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GENERALIZED PYTHAGOREAN FUZZY CLOSED SETS

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Abstract:

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1. INTRODUCTION

In 1965,fuzzy set theory first introduced by Zadeh [13].Fuzzy set theory was characterized by amembershipfunctionwhichassignstoeachtarge tamembershipvaluerangingbetween0and1. In 1968, the concept of fuzzy topological space was introduced by Chang [2]. Also generalizedsome basic notions of topology such as open set, closed set, continuity and compactness to fuzzytopologicalspaces.Atanassov[1]introduce dtheconceptofintuitionisticfuzzysets.Anintrodu ctionto intuitionistic fuzzy topological spaces was given by Coker [3] in 1997. Yager proposed another classof non-standard fuzzy sets, called Pythagorean fuzzy sets. The concept and notions of Pythagoreanfuzzytopologicalspaceswasintrodu cedbyMuratOlgun,MehmetUnverandSeyhmus Yardimici [6].In 2020,Naeem et.al [6].studied Pythagorean m-polar fuzzy topology with TOPSIS approach inexploringmosteffectualmethodforcuringfrom COVID-19 in 2020. TahaYasinOzturkandAdemYolcu [11] introduced some operations such as Pythagorean fuzzy interior, closure boundary on Pythagoreanfuzzy topological spaces.Also, Pythagorean fuzzy open (closed) functions and Pythagorean fuzzyhomeomorphismareintroducedandtheirba sicpropertiesareinvestigatedin 2020.

2. PRELIMINARIES

Definition 2.1. Let X be the non-empty universe of discourse.A fuzzy set A in X,$A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A: X \to [0, 1]$ is the membership function of the fuzzy setA;$\mu_A(x) \in [0,1]$isthemembershipof$x \in X$inA

Definition 2.2. Let X be the non-empty universe of discourse. An Intuitionistic fuzzyset (IFS) A in X is given by A= $\{x, \mu_A(x), \nu_A(x) : x \in X\}$ where the functions$\mu_A(x) \in [0, 1]$and$\nu_A(x) \in [0,1]$denotethedegreeofmembership anddegreeofnon- membershipofeachelement$x \in X$to the set A, respectively, and$0 \le \mu_A(x) + \nu_A(x) \le 1$for each$x \in X$ .The degree ofindeterminacy$I_A = 1 - (\mu_A(x) + \nu_A(x))$foreach $x \in X$.

Definition 2.3. Let (X, T) be an intuitionistic fuzzy topological space. An intuitionisticfuzzysetAin(X,T) issaidtobegeneralizedintuitionisticfuzzyclosed (inshortlyGIF-closed)if$IFcl(A) \subseteq G$whenever$A \subseteq G$and G is intuitionistic fuzzy open.The complement of aGIF-closedsetisGIF-open.

Definition 2.4. Let X be the non-empty universe of discourse. A Pythagorean fuzzy set(PFS)PinXisgivenbyP= $\{\langle x, \mu_P(x), \upsilon_P(x)\rangle : x \in X\}$ wherethefunctions $\mu_P(x) \in[0,1]$and $\upsilon_P(x)\in[0,1]$denotethedegreeofmembershipand degreeofnon- membershipofeachelement $x\in X$tothesetP,respectively,with the condition that $0 \le \mu_P^2(x) + \nu_P^2(x) \le 1$ foreach$x\in X$.Thedegreeofindeterminacy $I_P = \sqrt{1 - (\mu_P^2(x) + (\upsilon_P^2(x))}$foreach $x\in X$.

Definition2.5. Let$P_1=\{\langle x, \mu_{P_1}(x), \upsilon_{P_1}(x)\rangle : x \in X\}andP_2=\{\langle x, \mu_{P_2}(x), \upsilon_{P_2}(x)\rangle : x \in X\}$betwoPythagoreanfuzzysetsoverX.Then,

1. thePythagoreanfuzzycomplementof P_1isdefinedby
$P_1^c = \{\langle x, \upsilon_P(x), \mu_P(x)\rangle : x \in X\}$
2. thePythagoreanfuzzyintersectionofP_1andP_2isdefinedby
$P_1 \cap P_2 = \{\langle x, min\{\mu_{P_1}(x), \mu_{P_2}(x)\}, \max\{\upsilon_{P_1}(x), \upsilon_{P_2}(x)\}\rangle : x \in X\}$
3. thePythagoreanfuzzyunionofP_1andP_2isdefined by $P_1 \cup P_2 = \{\langle x, min\{\mu_{P_1}(x), \mu_{P_2}(x)\}, \max\{\upsilon_{P_1}(x), \upsilon_{P_2}(x)\}\rangle : x \in X\}$
4. wesayP_1isaPythagoreanfuzzysubsetofP_2andwewrite$P_1\subseteq P_2$if
$\mu_{P_1}(x) \le \mu_{P_2}(x)$ and $\upsilon_{P_1}(x) \ge \upsilon_{P_2}(x)$foreach$x\in X$,
5. $0_X=\{\langle x,0,1\rangle\}, x \in X$ }and$1_X=\{\langle x,1,0\rangle x \in X\}$.

Definition2.6. Let $(X, \tau)_P$be an Pythagorean Fuzzy topological space andP= $\{\langle x, \mu_P(x), \upsilon_P(x)\rangle : x \in X\}$ beaPythagoreanfuzzysetoverX.ThenthePythagoreanfuzzy interior,PythagoreanfuzzyclosureandPythagoreanfuzzyboundaryofParedefinedby;

a. $int\ (P)=\cup\{G: G is a PFOS in X and G\subseteq P\}$
b. $cl\ (P)=\cap\{K:$ KisaPFCSinXand $P\subseteq K\}$
c. $Fr\ (P)=cl\ (P)\cap cl\ (P^c)$

Remark2.7. Itisclearthat,

a. int(P)isthebiggestPythagoreanfuzzyopensetcontainedinP,
b. cl(P)isthesmallestPythagoreanfuzzyclosedsetcontainingP.

Remark 2.8. From the definition Pythagorean fuzzy union and intersection, it is obvious that Pythagoreanfuzzyinterior,closureandboundaryarePythagoreanfuzzysets.

3. GENERALIZED PYTHAGOREAN FUZZY CLOSED SETS

Definition 3.1. Let$(X, \tau)_P$be an Pythagorean Fuzzy topological space. A PythagoreanFuzzy set A in $(X, \tau)_P$is said to be generalized Pythagorean fuzzy closed (shortly GPFC) if$PFcl(A)\subseteq P$whenever$A\subseteq P$andPisPFO.

ThecomplementofGPFCisGPFO

Definition3.2. Let $(X,\tau)_P$beaPythagoreanFuzzytopologicalspace.LetAbeaPFSinX.ThenGeneralizedPythagoreanfuzzyclosureandGeneralizedPythagoreanFuzzyinteriorofAaredefinedby

(1) $GPFcl(A) = \cap \{G: G is GPF closedset in X and A\subseteq G\}$
(2) $GPFint(A) = \cup \{G: G is GPF openset in X and A\supseteq G\}$

Example 3.3. Let X = {x_1 , x_2}. Consider the following family of Pythagorean fuzzy sets τ ={0_X, A, B 1_X} where $\boldsymbol{A} = \{\langle \boldsymbol{x_1}, \mathbf{0.6}, \mathbf{0.4}\rangle, \langle \boldsymbol{x_2}, \mathbf{0.5}, \mathbf{0.6}\rangle\}$, $\boldsymbol{B} = \{\langle \boldsymbol{x_1}, \mathbf{0.7}, \mathbf{0.04}\rangle, \langle \boldsymbol{x_2}, \mathbf{0.8}, \mathbf{0.3}\rangle\}$.

Clearly $(X, \tau)_P$ is a Pythagorean fuzzy topological space. Here the set C = $\{\langle \boldsymbol{x_1}, \mathbf{0.7}, \mathbf{0.3}\rangle, \langle \boldsymbol{x_2}, \mathbf{0.5}, \mathbf{0.4}\rangle\}$ is aGeneralized Pythagorean fuzzy closed set.

Proposition 3.4. Let$(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A and B be anytwo Pythagorean fuzzy sets in$(X, \tau)_P$. Then the Generalized Pythagorean Fuzzy closure operatorsatisfiesthefollowingproperties.

i. A⊆GPFcl(A)
ii. GPFcl(GPFcl(A)) = GPFcl(A)
iii. A⊆B⇒GPFcl(A)⊆GPFcl(B)
iv. GPFcl(A∪B)=GPFcl(A)∪GPFcl(B)
v. GPFcl(1_X)=1_X; GPFcl(0_X)=0_X.

Proof.(i),(ii),(iii)and(v)canbeeasilyobtainedbythedefinitionoftheGPF closure.
(iv) From GPFcl$(A) \subseteq GPFcl(A \cup B)$. Weobtain $GPFcl(A) \cup GPFcl(B) \subseteq GPFcl(A \cup B)$.
On the other hand, from the facts A$\subseteq GPFcl(A)$ andB$\subseteq GPFcl(B)$ ⇒ A∪$B \subseteq GPFcl(A) \cup GPFcl(B)andGPFcl(A) \cup GPFcl(B) \in GPFCS$. Wehave$GPFcl(A \cup B) \subseteq GPFcl(A) \cup GPFcl(B)$.

Thus,proofoftheaxioms(iv)isobtainedfromthesetwoinequalities.

Proposition 3.5. Let$(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A and B be anytwo Pythagorean fuzzy sets in$(X, \tau)_P$ .Then the Generalized Pythagorean Fuzzy interior operatorsatisfiesthefollowingproperties.

i. $GPFint(A) \subseteq A$
ii. $GPFint(GPFint(A)) = GPFint(A)$
iii. $A \subseteq B \Rightarrow GPFint(A) \subseteq GPFint(B)$
iv. $GPFint(A \cap B) = GPFint(A) \cap GPFint(B)$
v. $GPFint(1_X) = 1_X$; $GPFint(0_X)) = 0_X$

Proof.(i),(ii),(iii)and(v)canbeeasilyobtainedfromthedefinitionoftheGeneralizedPythagoreanFuzzyinterior.
(iv) From GPFint$(A \cap B) \subseteq GPFint(A)$ and $GPFint(A \cap B) \subseteq GPFint(B)$.
We obtain GPFint$(A \cap B) \subseteq GPFint(A) \cap GPFint(B)$. On the other hand, from thefacts$GPFint(A) \subseteq A$and$GPFint(B) \subseteq B \Rightarrow GPFint(A) \cap GPFint(B) \subseteq A \cap BandGPFint(A) \cap GPFint(B) \in \tau_P$.We have$GPFint(A) \cap GPFint(B) \subseteq GPFint(A \cap B)$ .Thus,proofoftheaxioms(iv)isobtainedfromthesetwoinequalities.

Proposition3.6. Let$(X,\tau)_P$beaPythagoreanFuzzytopologicalspace.LetAandBbeanytwoPythagoreanfuzzysetsin$(X,\tau)_P$.Thenthefollowingpropertieshold.

1. 1 – GPFcl(A) = GPFint(1 – A)
2. 1 – GPFint(A) = GPFcl(1 – A)

Proposition3.7. IfAandBareGPF-closedsets,thenA∪B isaGPF-closedset.

Remark3.8. TheintersectionoftwoGPF-closedsetsneednotbeGPF-closedset.

Proposition 3.9. Let$(X, \tau)_P$be a Pythagorean Fuzzy topological space. If B is GPF-closedandB⊆A⊆PFcl(B)thenAisGPF-closed.

Proof. Let C be GPF-closed set in $(X, \tau)_P$ such that A ⊆C. Since B ⊆C and B is a GPF-closed set, PFcl(B) ⊆C. Since A ⊆PFcl(B), we have PFcl(A) ⊆PFcl(PFcl(B) =PFcl(B)⊆C. Hence PFcl(A) ⊆C which implies that A is a GPF closed in $(X, \tau)_P$.

Proposition3.10. In a Pythagorean fuzzy topological space$(X, \tau)_P$, τ_P= T_P (Thefamily of all Pythagorean fuzzy closed Sets) iff every Pythagorean fuzzy closed set of $(X, \tau)_P$is aGPFclosedset.

Proof.SupposethateveryPythagoreanfuzzysetAof$(X,\tau)_P$isGPFclosed. Let A$\in \tau_P$. Since$A$

$\subseteq A$and A is GPF-closed, $PFcl(A) \subseteq A$. But $A \subseteq PFcl(A)$. Hence, $PFcl(A) = A$. Thus,$A \in \tau_P$.Therefore,$\tau_P \subseteq T_P$.If$B \in T$,then$1_X - B \in \tau_P \subseteq T_P$andhence$B \in \tau_P$. That is$T_P \subseteq \tau_P$.Therefore$\tau_P = T_P$

Conversely,SupposethatAbea Pythagorean Fuzzy set in$(X, \tau)_P$.Let B be a Pythagoreanfuzzyopensetin$(X,\tau)_P$suchthat $A \subseteq B$. Byhypothesis,BisPythagoreanfuzzyclosedset. BythedefinitionofPythagoreanfuzzyclosure$PFcl(A) \subseteq B$) .ThereforeAisGPF-closed.

Proposition3.11. IfPFint(A)⊆B⊆AandifAis GPF-openthenBisalsoGPF-open.

Proposition 3.12. Let$(X, \tau)_P$be an Pythagorean fuzzy topological space. A PythagoreanfuzzysetAisGPF-openiff B⊆GPFint(A),wheneverBisPythagoreanfuzzyclosedand B⊆A.

Proof.Let A be a GPF-open set and B be a Pythagorean fuzzy closed set, such that B$\subseteq A$. Now,$B \subseteq A \Rightarrow 1_X - A \subseteq 1_X - Band1_X - A$isaGPF-closedset$\Rightarrow PFcl(1_X - A) \subseteq 1_X - B$.Thatis, $B = 1_X - (1_X - B) \subseteq 1_X - PFcl(1_X - A)$. But$1_X - PFcl(1_X - A) = PFint(A)$. Thus, $B \subseteq PFint(A)$.

Conversely, suppose that A be Pythagorean fuzzy set, such that$B \subseteq PFint(A)$ whenever B is Pythagorean fuzzy closed and B$\subseteq A$. Let $1_X - A \subseteq B$ wheneverB is Pythagorean fuzzy-open.

Now, $1_X - A \subseteq B \Rightarrow 1_X - B \subseteq A$. Hence by assumption,$1_X - B \subseteq PFint(A)$.Thatis,$1_X - PFint(A) \subseteq B$. But $1_X - PFint(A) = PFcl(1_X - A)$ .Hence,$PFcl(1_X - A) \subset B$.Thatis,$1_X - A$isGPF-closed.Therefore,AisGPF-open.

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