

# A Review on Compressive Sensing Signal Reconstruction Approaches in Cognitive Radio Networks

Lopesh Kumar Katre<sup>1</sup>, Mahendra Choudhary<sup>2</sup>, Gaurav Morghare<sup>3</sup>, Dr. Manish Sawle<sup>4</sup>

<sup>1</sup>M Tech Scholar, Department of Electronics and Communication Engineering, O.I.S.T, Bhopal, India

Email: (lopeshkatre6@gmail.com)

<sup>2</sup>Research Guide, Department of Electronics and Communication Engineering, O.I.S.T, Bhopal.

<sup>3</sup>Asst. Professor, Department of Electronics and Communication Engineering, O.I.S.T, Bhopal.

<sup>4</sup>Head and Professor, Department of Electronics and Communication Engineering, O.I.S.T, Bhopal.

\*\*\*\*\*

## Abstract:

Compressive Sensing (CS) is a new sensing modality which compresses the signal being received at the time of sensing. Signals can have sparse or compressible illustration either in original domain or in some seriously change domain. Relying on the sparsity of the signals, CS lets in us to pattern the signal at a price plenty beneath the Nyquist sampling rate. Also, the varied reconstruction algorithms of CS can faithfully reconstruct the unique sign lower back from fewer compressive measurements. This reality has encouraged lookup hobby closer to the use of CS in the various fields like magnetic resonance imaging, high speed video acquisition, ultrawideband (UWB) communication, etc. This survey paper opinions the primary theoretical concepts underlying CS. To bridge the hole between concept and practicality of CS, distinctive CS acquisition techniques and reconstruction approaches are elaborated systematically in this paper. The major application areas the place CS is presently being used are reviewed here.

**Keywords** — Compressive Sensing, Cognitive Radio, CS reconstruction, CS Application, Nyquist sampling rate etc.

\*\*\*\*\*

## I. INTRODUCTION

After the well-known Shanon sampling theorem, introduction of compressive sensing (CS) is like a important breakthrough in sign processing community. CS used to be introduced by Donoho, Candès, Romberg, and Tao in 2004 [1]–[3]. They have developed its mathematical foundation. CS is basically used for the acquisition of indicators which are both sparse or compressible. Sparsity is the inherent property of these signals for which, entire of the records contained in the sign can be represented solely with the assist of few substantial components, as in contrast to the complete size of the signal. Similarly, if the sorted aspects of a sign decay unexpectedly obeying power law, then these

alerts are known as compressible signals, refer Fig.1. A sign can have sparse/compressible representation either in unique area or in some seriously change domains like Fourier transform, cosine transform, wavelet transform, etc. A few examples of indicators having sparse illustration in certain domain are: herbal pics which have sparse representation in wavelet domain, speech sign can be represented by means of fewer components the usage of Fourier transform, higher mannequin for medical images can be got the usage of Radon transform, etc. A good introduction about basis, frames and dictionaries in which the sparsest feasible illustration of a sign can be obtained, is available in articles [12]–[16]. Acquisition of sparse signals using typical techniques requires: i)

sampling the use of Nyquist criterion, which outcomes in too many samples in contrast to the authentic statistics contents of the signal, ii) compressing the signal by means of computing vital radically change coefficients for all the samples, conserving solely large coefficients and discarding the smaller ones for storage/transmission purposes. CS simplifies the sign acquisition by taking some distance fewer random measurements. Fig.2 depicts the comparison between typical sampling and CS sampling schemes.

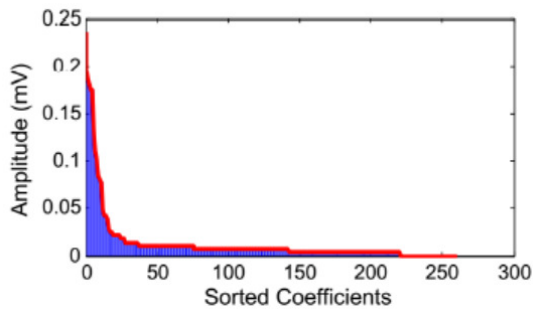


Figure 1. Rapid decay of coefficients of a signal when represented using suitable transform, obeying power law

Another quandary of sampling the use of Nyquist-rate is that the rate at which sampling has to be done, may additionally no longer be practical always. For example, in case of multiband indicators having wide spectral range, sampling price counseled through Nyquist-criterion may be orders of magnitude greater than the specifications of fine accessible analog-to-digital converter (ADC). The sampling rate the usage of Nyquist-criterion is determined with the aid of the highest frequency issue existing in signal, whereas, sampling rate in CS is ruled by means of the sign sparsity. The CS measurements are non-adaptive, i.e., no longer getting to know from previous measurements. The resulted fewer compressive measurements can be without difficulty saved or transmitted. This offers an impression of compressing the sign at the time of acquisition solely and hence the identify 'Compressive Sensing'. CS lets in the faithful reconstruction of the unique sign returned from fewer random measurements with the aid of making use of some non-linear reconstruction techniques. Because of all these features, CS finds its applications mainly in the areas i) where, quantity of sensors are confined due to excessive cost, e.g.,

non-visible wavelengths, ii) where, taking measurements is too expensive, e.g., excessive speed A/D converters, imaging by means of neutron scattering, iii) where, sensing is time consuming, e.g., scientific imaging, iv) where, sensing is strength constrained, etc. [4]–[7].

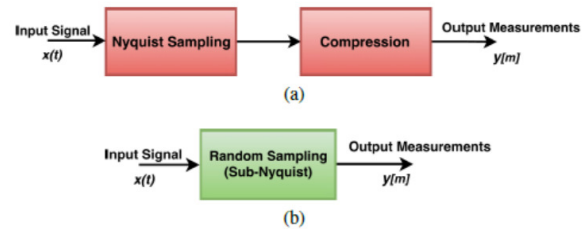


Figure 2. Comparison of sampling techniques: (a) traditional sampling, (b) compressive sensing.

## II. CS RECONSTRUCTION APPROACHES

CS reconstruction algorithms attempt to locate out the sparse estimation of the authentic enter signal, from compressive measurements, in some appropriate foundation or body or dictionary. A lot of lookup has been executed on this thing of CS, to come up with higher performing algorithms. The lookup using factors in this place are capacity to get better from minimal quantity of measurements, noise robustness, speed, complexity, performance guarantees, etc. [8]. The CS reconstruction algorithms are ordinarily labeled underneath six approaches. This part summarizes the famous algorithms below each approach.

### A. Convex Optimization Approach

This strategy poses the CS reconstruction hassle as a convex optimization trouble which can be solved by using utilizing solver from linear programming. The convex formulations proposed in literature, for acquiring the sparse representation of a signal, are mentioned below:

1) *Basis Pursuit*: Basis Pursuit (BP) used to be proposed by S. Chen et al., in 1999 [28]. It is a convex optimization problem, which searches for a answer having minimal  $l_1$  norm, situation to the equality constraint given in (1).

$$\hat{s} = \arg \min_s \|s\|_1; \quad \text{subject to } \Theta s = y. \quad (1)$$

BP is used in CS to discover the sparse approximation  $\hat{s}$  of input signal  $x$ , in dictionary or matrix  $\Theta$ , from compressive measurements  $y$ . BP can get better faithfully solely if, the measurements are noise-free.

2) *Denoising the use of Convex Approach:* If the measurements are corrupted through noise, then to suppress the noise, exact reconstruction is now not desired. The denoising can be achieved by enjoyable the equality constraint in (1) to account for measurement noise. The broadly used formulations for strong data recovery from noisy measurements are Dantzig selector, basis pursuit denoising (BPDN), whole variant (TV) minimization based denoising, etc.

• *Basis Pursuit Denoising:* BPDN used to be added by using S. Chen et al., in the discipline of computational harmonics [28]. This is equal as Least Absolute Shrinkage Selection operator (LASSO), which used to be brought by R. Tibshirani, in data [30]. To account for the noise in measurements, BPDN poses the sparse estimation problem, as an optimization hassle given by (12). It suggests that, BPDN searches for a answer having minimum  $\ell_1$ -norm difficulty to the at ease situation on constraint. The quadratic inequality constraint used by BPDN states that for the got solution, the squared  $\ell_2$ -norm of the error between  $y$  and  $s$  ought to be less than or equal to .

$$\hat{s} = \arg \min_s \|s\|_1; \quad \text{subject to} \quad \frac{1}{2} \|(y - \Theta s)\|_2^2 \leq \epsilon, \quad (2)$$

where,  $\ell_2$ , additionally recognized as euclidean norm, represents the length or dimension of a vector [17]. Some algorithms solve BPDN in its Lagrangian form, which is an unconstrained optimization trouble and can be rewritten as in (3).

$$\hat{s} = \arg \min_s \lambda \|s\|_1 + \frac{1}{2} \|(y - \Theta s)\|_2^2. \quad (3)$$

3) *Solvers for Convex Approach:* Solvers are required to solve the optimization troubles described above. The BP problem in (1) can be solved via linear programming algorithms like simplex algorithm recognized as BP-simplex, interiorpoint algorithm recognised as BP-interior. Here, simplex can be defined as a convex

polyhedron fashioned by way of the set of all feasible solutions (points) [28]. Apart from simplex and interior-point algorithms, the different famous algorithms for fixing convex optimization troubles are constant factor continuation (FPC), gradient projection for sparse illustration (GPSR), Bregman iteration algorithm, etc. Fig.13 suggests some famous solvers for fixing the convex optimization problems. The algorithmic steps of these solvers are described below:

- *P-Simplex Algorithm:* The primary steps for fixing the BP problem the use of simplex algorithm are proven in Fig.3(a) and are described below:

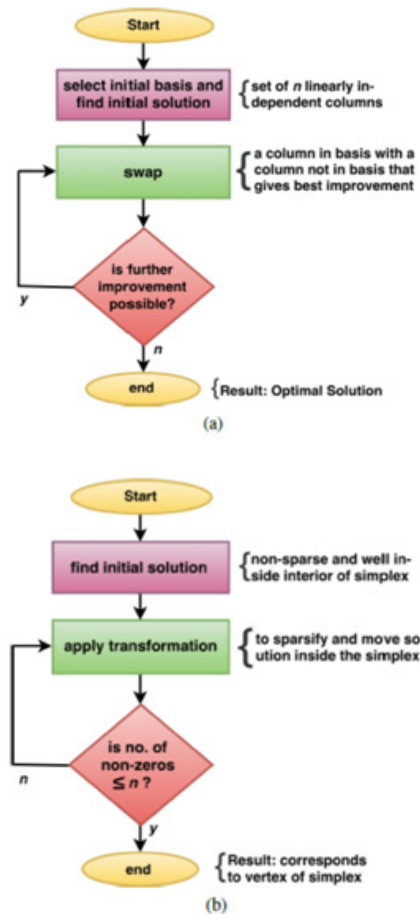


Figure 3 Algorithmic steps of solvers for basis pursuit namely: (a) simplex method and (b) interior-point method.

i). *Initial foundation selection:* preliminary foundation are a set of  $n$  linearly unbiased columns chosen from a dictionary. Using preliminary basis, locate the preliminary viable solution, which

corresponds to one of vertices of the simplex.

ii). *Swapping*: swap one column in modern-day foundation with the column no longer in the foundation that offers excellent improvement in goal function. This is equal to leaping on the vertices of simplex for looking the solution, in the direction of enhancing the goal function.

iii). Repeat step ii), till no in addition enchancement is possible. At last, the superior answer is achieved.

• *BP-Interior Algorithm*: The fundamental steps for fixing the BP hassle the usage of interior-point algorithm are proven in Fig.3(b) and are described below:

i). Initial solution: begin from a non-sparse preliminary solution which is properly inner the indoors of simplex.

ii). Apply transformation that sparsifies the solution. This corresponds to shifting the answer interior the simplex in the course of accomplishing to a vertex.

iii). Repeat step ii), till a answer having n significant non-zero entries, is reached. The end result so acquired is a feasible answer and corresponds to the vertex of simplex.

### *B. Greedy Approach*

The convex optimization strategy introduced above is a global optimization method. Different from that, the greedy approach is a step-by-step iterative method. In every iteration, the answer is up to date by using deciding on solely these columns of reconstruction matrix, which are fairly correlated with the measurements. The chosen columns are known as atoms. Generally, the atoms chosen once, are no longer protected in subsequent iterations of the algorithm. This thinking lowers the computational complexity of the algorithm. Here, the answer is approached in a grasping fassion and hence, the name. The advantages of this method are easy operation, low computational complexity and quicker execution. Drawback is, it requires knowledge of sparsity of the underlying signal, earlier than hand [8]. The algorithms the works on this method can be further classified into two categories:

1) *Serial Greedy Algorithms*: The algorithms that can be put below this class are matching pursuit (MP) proposed by Mallat et al., orthogonal matching pursuit (OMP) proposed with the aid of Y. C. Pati et al., and gradient pursuit (GP) proposed by using Bluemensath et al., Each Iteration of these algorithms selects solely one atom in each generation and computes the corresponding non-zero entry of answer vector. Therefore, these algorithms are termed as serial grasping algorithms.

2) *Parallel Greedy Algorithms*: The algorithms that can be put beneath this class are compressive sampling matching pursuit (CoSaMP) and subspace pursuit (SP). Instead of selecting only one atom from matrix , these algorithms operate by choosing okay or more than one of okay atoms at a time and hence termed as parallel grasping pursuits. Rest of the steps are same as described for serial grasping algorithms. These algorithms are extra effective than serial counterparts, due to the fact they have the functionality of doing away with the incorrect atoms chosen during previous iterations.

### *D. Combinatorial Approach*

Combinatorial algorithms have been firstly developed for solving sparse approximation troubles in team checking out to minimize the range of checks to be performed. The algorithmsthat come underneath this class are random Fourier sampling, heavy hitters on steroids (HHS), chaining pastimes and sparse sequential matching pursuit [27]. Reconstruction the use of these algorithms requires a precise dimension pattern. The measurement matrix ' is developed the usage of a set of discrete-valued functions, ensuing in a precise sample in ', like precisely equal number of ones in every column however allotted randomly. This means that every size  $y_j$  is received by using combining same range of samples of enter signal.

### *E. Non-Convex Approach*

All CS reconstruction algorithms tries to locate the sparsest possible answer from compressive measurements. An example explaining the capability of exclusive norms to reconstruct the sparsest answer is proven with the assist of unit normed-balls. In 2-D space, the unit normed-balls can be acquired by connecting all the factors for

which the price of their respective norm is equal to 1. In this example, the answer  $s$  is assumed to be sparse with  $k = 1$  and lies on the line intersecting the axes. To estimate the solution,  $l_2$  when  $l_1$ -ball is expanded, it touches the line at a factor which is now not sparse. On the different hand, each  $l_1$  and  $l_{1/2}$  balls are able to hit the favored result. As described earlier, the  $l_1$ -minimization, which is a convex optimization approach, searches for a solution with minimal  $l_1$ -norm. The non-convex method replaces  $l_1$ -norm by way of  $l_p$ -norm, where,  $0 < p < 1$ . This method is able to get better the sparse answer from a great deal fewer measurements compared to the convex approach. Another gain of nonconvex approach is that a

weaker model of RIP situation is sufficient for best reconstruction. The algorithms that come under this class are focal underdetermined gadget solution (FOCUSS), iteratively re-weighted least squares (IRLS), etc.

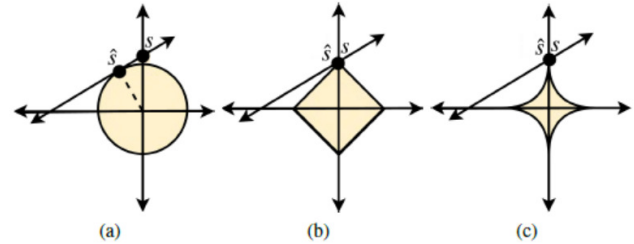


Figure 4. Example to illustrate sparse solution approximation using unit normed-balls in 2-D space: (a)  $l_2$ -ball, (b)  $l_1$ -ball, (c)  $l_{1/2}$ -ball.

TABLE I COMPARATIVE SUMMARY OF CS RECONSTRUCTION APPROACHES.

Approach	Complexity	Attributes	Pros	Cons
Convex	$\approx O(m^2n^3)$	<ul style="list-style-type: none"> <li>– global optimization method</li> <li>– minimizes <math>\ell_1</math>-norm to find solution</li> </ul>	<ul style="list-style-type: none"> <li>– noise robustness</li> <li>– ability to superresolve</li> </ul>	<ul style="list-style-type: none"> <li>– slower, Complex</li> <li>– difficult to implement for problems of larger size</li> </ul>
Greedy	<ul style="list-style-type: none"> <li>–serial version: <math>O(mnk)</math></li> <li>–parallel version: <math>O(mn.iter)</math></li> </ul>	<ul style="list-style-type: none"> <li>–correlation based step-by-step iterative method</li> </ul>	<ul style="list-style-type: none"> <li>–faster, low complexity and noise robustness</li> <li>–parallel versions has ability to discard wrong entries selected in previous iterations</li> </ul>	<ul style="list-style-type: none"> <li>–prior knowledge of signal sparsity is required</li> <li>– requires more measurements than convex counterparts</li> <li>–convergence issues</li> </ul>
Thresholding	$O(mn.iter)$	<ul style="list-style-type: none"> <li>–uses some nonlinear thresholding criteria to select atoms</li> </ul>	<ul style="list-style-type: none"> <li>–faster and low complexity</li> <li>– ability to add/discard multiple entries per iterations</li> </ul>	<ul style="list-style-type: none"> <li>–Convergence issue with IST</li> <li>–better performance requires adaptive step size which increases complexity</li> </ul>
Combinatorial	linear in $n$	<ul style="list-style-type: none"> <li>–computes min or median of measurements identified as consisting of a particular I/P sample</li> </ul>	<ul style="list-style-type: none"> <li>–faster and simpler</li> </ul>	<ul style="list-style-type: none"> <li>–requires noiseless and specific pattern in measurements</li> </ul>
Non-Convex	same as convex approaches	<ul style="list-style-type: none"> <li>–minimizes <math>\ell_p</math>-norm to find solution, where <math>0 &lt; p &lt; 1</math></li> <li>–global optimization method</li> </ul>	<ul style="list-style-type: none"> <li>– recovers from fewer measurements than <math>\ell_1</math> counterpart</li> <li>– functions under weaker RIP</li> <li>– no. of measurements and error decreases with <math>p</math></li> </ul>	<ul style="list-style-type: none"> <li>–slower, complex</li> <li>– difficult to implement for problems of larger size</li> </ul>
Bayesian	$O(nm^2)$	<ul style="list-style-type: none"> <li>–poses recovery as Bayesian inference problem</li> <li>–applicable for signals belonging to some known probability distribution</li> </ul>	<ul style="list-style-type: none"> <li>–faster and yields more sparser solution</li> <li>–estimates signal parameters without user intervention</li> </ul>	<ul style="list-style-type: none"> <li>–results are prior dependent which is difficult to select</li> <li>–high computational cost</li> </ul>

### F. Bayesian Approaches

Different from preceding procedures which reflect on consideration on the input sign to be deterministic, Bayesian method is applicable for the enter indicators which belongs to some known probability distribution. Hence, this strategy looks to be of more sensible interest. The distribution of coefficients of input signal can be two-state

Gaussian-mixture model, i.i.d. Laplace prior model, etc. This method pposes the reconstruction as Bayesian inference problem. The coefficients of enter signal can be estimated the usage of most possibility estimate (MLE) or most a posteriori (MAP) estimate. The algorithms that are used to clear up the Bayesian inference trouble are belief propagation, sparse Bayesian getting to know the usage of relevance vector machines, etc. These

algorithms are now not accompanied with the thought of reconstruction error. Another algorithm in this category is Bayesian compressive sensing

### III. CHALLENGES AND FUTURE SCOPE

CS has received a wider acceptance in a shorter time span, as a sampling approach for sampling the indicators at their information rate. CS takes the gain of sparsity or compressibility of the underlying sign to concurrently pattern and compress the signal. CS has a robust mathematical basis also. But, the growing recognition and acceptability of CS faces some challenges. We are highlighting some of the challenges, which additionally leads to some working instructions in the field.

- There is want for a easy and efficient, typical CS acquisition approach which is relevant to majority of the indicators and additionally leads to quicker acquisition.
- Similarly, a widespread CS reconstruction algorithm, which is faster, robust, much less complicated and offers assured convergence is needed.
- Searching a appropriate basis, in which sign to be acquired has sparsest viable representation, is itself a hard task.
- If one can become aware of the foundation in which sign has the sparsest possible representation, then it will assist in faithful reconstruction from similarly decreased CS measurements. So, a device wants to be developed, which can determine the sparsifying groundwork of signal.
- Development of rigorous overall performance bounds for the issues like minimal range of measurements and reconstruction iterations required for best reconstruction, guaranteed convergence, steady recovery, etc., are also workable areas in this field.

Also, lookup is being going on structured CS. The advantages of this strategy are quicker acquisition, lower complexity, less difficult to implement, etc. But the drawback is that the trustworthy reconstruction requires greater number of measurements. Also, it is tough to have structured

(BCS) algorithm, which can compute the error time period and as a consequence makes adaptive selections to discover the answer.

measurement matrices which obey RIP condition. Some proposals of CS have additionally been considered in literature, which can be labored in addition to take benefits of structured measurements in CS. The theoretical thought of CS described previously in this paper is the classical CS. There can be utility specific challenges, that wishes to be tackled through enhancing the classical version. Some of the highlights in this regard are presented below:

- In case of multidimensional signals, graph of an acquisition system and identification of a sparsifying groundwork is very difficult. Kronecker product matrices has been incorporated in CS to resolve these problems. Other methods can be tried in this situation.
- The kind of the alerts in which non-zero coefficients occur in clusters, are termed as block sparse signals. The challenges encountered in making use of CS to this type of the indicators are: block-sparsity and block coherence considerations for block primarily based acquisition, modifications in reconstruction algorithms to account for block sparsity and mannequin mismatches, etc.
- In some cases, CS measurements are gathered from multiple sources, which are associated in some sense. In this situation, Bayesian framework helps in lowering the number of measurements by using a criterion to give up acquisition when the enough variety of measurements have been taken. This additionally offers a way for sturdy records fusion from multiple sources. Another strategy is to use distributed coding algorithms, by way of exploring the joint sparsity in multiple signals. Applicability of different procedures can also be researched for this.
- Inference issues in sign processing like, detection, classification, estimation and filtering, do no longer require full sign reconstruction. Solving the inference problems from CS compressed measurements only, except reconstructing the sign is a larger challenge. This aids in reducing

dimension free in addition and lets in to get rid of complicated reconstruction process.

- Considering the significance of quantizing the CS measurements in lieu of finite precision, a distortion is introduced in CS measurements. Therefore, reconstruction algorithms wants to be modified to account for quantization error of CS measurements. Also, the research is progressing closer to improving the CS measurements which are quantized the usage of a single bit only. This 1-bit version of CS provides blessings like easy acquisition and robustness to gross non-linearities. This is additionally a promising route to discover further.

- Other challenges are, reconstruction from binary CS measurements, incorporating prior information to enhance reconstruction performance, addressing architectural issues for environment friendly hardware implementation, environment friendly software implementations, dimension strategies to further reduce minimal range of measurements, etc.

#### IV. CONCLUSION

Introduction of CS has revolutionized many areas in signal processing, the place there have been restricted scopes. Some of the major contributions are quicker MRI, excessive fine photograph and video acquisition the use of single pixel camera, acquisition of UWB indicators whilst notably lowering the electricity consumption, etc. This paper has introduced a systematical overview of CS. Considering its rigorous mathematics, which is every so often a barrier for many younger researchers, we introduced a simplified introduction of CS. For an convenient transition from principle with practicality, a precis of CS acquisition strategies and reconstruction procedures has additionally been presented. The CS acquisition method may additionally range from sign to signal. Similarly, the reconstruction method to be used is additionally incredibly signal dependent, which might also in addition wants to be modified to suit a specific situation. It will be notably advisable to have a universal CS acquisition and reconstruction strategy. A review of foremost utility areas the

place CS is presently being utilized has additionally been presented.

#### REFERENCES

- [1] E. J. Candès et al., "Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information," in *IEEE Trans. on Inf. Theory*, vol. 52, no. 2, pp. 489-509, Feb. 2016.
- [2] D. L. Donoho, "Compressed sensing," in *IEEE Trans. on Inf. Theory*, vol. 52, no. 4, pp. 1289-1306, April 2016.
- [3] E. J. Candès and T. Tao, "Near-Optimal Signal Recovery From Random Projections: Universal Encoding Strategies?," in *IEEE Trans. on Inf. Theory*, vol. 52, no. 12, pp. 5406-5425, Dec. 2016.
- [4] R. G. Baraniuk, "Compressive Sensing [Lecture Notes]," in *IEEE Sig. Process. Mag.*, vol. 24, no. 4, pp. 118-121, July 2017.
- [5] E. J. Candès and M. B. Wakin, "An Introduction to Compressive Sampling," in *IEEE Sig. Process. Mag.*, vol. 25, no. 2, pp. 21-30, March 2018.
- [6] E. J. Candès, "Compressive Sampling," in *Proc. of the Int. Cong. Of Mathematicians*, Vol. III, Madrid, Spain, pp. 1433-1452, Aug. 2016.
- [7] E. J. Candès and J. Romberg, "Sparsity and Incoherence in Compressive Sampling," in *Inverse Problems*, Vol. 23, No. 3, pp. 969-985, 2017.
- [8] R. Baraniuk et al., "An Introduction to Compressive Sensing," *OpenStax-CNX*, April 2, 2011. [Online]. Available: <http://legacy.cnx.org/content/col11133/1.5/>.
- [9] E. J. Candès, "The restricted isometry property and its implications for Compressed Sensing," in *C. R. Math. Acad. Sci.*, Paris, 346(9-10), pp. 589-592, 2018.
- [10] D. L. Donoho, "For Most Large Underdetermined Systems of Linear Equations the Minimal  $\ell_1$ -norm Solution is also the Sparsest Solution," in *Comm. on Pure and App. Math.*, vol. 59, no. 6, pp. 797-829, June 2016.
- [11] E. Candès and T. Tao, "Decoding by linear programming," in *IEEE Trans. Inform. Theory*, vol. 51, no. 12, pp. 4203-4215, Dec. 2015.
- [12] S. Mallat, "A Wavelet Tour of Signal Processing: The Sparse Way." 3rd Ed., Burlington, MA: Academic Press, 2019.
- [13] J. Kovacevic and A. Chebira, "Life Beyond Bases: The Advent of Frames (Part I)," in *IEEE Sig. Process. Mag.*, vol. 24, no. 4, pp. 86- 104, July 2017.
- [14] J. Kovacevic and A. Chebira, "Life Beyond Bases: The Advent of Frames (Part II)," in *IEEE Sig. Process. Mag.*, vol. 24, no. 5, pp. 115-125, Sept. 2017.
- [15] R. Rubinstein et al., "Dictionaries for Sparse Representation Modeling," in *Proc. of the IEEE*, vol. 98, no. 6, pp. 1045-1057, June 2010.
- [16] R. Gribonval and M. Nielsen, "Sparse representations in unions of bases," in *IEEE Trans. on Inf. Theory*, vol. 49, no. 12, pp. 3320-3325, Dec. 2013.
- [17] G. H. Golub and C. F. Van Loan, "Matrix Analysis", in *Matrix Computations*. 4th Ed., Baltimore, MD: The Johns Hopkins Univ. Press, 2013, ch.2, pp. 68-73.
- [18] J. N. Laska et al., "Theory and Implementation of an Analog-to-Information Converter using Random Demodulation," in *IEEE Int. Symp. on Cir. and Sys.*, New Orleans, LA, pp. 1959-1962, 2017.
- [19] J. A. Tropp et al., "Beyond Nyquist: Efficient Sampling of Sparse Bandlimited Signals," in *IEEE Trans. on Inf. Theory*, vol. 56, no. 1, pp. 520-544, Jan. 2020.

- [20] J. Yoo et al., "Design and implementation of a fully integrated compressed-sensing signal acquisition system," in IEEE Int. Conf. on Acou., Speech and Sig. Process. (ICASSP), Kyoto, pp. 5325-5328, 2012.
- [21] M. Mishali and Y. C. Eldar, "From Theory to Practice: Sub-Nyquist Sampling of Sparse Wideband Analog Signals," in IEEE J. of Sel. Topics in Sig. Process., vol. 4, no. 2, pp. 375-391, April 2010.
- [22] J. A. Tropp et al., "Random Filters for Compressive Sampling and Reconstruction," in IEEE Int. Conf. on Acou. Speech and Sig. Process. Proc., Toulouse, pp. III-III, 2016.
- [23] J. K. Romberg, "Compressive sensing by random convolution," in SIAM J. Imag. Sci., vol. 2, no. 4, pp. 1098-1128, Dec. 2019.
- [24] J. P. Slavinsky et al., "The compressive multiplexer for multi-channel compressive sensing," in IEEE Int. Conf. Acoust., Speech and Sig. Process. (ICASSP), Prague, Czech Republic, pp. 3980-3983, 2021.
- [25] Y. Zhao et al., "Enhanced Random Equivalent Sampling Based on Compressed Sensing," in IEEE Trans. on Inst. and Meas., vol. 61, no. 3, pp. 579-586, March 2021.
- [26] Y. Zhao et al., "Random Triggering-Based Sub-Nyquist Sampling System for Sparse Multiband Signal," in IEEE Trans. on Inst. And Meas., vol. 66, no. 7, pp. 1789-1797, July 2017.
- [27] T. Haque et al., "Theory and Design of a Quadrature Analog-to-Information Converter for Energy-Efficient Wideband Spectrum Sensing," in IEEE Trans. on Cir. and Sys. I: Reg. Papers, vol. 62, no. 2, pp. 527-535, Feb. 2015.
- [28] S. Chen et al., "Atomic Decomposition by Basis Pursuit," in SIAM J. Sci Comp., vol. 20, no. 1, pp. 33-61, 2019.
- [29] E. Candès, and T. Tao, "The Dantzig selector: statistical estimation when  $p$  is much larger than  $n$ ," in Annals of Stat., vol. 35, no. 6, pp. 2313-2351, 2017.
- [30] R. Tibshirani "Regression Shrinkage and Selection via the Lasso," in J. of the Royal Stat. Soc., vol. 58, pp. 267-288, 2016.