

## STUDY OF MAGNETOHYDRODYNAMICS (MHD) FLOW BY MATHEMATICAL METHODS

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### ABSTRACT

Magnetohydrodynamics (MHD) is the marriage of hydrodynamics to electromagnetism. Magnetohydrodynamics (MHD) is the study of the magnetic properties and behaviour of electrically conducting fluids. In this paper, I explore solutions of MHD flow; the governing fluid flow mathematical equations are utilized to study magnetic and electrical properties. The electrical and magnetic parameters are applied to analyze magnetic and electrical characteristics of MHD materials.

**Key words:** Applied Mathematics, MHD, Magnetic properties, Electrical properties.

### 1. INTRODUCTION

Magnetohydrodynamics (MHD; also called magneto-fluid dynamics or hydromagnetics) is the study of the magnetic properties and behaviour of electrically conducting fluids. Examples of such magnetofluids include plasmas, liquid metals, salt water, and electrolytes. The word magnetohydrodynamics is derived from magneto- meaning magnetic field, hydro- meaning water, and dynamics meaning movement. Problems involving MHD boundary layer flow of a fluid of varying viscosity subject to thermal radiation and Newtonian heating are of great importance to engineering and industrial applications due to their vast applications in thermal insulation, heat exchangers, geothermal reservoir, cooling of nuclear reactors, enhanced oil recovery, solar energy collection, designing of cooling systems for electronic devices, packed-bed catalytic reactors etc. Heat transfer by thermal radiation is also of great significance to engineering processes occurring at high temperatures and it is important in the designing of equipment in Nuclear power plants, gas turbines, and propulsion devices for aircraft, missiles, satellites and space vehicles.

The study of heat and mass transfer in the magneto hydrodynamic (MHD) flow has been considered by many Workers due to its applications in various areas. MHD flow plays vital roles in the field of medicine, for example in cancer tumor treatment causing hypothermia, reducing bleeding in serious injuries and magnetic resonance imaging [1]. The main applications of MHD flows include cooling of nuclear reactors, combustion modeling, and geophysics and plasma analyses. These applications are affected quantitatively by heat transfer enhancement [2]. The application of MHD flows induced by a magnetic field is widely used in semiconductor industries. A study on the MHD viscous fluid flow passing through a moving surface was conducted by Fang et al. [3]. Countless studies have been reported on MHD flows under enormous physical conditions.

The present work investigates the electrical and magnetic characteristics of MHD fluid flow problems by solving analytically the governing nonlinear differential equations using suitable asymptotic methods. The magnetization and electrical conductivity of magneto hydrodynamics (MHD) are studied by mathematical formulation of the problem solving.

## **2. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION**

We consider the flow of an incompressible Newtonian fluid through a rectangular miniature cylinder with the z-axis being the pivotal way. The differential conditions overseeing the flow incorporate the coherence condition and the Navier–Stirs conditions as follows

$$\frac{\partial U_j}{\partial x_j} = 0 \tag{1}$$

$$p \left( \frac{\partial y_j}{\partial x_j} + y_i \frac{\partial y_j}{\partial x_i} \right) = - \frac{\partial p}{\partial x_j} + \mu \frac{\partial^2 y}{\partial x_i \partial x_j} + p g_i, [i = 1,2,3; j = 1,2,3] \tag{2}$$

Where  $p$  and  $y_i$  are respectively the fluid pressure and velocity vector,  $g_j$  is the gravitational acceleration,  $\rho$  and  $\mu$  are respectively the fluid density and viscosity and  $x_i$  denotes coordinates. As the flow is axially symmetric, the velocity components in the x and y directions vanish, namely

$y_1 = y_x = 0$  and  $y_2 = y_y = 0$ . Thus the continuity equation (1) becomes

$$\frac{\partial y_3}{\partial x_3} = \frac{\partial y_z}{\partial z} = 0$$

Which gives rise to  $y_3 = v = v(x, y, t)$

As the flow is horizontal,  $g_3 = g_z = 0$ , and hence eq. (2) becomes

$$p \left( \frac{\partial v}{\partial t} \right) = \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\partial p}{\partial z}$$

We consider the liquid stream driven by the weight field with a pressure angle  $q(t)$  which can be communicated by a Fourier arrangement, to be specific

$$\frac{\partial p}{\partial z} = q(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)] \quad (3)$$

As the issue is pivotally symmetric, we just need to think about a quadrant of the cross-segment in the calculation.

By applying the Navier slip conditions in the primary quadrant of the rectangular cross segment, as in the paper by Wu *et al.*, (2008) [4] for each time  $t$ , we have

$$\begin{aligned} \frac{\partial v}{\partial y}(x, 0) &= 0; 0 \leq x \leq a \\ \frac{\partial v}{\partial x}(0, y) &= 0; 0 \leq y \leq b \\ v(x, b) + l \frac{\partial v}{\partial y}(x, b) &= 0; 0 \leq x \leq a \\ v(a, y) + l \frac{\partial v}{\partial x}(a, y) &= 0; 0 \leq y \leq b \end{aligned} \quad (4)$$

### 3. EQUATIONS GOVERNING MHD FLUID FLOW

If the flowing fluid happens to be in a magnetic field, then the equations governing such a flow are Navier-Stokes (momentum) equation and Maxwell's equations of electro magnetism. The Maxwell's equations of electro magnetism [5] are:

$$\nabla \times B = \mu_0 j \quad (5)$$

$$\frac{\partial B}{\partial t} = -\nabla \times E \quad (6)$$

$$j = \sigma (E + \nabla \times B) \quad (7)$$

Equations (5), (6) and (7) are the Ampere's law, the Faraday's law and the Ohm's law respectively. Here,  $\sigma$  is magnetic permeability,  $\sigma$  is the electrical conductivity of the fluid,  $j$  represents electric current density,  $E$  stands for electric field and  $B$  is the magnetic field.

These differential equations governing the flow should be solved simultaneously, either analytically or numerically.

#### **4. IMPORTANCE OF THE STUDY**

Problems involving MHD boundary layer flow of a fluid of varying viscosity subject to thermal radiation and Newtonian heating are of great importance to engineering and industrial applications due to their vast applications in thermal insulation, heat exchangers, geothermal reservoir, cooling of nuclear reactors, enhanced oil recovery, solar energy collection, designing of cooling systems for electronic devices, packed-bed catalytic reactors etc [6]. Heat transfer by thermal radiation is also of great significance to engineering processes occurring at high temperatures and it is important in the designing of equipment in Nuclear power plants, gas turbines, and propulsion devices for aircraft, missiles, satellites and space vehicles.

#### **CONCLUSION**

For the problems of the Magnetohydrodynamics (MHD) flow over the electric and magnetic field is studied and concluded that the both electric and magnetic field are both determining factors of the flow field. The governing fluid flow mathematical equations are utilized to study magnetic and electrical properties of MHD. The electrical and magnetic parameters are applied to analyze magnetic and electrical characteristics of MHD materials.

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