

An Approach for Engineering Tuning of PI-Controller with Dynamic Object Series-Connected Aperiodic and Integrating Unit

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Abstract:

An approach is proposed for engineering adjustment of the PI-controller with a dynamic object series-connected aperiodic and integrating unit. There is a proposal to solve the problem by solving the characteristic equation. As a result of the third row dynamic system analysis, the adjustment parameters of the PI-controller are calculated. The transitional processes of the closed system (object-controller) are dealt with by assignment and disturbance. For the transitional process by assignment, overshoot $\sigma=17,35\%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 2,65 % inaccuracy is observed in theory. Therefore, the proposed approach for engineering adjustment for a PI-controller with a second-order dynamic object series-connected aperiodic and integrating unit is suitable for use in third-order dynamic systems analysis.

Keywords —PI-controller, tuning, dynamic system, second order object, transfer function.

I. INTRODUCTION

Third order dynamic systems can be obtained in the following cases [1,2,3]:

- object one aperiodic and integrating unit operating with PI-controller;
- object two aperiodic unit operating with I-controller;
- object two aperiodic units working with PI-controller;
- object two aperiodic units, working with PID-controller with perfect differentiation;
- object three aperiodic units operating with a P-controller;
- object three aperiodic units, working with a PD-controller with perfect differentiation;
- object oscillating unit operating unit with PI-controller;
- object oscillating unit operating with PID-controller with ideal differentiation and
- object oscillating unit operating with a real-time differentiation first-order PD-controller.

The operation of the so-defined objects with stated linear controllers results in dynamic systems of third order. In the study of these systems - an analysis of the dynamic characteristics and determination of the desired adjustment of the controller, two approaches are used. The first approach addresses the universal methodologies and guidelines developed for more sophisticated systems. A great deal of these methodologies are also applicable to lower order systems. The second approach explores the differential equations of the second or third order, which is considered to be easy as the equations are relatively well studied.

As a disadvantage of the first approach, it can be noted that it does not always give accurate results. In some cases, it cannot be used. Its advantage is that it is easier and more suitable for engineering work.

As a disadvantage of the second approach (above all for third order systems) it can be noted the relative complexity of the research in engineering work. An important advantage here is higher accuracy.

The theoretical investigation of third order systems is generally not complicated. For conducting quick and accurate engineering calculations related to tuning the controllers, there are not always suitable nomograms and formulas from the first approach.

PROBLEMS WITH THE TUNING OF CONTROLLERS IN THIRD ORDER SYSTEMS

Third and higher order dynamic systems are often used in industrial automation systems for a variety of production processes, but due to their complexity, few authors have attempted to do theoretical research on them [6,7]. The complexity is that the roots of the characteristic equation of the closed ACS (automatic control system) is three, and it is not clear how the third real root influences the stability of the system, and hence the indicators of quality of the transitional processes.

POSSIBLE OPTIONS FOR SOLUTION OF THE ASSIGNED TASK

In analysing third order dynamic systems, the determination of dependencies between quality indicators and system parameters is considerably more complicated. One of the possible options for solving the task is through the use of Prof.Vishnegradski's diagram [1]. The diagram he suggests allows to judge not only sustainability but also some key quality indicators. In the study of dynamic systems of third order, he concluded that the nature of the transitional process can be determined without solving the characteristic equation of the system. For this purpose, it is sufficient for hyperbola built according to its parameters - X and Y to be supplemented with three auxiliary curves [1]. He has given an original word formulation of his criterion, which states: To be a dynamic third-order system sustainable, it is both necessary and sufficient to fulfill the following two conditions: 1. All the coefficients of the characteristic equation must be positive; 2. The average output minus the output of the final coefficients of the characteristic equation of the

system must be positive. Failure to comply with these conditions will make the third order dynamic system unstable or at the limit of resistance.

Other possible options for solving this task are by using Ziegler & Nicols first method, Koppelovich's nomograms and nomograms given in [2]. These are methods for determining the parameters for adjusting the controllers by known data for the transitional characteristic of the control object [3,5].

The purpose of this paper is to offer an engineering adjustment for a proportional-integral PI-controller with a dynamic second order object series-connected aperiodic and integrating unit by solving the characteristic equation of the closed system.

PROPOSAL FOR SOLVING THE PROBLEM BY SOLVING THE CHARACTERISTIC EQUATION

Figure 1 shows the structural diagram of a ACS comprising a second order object (aperiodic and integrating units) and a PI-controller.

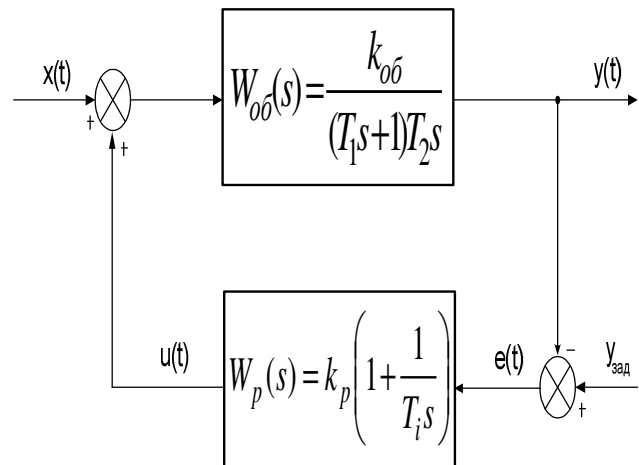


Fig.1. A system with a second order object and a PI-controller

The transfer function of the closed system (fig.1) regarding the assignment is the type

$$\begin{aligned}
 W_{3ad}(s) &= \frac{Y(s)}{Y_{3ad}(s)} = \frac{W_{o\delta}(s) \cdot W_p(s)}{1 + W_{o\delta}(s) \cdot W_p(s)} = \\
 &= \frac{\frac{k_{o\delta}}{(T_1s + 1)T_2s} \cdot k_p \left(\frac{T_i s + 1}{T_i s} \right)}{1 + \frac{k_{o\delta}}{(T_1s + 1)T_2s} \cdot k_p \left(\frac{T_i s + 1}{T_i s} \right)} = \quad (1) \\
 &= \frac{T_i s + 1}{\frac{T_1 T_2 T_i}{k_{o\delta} k_p} s^3 + \frac{T_2 T_i}{k_{o\delta} k_p} s^2 + T_i s + 1}
 \end{aligned}$$

The transfer function of the closed system (fig.1) regarding the disturbance is the type

$$\begin{aligned}
 W_x(s) &= \frac{Y(s)}{X(s)} = \frac{W_{o\delta}(s)}{1 + W_{o\delta}(s) \cdot W_p(s)} = \\
 &= \frac{\frac{k_{o\delta}}{(T_1s + 1)T_2s}}{1 + \frac{k_{o\delta}}{(T_1s + 1)T_2s} \cdot k_p \left(\frac{T_i s + 1}{T_i s} \right)} = \quad (2) \\
 &= \frac{T_i}{k_p} \cdot \frac{s}{\frac{T_1 T_2 T_i}{k_{o\delta} k_p} s^3 + \frac{T_2 T_i}{k_{o\delta} k_p} s^2 + T_i s + 1}
 \end{aligned}$$

We propose that the analysis of the third-order dynamic system be carried out with a successively connected oscillating and aperiodic link, i.

$$W(s) = \frac{1}{T_o^2 s^2 + 2\xi T_o s + 1} \cdot \frac{1}{T_i s + 1} \quad (3)$$

Assuming that the time constant of the aperiodic link (first order low pass filter) is equal to the time constant of the oscillating link, i. $T = T_o$ is obtained

$$W(s) = \frac{1}{T_o^2 s^2 + 2\xi T_o s + 1} \cdot \frac{1}{T_o s + 1} \quad (4)$$

For the polynomial in the denominator of expression (4) the characteristic equation is obtained

$$(T_o^2 s^2 + 2\xi T_o s + 1)(T_o s + 1) = T_o^3 s^3 + (2\xi + 1)T_o^2 s^2 + (2\xi + 1)T_o s + 1 \quad (5)$$

If we equal the corresponding coefficients in front of s^3 , s^2 etc. from the characteristic equation (5) to the coefficients of s^3 , s^2 etc. of the polynomial in

the denominator of expression (1), the transfer function of the closed system regarding the assignment will have the final appearance

$$W_{3ad}(s) = k_{3ad} \cdot \frac{T_i s + 1}{T_o^3 s^3 + (2\xi + 1)T_o^2 s^2 + (2\xi + 1)T_o s + 1} \quad (6)$$

where $k_{3ad} = 1$ is called a coefficient of the system assignment.

The transfer function of the closed disturbance system will have the final appearance

$$W_x(s) = k_x \cdot \frac{T_o s}{T_o^3 s^3 + (2\xi + 1)T_o^2 s^2 + (2\xi + 1)T_o s + 1} \quad (7)$$

$$\text{where } k_x = \frac{T_i}{k_p} \cdot \frac{1}{T_o} = \frac{T_i}{k_p} \cdot \sqrt[3]{\frac{k_{o\delta} k_p}{T_1 T_2 T_i}} = \sqrt[3]{\frac{T_i^2 k_{o\delta}}{k_p^2 T_1 T_2}}$$

is called the system disturbance factor.

By comparing the coefficients in front of the corresponding degrees of s in the polynomials of expressions (1) and (6), dependencies between the parameters of the transition process and the parameters of the system can be determined. Equivalent time constant is

$$T_o = \sqrt[3]{\frac{T_1 T_2 T_i}{k_{o\delta} k_p}} \quad (8)$$

Similarly, the attenuation coefficient ξ is determined. For it two expressions of s^2 and s of (6) are obtained, i.e.

$$\begin{aligned}
 \text{The first expression that can be determined } \xi \text{ is} \\
 (2\xi + 1)T_o^2 = \frac{T_2 T_i}{k_{o\delta} k_p} \quad (9)
 \end{aligned}$$

If we only express ξ we obtained

$$\xi = \frac{1}{2} \left[\frac{T_2 T_i}{T_o^2 k_{o\delta} k_p} - 1 \right] \quad (10)$$

The second expression from which can be determined ξ is

$$(2\xi + 1)T_o = T_i \quad (11)$$

If we express only ξ it is obtained

$$\xi = \frac{1}{2} \left[\frac{T_i}{T_0} - 1 \right] \quad (12)$$

If the expressions (9) and (11) are divided into one another, it is obtained

$$T_o = \frac{T_2}{k_{o\sigma} k_p} \quad (13)$$

If the expressions (10) and (12) are equal to one another, i.

$$\frac{1}{2} \left[\frac{T_2 T_i}{T_0^2 k_{o\sigma} k_p} - 1 \right] = \frac{1}{2} \left[\frac{T_i}{T_0} - 1 \right] \quad (14)$$

and then simplified, an expression of the type (13) is obtained. This confirms that the expressions (8) and (13) are equal, i

$$T_o = \sqrt[3]{\frac{T_1 T_2 T_i}{k_{o\sigma} k_p}} = \frac{T_2}{k_{o\sigma} k_p} \quad (15)$$

If an expression (15) is solved regarding the time constant of integration T_i , it is obtained

$$T_i = \frac{T_2^2}{k_{o\sigma}^2 k_p^2 T_1} \quad (16)$$

If we replace equations (13) and (16) in equation (11) and then simplify, we obtain the equation by which the coefficient of proportionality k_p is calculated i.e.

$$k_p = \frac{T_2}{(2\xi + 1) k_{o\sigma} T_1} \quad (17)$$

The transition process of the control object has the following form

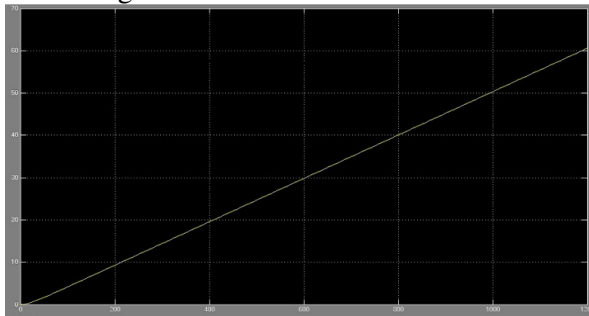


Fig.2. Transitional process of control object

The transfer function of the control object has the form

$$W_{o\sigma}(s) = \frac{k_{o\sigma}}{(T_1 s + 1) T_2 s} = \frac{1}{(19,5s + 1) \cdot 19,5s}$$

At a value of the attenuation coefficient $\xi = 0,456$, which corresponds to over-regulation $\sigma = 20\%$ by equation (17) we calculate the coefficient of proportionality of the PI-controller, i.e for k_p we can write

$$k_p = \frac{19,5}{(2 \cdot 0,456 + 1) \cdot 1 \cdot 19,5} = 0.523$$

We use equation (16) to calculate the value of the integration time constant. Once we replace in it we get

$$T_i = \frac{19,5^2}{1^2 \cdot 0,523^2 \cdot 19,5} = 71,3 \text{ sec}$$

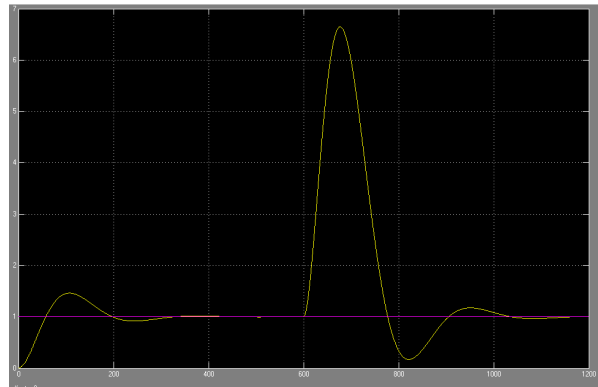


Fig.3. Transitional processes by assignment and by disturbance

By the expression (12) the damping factor ξ is calculated and approximately what is the value of the overshoot σ from [3]

$$\xi = \frac{1}{2} \left[\frac{T_i}{T_0} - 1 \right] = \frac{1}{2} \left[\frac{71,3}{37,29} - 1 \right] = 0,456$$

$$\sigma^2 = \exp\left(-\frac{\zeta}{\sqrt{1-\zeta^2}} \cdot 2\pi\right) = \exp\left(-\frac{0,456}{\sqrt{1-0,456^2}} \cdot 2\pi\right) = 0,04$$

or only $\sigma = 20\%$.

Determine the maximum dynamic deviation y_1 in the expression given in [3]

$$y_1 = \exp\left(-\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = \exp\left(-\frac{0,456}{\sqrt{1-0,456^2}}\right) = 0,6.$$

If any of the above two parameters does not meet the prerequisites for quality, adjust the controller.

The transitional processes of the closed system (fig.1) by assignment and by disturbance are shown in fig.3. For the transitional process by assignment, overshoot $\sigma = 17,35\%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, $2,65\%$ inaccuracy is observed in theory. Therefore, the proposed sub-process for engineering adjustment of a PI-controller with a dynamic object series-connected aperiodic and integrating unit is suitable for use in the analysis of third-order dynamic systems.

CONCLUSIONS

An approach is proposed for engineering adjustment of the PI-controller with a dynamic object series-connected aperiodic and integrating unit. There is a proposal to solve the problem by solving the characteristic equation.

As a result of the analysis of the third order dynamic system, the adjustment parameters of the PI-controller are recalculated.

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