

$$\Psi(\xi) = -A^2\frac{e^{i\frac{4\pi}{\lambda}f}}{\lambda^2 f^2}\, e^{i\frac{\pi}{\lambda f}\xi^2}\int_{-\infty}^{+\infty} f(x)\, e^{i\frac{\pi}{\lambda f}x^2}\int_{-\infty}^{+\infty} e^{-\pi\left[\sqrt{\frac{1}{i\lambda f}}y\right]^2} * e^{-i*2\pi\frac{(x+\xi)}{\lambda f}y}\, dy\, dx \qquad (5)$$

And by evaluating the nested integral, the following is obtained:

$$\boldsymbol{\Psi(\xi) = -A^2\frac{\sqrt{i\lambda f}}{\lambda^2 f^2}\, e^{i\frac{4\pi}{\lambda}f}\, e^{i\frac{2\pi}{\lambda f}\xi^2}\int_{-\infty}^{+\infty} f(x)\, e^{i\frac{\pi}{\lambda f}x^2} * e^{-i*\pi\frac{(x^2+2\xi x)}{\lambda f}}\, dx} \qquad (6)$$

$$\Psi(\xi) = -A^2\frac{\sqrt{i\lambda f}}{\lambda^2 f^2}\, e^{i\frac{4\pi}{\lambda}f}\, e^{i\frac{2\pi}{\lambda f}\xi^2}\int_{-\infty}^{+\infty} f(x)\, e^{-i*2\pi\frac{\xi}{\lambda f}x}\, dx \qquad (7)$$

And here it is clearer that the resulting integral is the Fourier transform of the function f(x), that is, of the object, so that:

$$\Psi(\xi) = -A^2\frac{\sqrt{i\lambda f}}{\lambda^2 f^2}\, e^{i\frac{4\pi}{\lambda}f}\, e^{i\frac{2\pi}{\lambda f}\xi^2}\, F\left(\frac{x}{\lambda f}\right) \qquad (8)$$

II. DEVELOPMENT OF THE EXPERIMENT

Firstly, all the elements and materials were located to proceed with the setup of our experiment. The materials provided for carrying out the practice are:

- He-Ne Laser 10 mW (632.8 nm)
- Spatial filter, with 20X microscope objective and small aperture (pinhole)
- Positive lens (collimator) with f = 10.7 cm
- Object to process two small pitch Ronchi gratings
- Two positive lenses with a focal length of f=85 cm
- Aluminium foil to make filters
- Digital camera
- 10x microscope objective
- 1 mirror with a diameter of 3 inches
- 1 mirror with a diameter of 5 cm
- White light lamp.

Having all the elements and materials, we proceeded to set up the experimental arrangement shown in Figure 2:

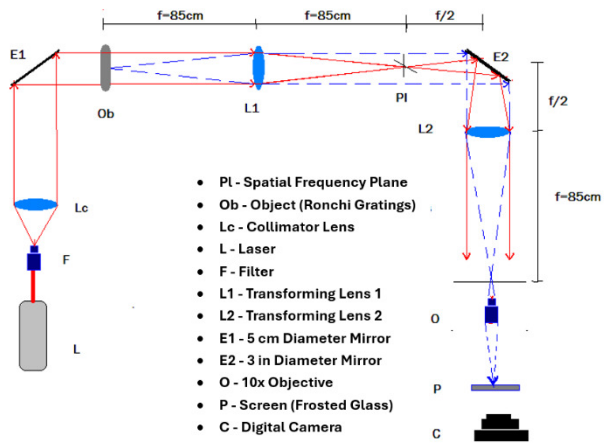
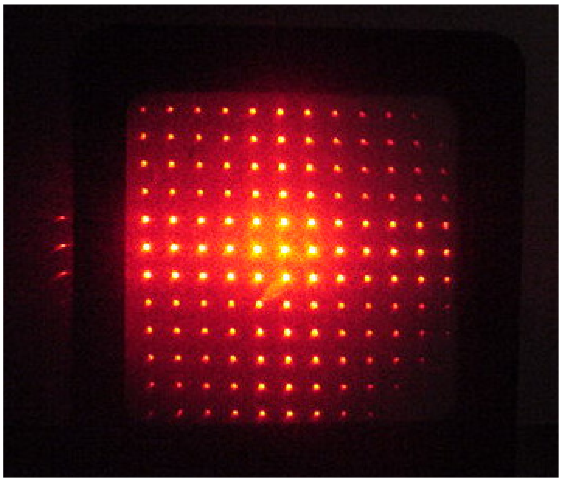


Figure 2. Experimental Arrangement for Performing Spatial Filtering.

During the setup of the experimental arrangement, the following considerations or observations were made:

- A 20x objective was used along with the pinhole for spatial filtering (F); the reason for using a 20x objective was to avoid expanding the beam too much.

- A collimating lens (Lc) with a focal length of 10.7 cm was used. Having a small focal length compared to the lenses with f = 85 cm allowed us to save space on our workbench.
- The mirror with a diameter of 5 cm was mounted as (E1) because the beam incident on the mirror is collimated, and the beam spot radius does not vary; the diameter of the spot was smaller than that of the 5 cm mirror.
- The object (Ob) consisted of two Ronchi gratings oriented at 90º to each other, which served as the input plane for the optical processor.
- The lens (L1) was placed; this lens is responsible for performing the Fourier transform of the spatial light distribution in front of it, in this case, the object. In the focal plane behind this lens, the spatial frequency plane is located, where the Fourier transform is also found. Various filters made from aluminum foil will be placed in this same plane to allow certain spatial frequencies to pass; this will be explored further later.
- The 3-inch diameter mirror (E2) was mounted because after the back focal point of lens (L1), the object in the Fourier space is amplified.
- The lens (L2) also acts as a transforming lens, but in this case, it performs the inverse Fourier transform of the transform in front of it. Therefore, after the back focal point of lens (L2), we obtain the object again, which can be viewed on a screen.
- If the object cannot be clearly observed, it is necessary to insert an objective at a small distance after the back focal point of lens (L2). In our case, a 10x objective was incorporated to amplify the image of the object tenfold.
- To find the observation plane where the screen was placed, the laser beam was replaced with a white light source, and the screen was moved out from the objective until a good focus was achieved. Once the observation plane was found, the laser beam was reintroduced.



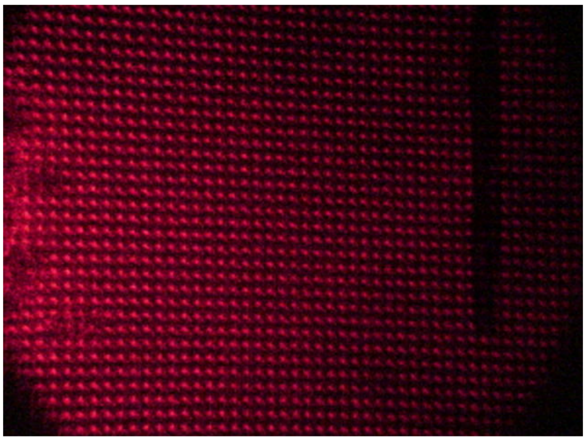
Photograph 1. Spatial Frequency Plane: the Fourier Transform is located in this same plane.

In this way, the setup of the experimental arrangement was completed. The next step was the fabrication of various filters using aluminum foil, which were placed in the spatial frequency plane, that is, at the back focal point of lens (L1). The purpose of these filters is to allow certain spatial frequencies to pass, which causes a change in intensity or phase that can be observed in the observation plane.

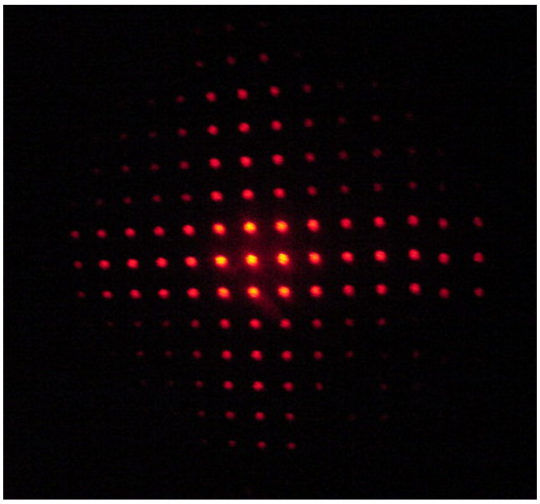
Therefore, various spatial filters were fabricated and placed in the Fourier transform plane. They were placed one by one to observe the differences or effects caused by each filter made. Photographs of each of the filters used were taken, which clearly demonstrate the changes caused by these filters. These photographs will be presented in the following section of this document.

III. RESULTS

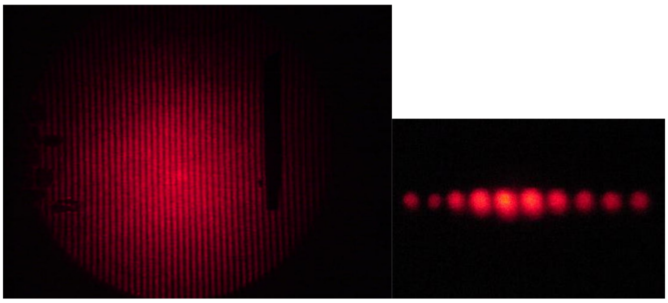
The following series of photographs were obtained, showing on the right the filter used placed in the Fourier transform plane, and on the left the image of the object in the observation plane captured by the digital camera:



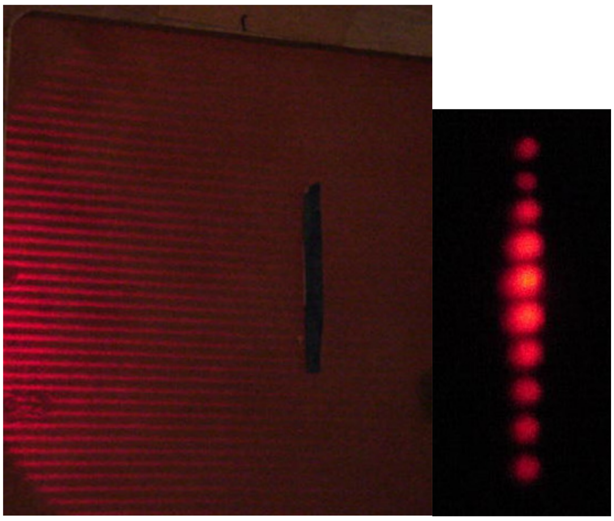
Photograph 2. Image of the object in the observation plane without a filter.



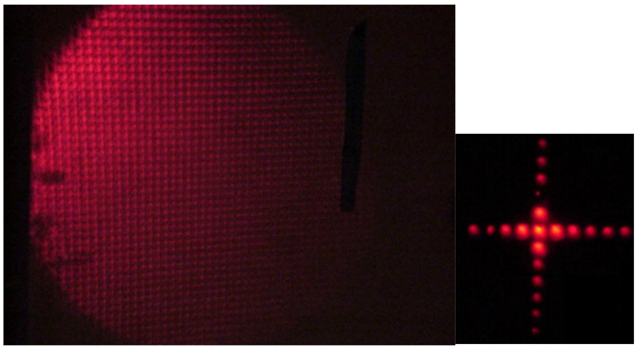
Photograph 3. Plane where the Fourier Transform is located (without filter).



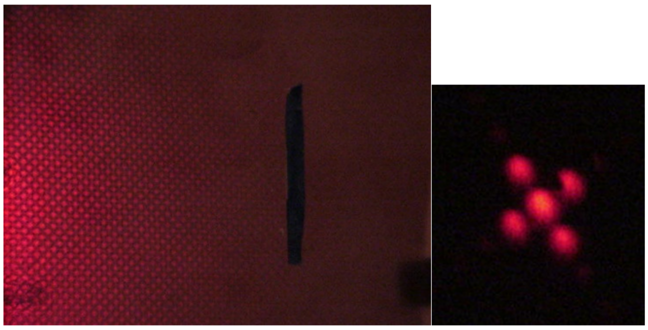
Photograph 4. Image of the object in the observation plane with a filter that allows the central horizontal to pass.



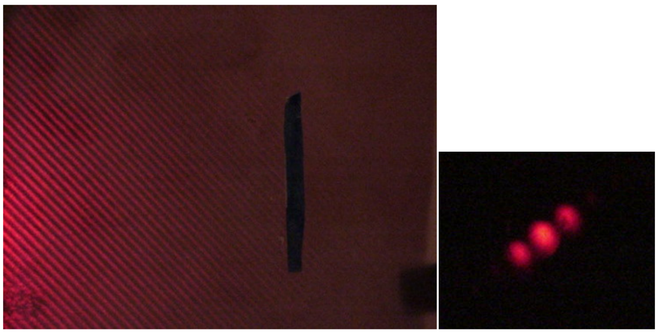
Photograph 5. Image of the object in the observation plane with a filter that allows only the central vertical to pass.



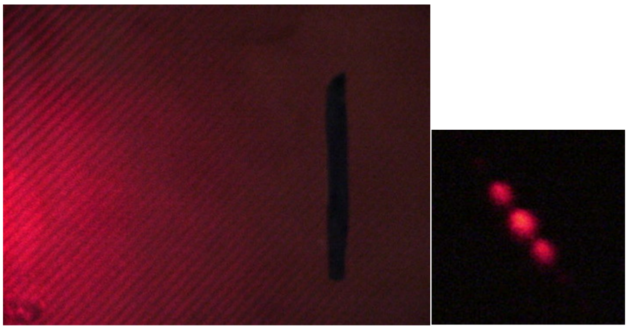
Photograph 6. Image of the object in the observation plane with a filter that allows only the central cross to pass.



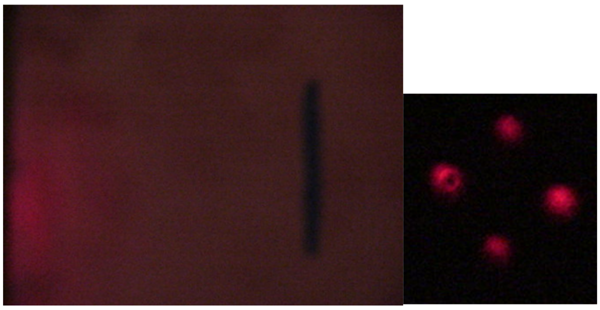
Photograph 7. Image of the object in the observation plane with a filter that allows only the central dot to pass.



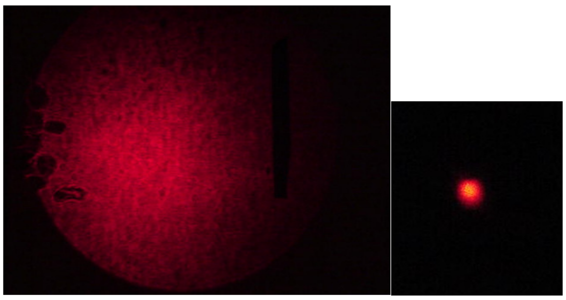
Photograph 8. Image of the object in the observation plane with a filter that allows only an upward diagonal to the right to pass.



Photograph 9. Image of the object in the observation plane with a filter that allows only an upward diagonal to the left to pass.



Photograph 10. Image of the object in the observation plane with a filter that allows only order 2 to pass.



Photograph 11. Image of the object in the observation plane with a filter that allows only order zero to pass.

IV. CONCLUSIONS

During the experiment, we were able to conclude the following:

- It was possible to verify and physically observe one of the properties of lenses, which is to perform the Fourier transform of a spatial light distribution in front of it. With the help of another lens, the inverse transform of the Fourier transform can be achieved.

- The image of the object viewed on the screen follows a behavioral pattern with the filters. In the observation plane, stripes tend to form perpendicular to the direction allowed by the filter; that is, if a vertical filter is used, vertical stripes appear in the observation plane. If a horizontal filter is used, vertical stripes are formed. If both vertical and horizontal filters are used, squares are formed in the observation plane. This behavioral pattern is easily observed in the photographs presented.

- In the experimental arrangement, it was confirmed that when the light emitted by the object passes through the transformation lens, its Fourier transform is performed. Then, when it passes through the other transformation lens, the inverse transform is performed, and the image of the object is recovered, which can be viewed on a screen, with the only difference being that the output image is inverted compared to the input image.

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