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Analytical Study of the Hepatitis E Virus Model (HEV) Via Hybrid Semi-Analytical Laplace Transform and Adomian Decomposition Method (LADM)

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Abstract. This article is aimed at making comprehensive study of the HEV Model comprising of approximate solution using Laplace-Adomian decomposition method (LADM). The governing nonlinear differential equation describing the model are first reduced via the novel semi-analytical procedure before applying the proposed iterative method to the resulting equations to recursively find the solutions to the parameters of the model. The result obtained showed, the proposed methods is reliable, efficient, computationally convenient, accurate and convergent to the exact solution.

Keywords. HEV Model, Laplace Adomian decomposition Method (LADM), Pade approximant, Hybrid Semi-analytical

1 INTRODUCTION

Hepatitis E virus is a variant of the Hepatitis disease. This disease causes the inflammation of the liver and pose danger to the victim if left unchecked early.It exists in three distinct types 1,2,3,4. The first two occur mainly among humans whereas, the latter two is found among animals especially pigs and deer. Reports from World Health organization (WHO) showed a staggering 20million cases of HEV cases occur yearly with a whooping 3.3million symptomatic cases. The report also reveals a total of 44,000 fatalities occur only in 2015with most coming from the east and south China where the disease is most prevalent [1-6]

The Chinese in a bid to nip in the bud were the first to develop the first known treatment for the Hepatitis E Virus with ground-breaking research. There is plethora of factors that influenced the virus, among them are poor sanitation, meat from infected animals, meat from infected persons, shell from an uncooked animal, blood transfusion from infected persons and faecal contamination of drinking water. Several studies have been devoted to study the prevailing causes and possible treatments of the virus. Copious studies of the virus can be found in the following [7-12].

The LaplaceAdomian decomposition method is a hybrid semi-analytical method obtained from the coupling of the famous Adomian decomposition method and Laplace transformation. This method makes the solution converges faster as it does not require linearization and selection of parameters. LADM have been extensively applied to solve diverse problems ranging from linear and nonlinear differential equations, integro-differential equations, thin flow problems, flow problems. Ebiwaremeand Da-Wariboko [13] have examined the crime deterrence model using modified Adomian decomposition method with Pade approximant for numerical solutions to the model parameters. The results show, the state variables greatly influenced the model.The Magnetohydrodynamics viscous flow over a stretching sheet have been investigated analytically by Roohani et al. [14].The study used Pade approximant to enlarge the convergence domain and improve its convergence. Kumar et al. [15] used Laplace decomposition method to investigate the effect of governing parameters on the transport of chemical ion through the soil. In this study, only small Reynold number is used as it has minimal effect on the system.

Several other authors have applied this novel hybrid semi-analytical method to solve many problems. Forexample, Doan [16] studies the systems of ordinary differential equation. Khan and Hussain [17] have applied LADM to study convection-diffusion equation in semi-infinite domain. Youssouf et al [18] have examined convection diffusion-dissipation equation employing Laplace Adomian decomposition method. Islam et al [19] employed LADM to obtain numerical solution to the logistic differential equation. Mohammed and Torky [20] have investigated systems of nonlinear partial differential equations using LADM and improved upon the result using padeapproximation. The system of ordinary deferential equationhas been studied using LADM by Dogan [21]. Pue-on [22] did an analytical study of the Newell-Whitehead-Segel equation. The HIV infection CD4+ T cells have been considered for the state variables using LADM by Ongun [23]. Hussain et al [24] have examined coupled partial differential equations. Khuri [25] developed an algorithm to study nonlinear differential equation using LADM. Systems of linear and nonlinear partial differential equation have been studied by Fadaei [26]

The motivation in this present paper is to study the dynamics of the model parameters (S,X,I,R,U,V,W) of the Hepatitis E virus model using the hybrid semi-analytical method. The model consists of seven state parameters and eight state variables. The article is organized as follows: section 2 gives the in-depth description of the model. The proposed solution technique LADM is explained in detail and contained is given in section 3. Analysis of the model via the application of the proposed method where the state variables are ascertained is presented in section 4.The numerical simulation of the model parameters is presented in section 5. Section 6 gives the results presented in tables and graphs as well as their discussion. The conclusion of the study is drawn and given in section 7.

II MATHEMATICAL DESCRIPTION OF THE HEV MODEL

The nonlinear model system consists of five ODEs given by

$$\left.\begin{aligned}\frac{dS}{dt} &= \wp(1-\ell I) - (\alpha I + \gamma_E P)S - \vartheta S\\ \frac{dE}{dt} &= (\alpha I + \gamma_E P)S - (\vartheta + \rho)E + \wp\ell I\\ \frac{dR}{dt} &= \rho E - (\vartheta + \sigma)I\\ \frac{dR}{dt} &= \sigma I - \vartheta R\\ \frac{dP}{dt} &= \varphi I - \lambda P\end{aligned}\right\} \tag{1}$$

Subject to the initial conditions

$$S(0) = S_0, E(0) = E_0, I(0) = I_{0,} R(0) = R_0, P(0) = P_0 \tag{2}$$

where the model variables and parameters are described in the table below.

Table 1. Description of the parameters and variables for the HEV model.

Variables	Description
S	Population of susceptible individuals

E	Population of Exposed Individuals
I	Population of Infected individuals
R	Population of recovered individuals
P	Density of viral load in the environment

Parameters	Description
$\wp$	Recruitment rate
α	Contact rate
ℓ	transfer rate of infection from infected mother to her child.
ρ	Rate of infection
σ	Recovery rate
ϑ	natural death rate of humans
φ	rate of the virus transfer from infected people to the environment
γ_E	Contact rate between S and the environment P
λ	Virus decay in the environment

III LAPLACE ADOMIAN DECOMPOSITION METHOD (LADM)

In thissection, we outline the fundamentals of the hybrid Laplace transformation and Adomian decomposition method (LADM)

Consider a functional differential equation of the form

$$L[u(x)] + R[u(x)] + N[u(x)] = g(x) \quad (3)$$

Subject to the initial condition

$$u(x,0) = f(x),\ \frac{\partial u(x,0)}{\partial t} = h(x) \quad (4)$$

Rearranging the above, we obtain the following relation for $L[u(x)]$

$$L[u(x)] = g(x) - R[u(x)] - N[u(x)] \quad (5)$$

Applying Laplace transform on both sides of Eq. (3), supposing the highest differential operator is of order two and using the differentiation property, we get

$$s^2\mathcal{L}\{u(x)\} - sh(x) - f(x) = \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\}$$

$$s^2\mathcal{L}\{u(x)\} = sh(x) + f(x) + \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\}$$

$$\mathcal{L}\{u(x)\} = \frac{h(x)}{s} + \frac{f(x)}{s^2} + \frac{1}{s^2}\mathcal{L}\{g(x)\} - \frac{1}{s^2}\mathcal{L}\{Ru(x)\} - \frac{1}{s^2}\mathcal{L}\{Nu(x)\} \quad (6)$$

Next, we apply the inverse transform on both sides of Eq. (14), we obtain

$$u(x) = \phi(x) - \mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\{Ru(x)\} - \frac{1}{s^2}\mathcal{L}\{Nu(x)\}\right] \quad (7)$$

Where $\phi(x)$ is the term arising from the first three terms on the right-hand side of Eq. (7)

Next, we assume the solution as decomposing series in the form

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \quad (8)$$

Similarly, the nonlinear terms are written in terms of the Adomian polynomials

$$Nu(x) = \sum_{n=0}^{\infty} A_n \quad (9)$$

Where the $A_n^{\prime s}$ represents the Adomian polynomials defined in the form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\sum_{k=0}^{\infty} \lambda^i y_i)]_{i=0}, n = 0,1,2,3 \quad (10)$$

Plugging Eqs (8) and (9) into Eq. (7), we obtain

$$\sum_{n=0}^{\infty} u_n(x) = \phi(x) - \mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\{R\sum_{n=0}^{\infty} u_n(x)\} - \frac{1}{s^2}\mathcal{L}\{N\sum_{n=0}^{\infty} A_n\}\right] (11)$$

Matching both sides of Eq. (11), we obtain an iterative algorithm in the form

$$\begin{aligned}
u_0(x) &= \phi(x) \\
u_1(x) &= -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_0(x)\right\} - \frac{1}{s^2}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_0\right\}\right] \\
u_2(x) &= -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\{R\sum_{n=0}^{\infty} u_1(x)\} - \frac{1}{s^2}\mathcal{L}\{N\sum_{n=0}^{\infty} A_1\}\right] \quad (12) \\
u_3(x) &= -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_2(x)\right\} - \frac{1}{s^2}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_2\right\}\right] \\
&\vdots \\
u_{n+1}(x) &= -\mathcal{L}^{-1}\left[\frac{1}{s^2}\mathcal{L}\left\{R\sum_{n=0}^{\infty} u_n(x)\right\} - \frac{1}{s^2}\mathcal{L}\left\{N\sum_{n=0}^{\infty} A_n\right\}\right]
\end{aligned}$$

Then the solution of the differential equation is obtained as the sum of decomposed series in the form

$$u(x) \approx u_0(x) + u_1(x) + u_2(x) + \cdots \quad (13)$$

IV MATHEMATICAL ANALYSIS OF THE MODEL VIA LADM

Taking the Laplace transform of both sides of Eq. (1), we get

$$\left.\begin{aligned}
\mathcal{L}\left\{\frac{dS}{dt}\right\} &= \mathcal{L}\{\wp(1-\ell I)-(\alpha I+\gamma_E P)S-\vartheta S\} \\
\mathcal{L}\left\{\frac{dE}{dt}\right\} &= \mathcal{L}\{(\alpha I+\gamma_E P)S-(\vartheta+\rho)E+\wp\ell I\} \\
\mathcal{L}\left\{\frac{dI}{dt}\right\} &= \mathcal{L}\{\rho E-(\vartheta+\sigma)I\} \\
\mathcal{L}\left\{\frac{dR}{dt}\right\} &= \mathcal{L}\{\sigma I-\vartheta R\} \\
\mathcal{L}\left\{\frac{dP}{dt}\right\} &= \mathcal{L}\{\varphi I-\lambda P\}
\end{aligned}\right] \quad (14)$$

Applying the formulas for Laplace transformation, we obtain the equivalent form as follows

$$\begin{aligned}
&w\mathcal{L}\{S\}-S(0)=\mathcal{L}\{\wp\}-\ell\mathcal{L}\{I\}-\alpha\mathcal{L}\{IS\}-\gamma_E\mathcal{L}\{PS\}-\vartheta\mathcal{L}\{S\} \\
&w\mathcal{L}\{E\}-E(0)=\alpha\mathcal{L}\{IS\}+\gamma_E\mathcal{L}\{PS\}-(\vartheta+\rho)\mathcal{L}\{E\}+\wp\ell\mathcal{L}\{I\} \\
&w\mathcal{L}\{I\}-I(0)=\rho\mathcal{L}\{E\}-(\vartheta+\sigma)\mathcal{L}\{I\} \\
&w\mathcal{L}\{R\}-R(0)=\sigma\mathcal{L}\{I\}-\vartheta\mathcal{L}\{R\} \qquad (15) \\
&w\mathcal{L}\{P\}-P(0)=\varphi\mathcal{L}\{I\}-\lambda\mathcal{L}\{P\}
\end{aligned}$$

Using the initial condition in Eq. (4), we get the reduced form as follows

$$\begin{aligned}
&\mathcal{L}\{S\}=\frac{S(0)}{w}+\frac{\wp}{w^2}-\frac{\ell}{w}\mathcal{L}\{I\}-\frac{\alpha}{w}\mathcal{L}\{A\}-\frac{\gamma_E}{w}\mathcal{L}\{B\}-\frac{\vartheta}{w}\mathcal{L}\{S\} \\
&\mathcal{L}\{E\}=\frac{E(0)}{w}+\frac{\alpha}{w}\mathcal{L}\{A\}+\frac{\gamma_E}{w}\mathcal{L}\{B\}-\frac{(\vartheta+\rho)}{w}\mathcal{L}\{E\}+\frac{\wp\ell}{w}\mathcal{L}\{I\} \\
&\mathcal{L}\{I\}=\frac{I(0)}{w}+\frac{\rho}{w}\mathcal{L}\{E\}-\frac{(\vartheta+\sigma)}{w}\mathcal{L}\{I\} \\
&\mathcal{L}\{R\}=\frac{R(0)}{w}+\frac{\sigma}{w}\mathcal{L}\{I\}-\frac{\vartheta}{w}\mathcal{L}\{R\} \qquad (16) \\
&\mathcal{L}\{P\}=\frac{P(0)}{w}+\frac{\varphi}{w}\mathcal{L}\{I\}-\frac{\lambda}{w}\mathcal{L}\{P\}
\end{aligned}$$

Where $A=IS, B=PS$ are nonlinear terms

Next, we decompose the parameters of interest, (S,E,I,R,P)as infinite series of the form

$$S=\sum_{n=0}^{\infty}S_n,\ \ E=\sum_{n=0}^{\infty}E_n,\ \ I=\sum_{n=0}^{\infty}I_n\,, R=\sum_{n=0}^{\infty}R_n,\ \ P=\sum_{n=0}^{\infty}P_n, \qquad (17)$$

Where the terms S_n, E_n, I_n, R_n and P_n are to be determined via recursion.

Similarly, the nonlinear termsare equally decomposed in the form

$$A=\sum_{n=0}^{\infty}A_n, B=\sum_{n=0}^{\infty}B_n, \qquad (18)$$

where $A_n$and B_n are the so-calledAdomian polynomials. The first seven polynomials are considered as follows

$$A = IS$$

$$A_0 = I_0 S_0$$

$$A_1 = I_0 S_1 + I_1 S_0$$

$$A_2 = I_0 S_2 + I_1 S_1 + I_2 S_0$$

$$A_3 = I_0 S_3 + I_1 S_2 + I_2 S_1 + I_3 S_0 \qquad (19)$$

$$A_4 = I_0 S_4 + I_1 S_3 + I_2 S_2 + I_3 S_1 + I_4 S_0$$

$$A_5 = I_0 S_5 + I_1 S_4 + I_2 S_3 + I_3 S_2 + I_4 S_1 + I_5 S_0$$

$$A_6 = I_0 S_6 + I_1 S_5 + I_2 S_4 + I_3 S_3 + I_4 S_2 + I_5 S_1 + I_6 S_0$$

$$A_7 = I_0 S_7 + I_1 S_6 + I_2 S_5 + I_3 S_4 + I_4 S_3 + I_5 S_2 + I_6 S_1 + I_7 S_0$$

$$B = PS$$

$$B_0 = P_0 S_0$$

$$B_1 = P_0 S_1 + P_1 S_0$$

$$B_2 = P_0 S_2 + P_1 S_1 + P_2 S_0$$

$$B_3 = P_0 S_3 + P_1 S_2 + P_2 S_1 + P_3 S_0 \qquad (20)$$

$$B_4 = P_0 S_4 + P_1 S_3 + P_2 S_2 + P_3 S_1 + P_4 S_0$$

$$B_5 = P_0 S_5 + P_1 S_4 + P_2 S_3 + P_3 S_2 + P_4 S_1 + P_5 S_0$$

$$B_6 = P_0 S_6 + P_1 S_5 + P_2 S_4 + P_3 S_3 + P_4 S_2 + P_5 S_1 + P_6 S_0$$

$$B_7 = P_0 S_7 + P_1 S_6 + P_2 S_5 + P_3 S_4 + P_4 S_3 + P_5 S_2 + P_6 S_1 + P_7 S_0$$

Plugging Eqs. (17) and (18) into Eq. (16), we have the following system of equations

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} S_n\right\} = \frac{S(0)}{w} + \frac{\wp}{w^2} - \frac{\ell}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\} - \frac{\alpha}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\} - \frac{\gamma_E}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} B_n\right\} - \frac{\vartheta}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} S_n\right\}$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} E_n\right\} = \frac{E(0)}{w} + \frac{\alpha}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\} + \frac{\gamma_E}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} B_n\right\} - \frac{(\vartheta+\rho)}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} E_n\right\} + \frac{\wp\ell}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\}$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\} = \frac{I(0)}{w} + \frac{\rho}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} E_n\right\} - \frac{(\vartheta+\sigma)}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\}$$

$$\mathcal{L}\{\textstyle\sum_{n=0}^{\infty} R_n\} = \frac{R(0)}{w} + \frac{\sigma}{w}\mathcal{L}\{\sum_{n=0}^{\infty} I_n\} - \frac{\vartheta}{w}\mathcal{L}\{\sum_{n=0}^{\infty} R_n\} \qquad (21)$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} P_n\right\} = \frac{P(0)}{w} + \frac{\varphi}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\} - \frac{\lambda}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} P_n\right\}$$

Taking the initial condition in Eq. (2) of the form

$$S(0) = S_0 \geq 1, E(0) = E_0 \geq 1, I(0) = I_0 = 1, R(0) = R_0 \geq 1, P(0) = P_0 \geq 1 \quad (22)$$

Substituting Eq. (22) into Eq. (21), the reduced form become

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} S_n\right\} = \frac{1}{w} + \frac{\wp}{w^2} - \frac{\ell}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\} - \frac{\alpha}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\} - \frac{\gamma_E}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} B_n\right\} - \frac{\vartheta}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} S_n\right\}$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} E_n\right\} = \frac{1}{w} + \frac{\alpha}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\} + \frac{\gamma_E}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} B_n\right\} - \frac{(\vartheta+\rho)}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} E_n\right\} + \frac{\wp\ell}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\}$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\} = \frac{1}{w} + \frac{\rho}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} E_n\right\} - \frac{(\vartheta+\sigma)}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\}$$

$$\mathcal{L}\{\textstyle\sum_{n=0}^{\infty} R_n\} = \frac{1}{w} + \frac{\sigma}{w}\mathcal{L}\{\textstyle\sum_{n=0}^{\infty} I_n\} - \frac{\vartheta}{w}\mathcal{L}\{\textstyle\sum_{n=0}^{\infty} R_n\} \quad (23)$$

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} P_n\right\} = \frac{1}{w} + \frac{\varphi}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} I_n\right\} - \frac{\lambda}{w}\mathcal{L}\left\{\sum_{n=0}^{\infty} P_n\right\}$$

Matching both sides of Eq. (23) yield the iterative algorithm as follows.

$$\mathcal{L}\{S_0\} = \frac{1}{w} + \frac{\wp}{w^2}$$

$$\mathcal{L}\{S_1\} = -\frac{\ell}{w}\mathcal{L}\{I_0\} - \frac{\alpha}{w}\mathcal{L}\{A_0\} - \frac{\gamma_E}{w}\mathcal{L}\{B_0\} - \frac{\vartheta}{w}\mathcal{L}\{S_0\}$$

$$\mathcal{L}\{S_2\} = -\frac{\ell}{w}\mathcal{L}\{I_1\} - \frac{\alpha}{w}\mathcal{L}\{A_1\} - \frac{\gamma_E}{w}\mathcal{L}\{B_1\} - \frac{\vartheta}{w}\mathcal{L}\{S_1\}$$

$$\mathcal{L}\{S_3\} = -\frac{\ell}{w}\mathcal{L}\{I_2\} - \frac{\alpha}{w}\mathcal{L}\{A_2\} - \frac{\gamma_E}{w}\mathcal{L}\{B_2\} - \frac{\vartheta}{w}\mathcal{L}\{S_2\} \quad (24)$$

$$\vdots$$

$$\mathcal{L}\{S_{n+1}\} = -\frac{\ell}{w}\mathcal{L}\{I_n\} - \frac{\alpha}{w}\mathcal{L}\{A_n\} - \frac{\gamma_E}{w}\mathcal{L}\{B_n\} - \frac{\vartheta}{w}\mathcal{L}\{S_n\}$$

$$\mathcal{L}\{E_0\} = \frac{1}{w}$$

$$\mathcal{L}\{E_1\} = \frac{\alpha}{w}\mathcal{L}\{A_0\} + \frac{\gamma_E}{w}\mathcal{L}\{B_0\} - \frac{(\vartheta+\rho)}{w}\mathcal{L}\{E_0\} + \frac{\wp\ell}{w}\mathcal{L}\{I_0\}$$

$$\mathcal{L}\{E_2\} = \frac{\alpha}{w}\mathcal{L}\{A_1\} + \frac{\gamma_E}{w}\mathcal{L}\{B_1\} - \frac{(\vartheta+\rho)}{w}\mathcal{L}\{E_1\} + \frac{\wp\ell}{w}\mathcal{L}\{I_1\}$$

$$\mathcal{L}\{E_3\} = \frac{\alpha}{w}\mathcal{L}\{A_2\} + \frac{\gamma_E}{w}\mathcal{L}\{B_2\} - \frac{(\vartheta+\rho)}{w}\mathcal{L}\{E_2\} + \frac{\wp\ell}{w}\mathcal{L}\{I_2\} \quad (25)$$

$$\vdots$$

$$\mathcal{L}\{E_{n+1}\} = \frac{\alpha}{w}\mathcal{L}\{A_n\} + \frac{\gamma_E}{w}\mathcal{L}\{B_n\} - \frac{(\vartheta+\rho)}{w}\mathcal{L}\{E_n\} + \frac{\wp\ell}{w}\mathcal{L}\{I_n\}$$

$$\mathcal{L}\{I_0\} = \frac{1}{w}$$

$$\mathcal{L}\{I_1\} = \frac{\rho}{w}\mathcal{L}\{E_0\} - \frac{(\vartheta+\sigma)}{w}\mathcal{L}\{I_0\}$$

$$\mathcal{L}\{I_2\} = \frac{\rho}{w}\mathcal{L}\{E_1\} - \frac{(\vartheta+\sigma)}{w}\mathcal{L}\{I_1\}$$

$$\mathcal{L}\{I_3\} = \frac{\rho}{w}\mathcal{L}\{E_2\} - \frac{(\vartheta+\sigma)}{w}\mathcal{L}\{I_2\} \quad (26)$$

$$\vdots$$

$$\mathcal{L}\{I_{n+1}\} = \frac{\rho}{w}\mathcal{L}\{E_n\} - \frac{(\vartheta+\sigma)}{w}\mathcal{L}\{I_n\}$$

$$\mathcal{L}\{R_0\} = \frac{1}{w}$$

$$\mathcal{L}\{R_1\} = \frac{\sigma}{w}\mathcal{L}\{I_0\} - \frac{\vartheta}{w}\mathcal{L}\{R_0\}$$

$$\mathcal{L}\{R_2\} = \frac{\sigma}{w}\mathcal{L}\{I_1\} - \frac{\vartheta}{w}\mathcal{L}\{R_1\}$$

$$\mathcal{L}\{R_3\} = \frac{\sigma}{w}\mathcal{L}\{I_2\} - \frac{\vartheta}{w}\mathcal{L}\{R_2\} \quad (27)$$

$$\vdots$$

$$\mathcal{L}\{R_{n+1}\} = \frac{\sigma}{w}\mathcal{L}\{I_n\} - \frac{\vartheta}{w}\mathcal{L}\{R_n\}$$

$$\mathcal{L}\{P_0\} = \frac{1}{w}$$

$$\mathcal{L}\{P_1\} = \frac{\varphi}{w}\mathcal{L}\{I_0\} - \frac{\lambda}{w}\mathcal{L}\{P_0\}$$

$$\mathcal{L}\{P_2\} = \frac{\varphi}{w}\mathcal{L}\{I_1\} - \frac{\lambda}{w}\mathcal{L}\{P_1\}$$

$$\mathcal{L}\{P_3\} = \frac{\varphi}{w}\mathcal{L}\{I_2\} - \frac{\lambda}{w}\mathcal{L}\{P_2\} \quad (28)$$

$$\vdots$$

$$\mathcal{L}\{P_{n+1}\} = \frac{\varphi}{w}\mathcal{L}\{I_n\} - \frac{\lambda}{w}\mathcal{L}\{P_n\}$$

Taking the inverse Laplace transform of the first equations in Eqs. (24)-(28), we obtain

$$\mathcal{L}\{S_0\} = \frac{1}{w} + \frac{\wp}{w^2}$$
$$\mathcal{L}\{E_0\} = \frac{1}{w}$$
$$\mathcal{L}\{I_0\} = \frac{1}{w} \quad (29)$$
$$\mathcal{L}\{R_0\} = \frac{1}{w}$$
$$\mathcal{L}\{P_0\} = \frac{1}{w}$$

Substituting the values of $S_0, E_0, I_{0,} R_0$and P_0 into the first iterates of Eqs. (24) – (28), we get the algorithms

$$\mathcal{L}\{S_1\} = -\frac{(\ell + \alpha + \gamma_E)}{w^3} - \frac{(\gamma_E - \alpha\wp - \vartheta\wp)}{w^4}$$
$$\mathcal{L}\{E_1\} = \frac{(\wp\ell - \vartheta - \rho)}{w^2} + \frac{\alpha + \gamma_E}{w^3} + \frac{\wp(\alpha + \gamma_E)}{w^4}$$
$$\mathcal{L}\{I_1\} = \frac{\rho}{w^2} - \frac{(\vartheta + \sigma)}{w^2} \quad (30)$$
$$\mathcal{L}\{R_1\} = \frac{\sigma}{w^2} - \frac{\vartheta}{w^2}$$
$$\mathcal{L}\{P_1\} = \frac{\varphi}{w^2} - \frac{\lambda}{w^2}$$

Similarly, plugging the values $S_1, E_1, I_{1,} R_1$and P_1 to the second iterates, we obtain

$$\mathcal{L}\{S_2\} = \frac{(\ell + \alpha)(\vartheta + \sigma - \rho)}{w^3} + \frac{\gamma_E(\lambda - \varphi)}{w^3} + \frac{\vartheta(\ell + \alpha + \gamma_E + \vartheta)}{w^3} + \frac{\vartheta^2\wp\ell}{w^4}$$
$$\mathcal{L}\{E_2\} = \frac{\alpha}{w}\left[\frac{1}{w}\left\{\frac{-(\ell + \alpha + \gamma_E + \vartheta)}{w^2}\right\} - \frac{\vartheta\wp\ell}{w^3} + \left(\frac{1}{w} + \frac{\wp\ell}{w^2}\right)\left[\frac{\rho}{w^2} - \frac{(\vartheta + \sigma)}{w^2}\right]\right]$$
$$+ \frac{\gamma_E}{w}\left[-\frac{1}{w}\left\{\frac{(\ell + \alpha + \gamma_E + \vartheta)}{w^2} + \frac{\wp\ell\vartheta}{w^3}\right\} + \left(\frac{1}{w} + \frac{\wp}{w^2}\right)\left\{\frac{\varphi}{w^3} - \frac{\lambda}{w^2}\right\}\right]$$
$$- \frac{(\vartheta + \rho)}{w}\left[\frac{(\wp\ell - \vartheta - \rho)}{w^2} + \frac{\gamma_E}{w^3} + \frac{\alpha + \alpha\wp\ell + \gamma_E\wp\ell}{w^4}\right] + \frac{\wp\ell}{w}\left[\frac{\rho}{w^2} - \frac{(\vartheta + \sigma)}{w^2}\right]$$
$$\mathcal{L}\{I_2\} = \frac{\rho}{w}\left[\frac{\wp\ell - \vartheta - \rho}{w^2} + \frac{\gamma_E}{w^3} + \frac{\alpha + \wp\ell(\alpha + \gamma_E)}{w^4}\right] - \frac{(\vartheta + \sigma)}{w}\left[\frac{\rho}{w^2} - \frac{(\vartheta + \sigma)}{w^2}\right] \quad (31)$$
$$\mathcal{L}\{R_2\} = \frac{\sigma\rho}{w^3} - \frac{\sigma(\vartheta + \sigma)}{w^3} - \frac{\sigma\vartheta}{w^3} + \frac{\vartheta^2}{w^3}$$
$$\mathcal{L}\{P_2\} = \frac{\varphi\rho}{w^3} - \frac{\varphi(\vartheta + \sigma)}{w^3} - \frac{\varphi\lambda}{w^3} + \frac{\lambda^2}{w^3}$$

Substituting the Laplace transform of the quantities on the right-hand side of Eqs (29) – (31) and applying the inverse Laplace transform, we obtain the values$S_3(t), E_3(t), I_3(t), R_3(t), P_3(t)$. Subsequent terms

$S_4(t), S_5(t) \dots S_n(t), E_4(t), E_5(t) \dots E_n(t), I_4(t), I_5(t) \dots I_n(t), R_4(t), R_5(t) \dots R_n(t)$ and $P_4(t), P_5(t) \dots P_n(t)$ can be recursively obtained.

V NUMERICAL SIMULATION

In this subsection, we apply the LADM to the HEV model and determine solutions to the parameters of interest in explicit form by setting $\lambda = 0.03, \varphi = 0.02, \sigma = 0.023801429, \rho = 0.02, \vartheta = 1/67.7, \gamma_E = 0.0005, \alpha = 0.0004, \ell = 0.02, \wp = 0.8$ via simulation and the first few calculations for $S(t), E(t), I(t), R(t), P(t)$ are calculated and presented below.

$$\mathcal{L}\{S_0\} = \frac{1}{w} + \frac{0.8}{w^2}, \mathcal{L}\{E_0\} = \frac{1}{w}, \mathcal{L}\{I_0\} = \frac{1}{w}, \mathcal{L}\{R_0\} = \frac{1}{w}, \mathcal{L}\{P_0\} = \frac{1}{w} \quad (32)$$

$$\mathcal{L}\{S_1\} = -\frac{0.035671}{w^2} - \frac{0.000236337}{w^3}$$

$$\mathcal{L}\{E_1\} = -\frac{0.022572}{w^2} + \frac{0.0005}{w^3} + \frac{0.0004144}{w^4}$$

$$\mathcal{L}\{I_1\} = \frac{0.02}{w^2} - \frac{0.038572}{w^3} \qquad \mathcal{L}\{R_1\} = \frac{0.023801}{w^2} - \frac{0.014771}{w^2} \quad (33)$$

$$\mathcal{L}\{P_1\} = \frac{0.02}{w^2} - \frac{0.03}{w^2}$$

Taking the inverse Laplace transform of both sides of Eq. (32), we obtain the analytical solutions for the parameters as follows

$$S(t) = 1 + 0.764329t - 0.000118169t^2$$

$$E(t) = 1 - 0.022572t + 0.00025t^2 + 0.0000690667t^3$$

$$I(t) = 1 - 0.02t - 0.019286t^2$$

$$R(t) = 1 + 0.023801t - 0.0073855t^2 \quad (34)$$

$$P(t) = 1 + 0.02t - 0.015t^2$$

Using symbolic computational software,Maple 20 we calculate the $[4/4]$ Pade approximant of the infinite series of Eq. (34) which gives the following rational approximations to the solution.

$$S_{Pade}(t) = \frac{0.999 + 0.4402352014t - 0.151940t^2 + 0.0733310t^3 - 0.01962t^4}{1 - 0.324t + 0.09t^2 + 3.008 \times 10^{-15}t^3 + 1.0949594438 \times 10^{-17}t^4}$$

$$E_{Pade}(t) = \frac{1 - 0.0112t - 0.0047461t^2 + 0.071888t^3 + 7.79482454 \times 10^{-7}t^4}{1 + 0.01128t + 6.2 \times 10^{-18}t^2 - 4.06795 \times 10^{-18}t^3 - 3.82 \times 10^{-18}t^4}$$

$$I_{Pade}(t) = \frac{1 - 0.0101011t - 0.00974765t^2 - 0.00038563t^3 - 0.01877468658t^4}{1 + 0.009898t + 0.009736t^2 + 1.63239 \times 10^{-16}t^3 - 6.1549 \times 10^{-17}t^4}$$

$$R_{Pade}(t) = \frac{1 + 0.011946811001t - 0.003836t^2 + 0.000178t^3 - 0.000028298t^4}{1 - 0.01185t + 0.00383158t^2 + 1.2402 \times 10^{-17}t^3 + 1.349 \times 10^{-20}t^4}$$

$$P_{Pade}(t) = \frac{1 + 0.01007t - 0.00760307894274t^2 + 0.00030t^3 - 0.0001139213t^4}{1 - 0.009921t + 0.0075953t^2 - 1.583238 \times 10^{-17}t^3 - 4.70 \times 10^{-19}t^4}$$

VI RESULTS AND DISCUSSION

In this subsection, the obtained results of the model in Eq. (1) are presented in Tables (1-6) and Figures (1-6). The accuracy and efficiency of the hybrid semi-analytical iterative method is confirmed when compared with results from other semi-analytical and numerical methods like Runge-Kutta and Temimi-Ansari method(TAM). The results obtained is in excellent agreement.

Table 1: Comparison of solutions of LADM, LADM-Pade& VHPM for Susceptible Individuals, S(t)

t	LADM	LADM-Pade	VHPM
0	1.0000	1.0000	1.0000
0.2	1.15286	1.15286	1.15286
0.4	1.30571	1.30570	1.30571
0.6	1.45855	1.45856	1.45855
0.8	1.61139	1.61129	1.61129
1.0	1.76421	1.76420	1.76420
1.2	1.91702	1.191702	1.191701
1.4	2.06983	2.06982	2.06981
1.6	1.21262	2.22261	2.22260
1.8	2.37541	2.37540	2.37542
2.0	2.52819	2.52890	2.52821

Table 2: **Comparison of solutions of LADM,LADM-Pade& VHPM for Exposed Individuals, E(t)**

t	LADM	LADM-Pade	VHPM
0	1.0000	1.0000	0.99999
0.2	0.995496	0.995410	0.995510
0.4	0.991016	0.991015	0.991020
0.6	0.986562	0.986561	0.98660
0.8	0.982138	0.9982129	0.982190
1.0	0.977747	0.977750	0.977740
1.2	0.973393	0.973310	0.973320
1.4	0.969079	0.969081	0.969080

1.6	0.964808	0.964807	0.964806
1.8	0.960583	0.960590	0.960499
2.0	0.956409	0.956410	0.956420

Table 3: Comparison of solutions of LADM, LADM-Pade& VHPM for Infected Individuals, I(t)

t	LADM	LADM-Pade	VHPM
0	1.0000	1.0000	1.0000
0.2	0.995229	0.995210	0.995220
0.4	0.988914	0.988910	0.988915
0.6	0.981057	0.981049	0.981058
0.8	0.971657	0.971649	0.971642
1.0	0.960714	0.960799	0.960699
1.2	0.948228	0.948210	0.948221
1.4	0.934199	0.934190	0.934210
1.6	0.918628	0.918612	0.918619
1.8	0.901513	0.901511	0.901519
2.0	0.882856	0.882799	0.882810

Table 4: **Comparison of solutions of LADM, LADM-Pade& VHPM for RecoveredIndividuals, R(t)**

t	LADM	LADM-Pade	VHPM
0	1.0000	1.0000	1.0000
0.2	1.00446	1.00332	1.00439
0.4	1.00834	1.00829	1.00831
0.6	1.01162	1.01160	1.011590
0.8	1.01431	1.01429	1.014100
1.0	1.01642	1.016410	1.016390
1.2	1.01793	1.017920	1.017899
1.4	1.01885	1.018790	1.018520
1.6	1.01917	1.01902	1.019011
1.8	1.01891	1.01899	1.018920
2.0	1.01806	1.01802	1.018039

Table 5: Comparison of solutions of LADM, LADM-Pade& VHPM for Density of Viral load, P(t)

t	LADM	LADM-Pade	VHPM
0	1.0000	1.0000	1.0000
0.2	1.00340	1.00330	1.00330
0.4	1.00560	1.00590	1.00499
0.6	1.00660	1.00680	1.00599
0.8	1.00640	1.00640	1.00590
1.0	1.00500	1.00490	1.00550
1.2	1.00240	1.002390	1.00241
1.4	0.99860	0.99791	0.998590
1.6	0.99360	0.99359	0.99290
1.8	0.98740	0.987300	0.98720
2.0	0.98000	0.98010	0.98002

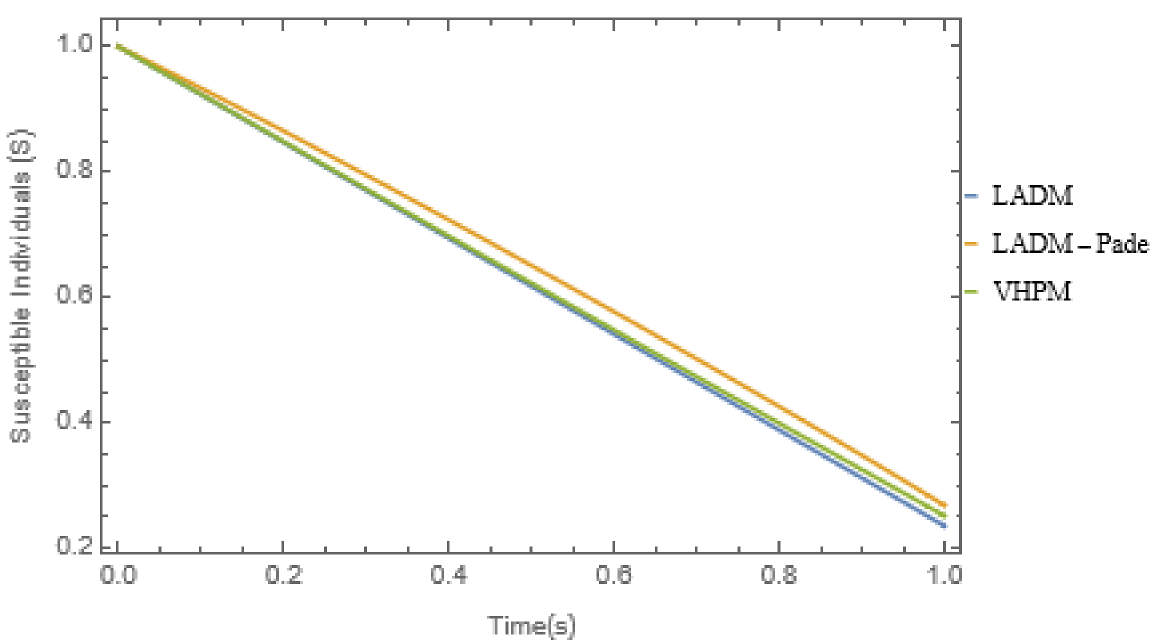

Figure 1. Variation of Susceptible Individuals with Time

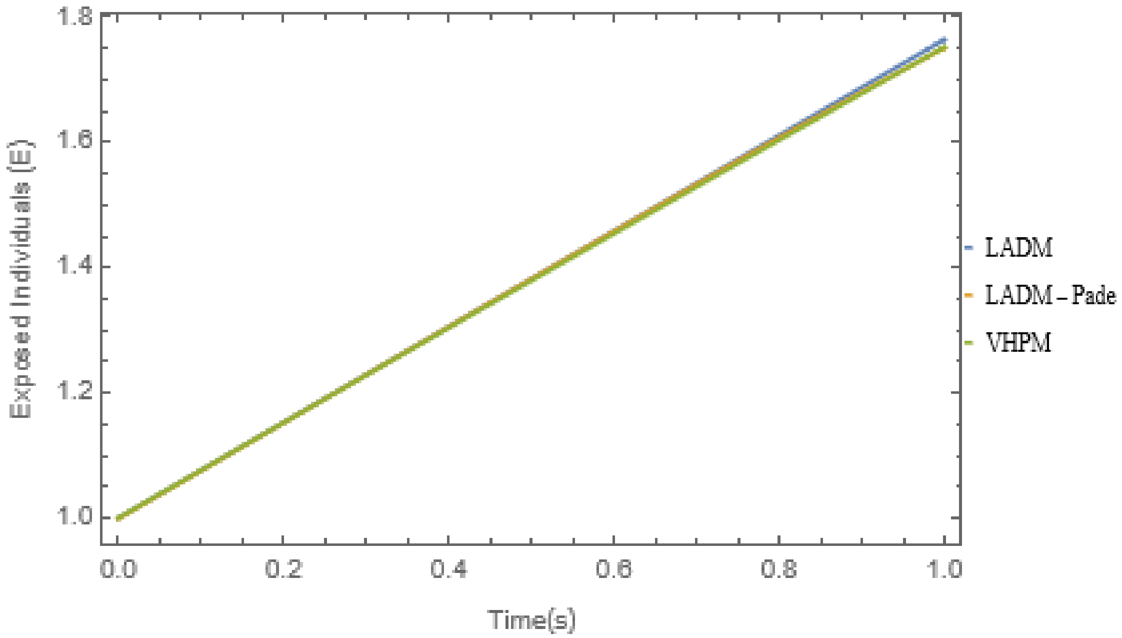

Figure 2. Variation of Exposed Individuals with Time

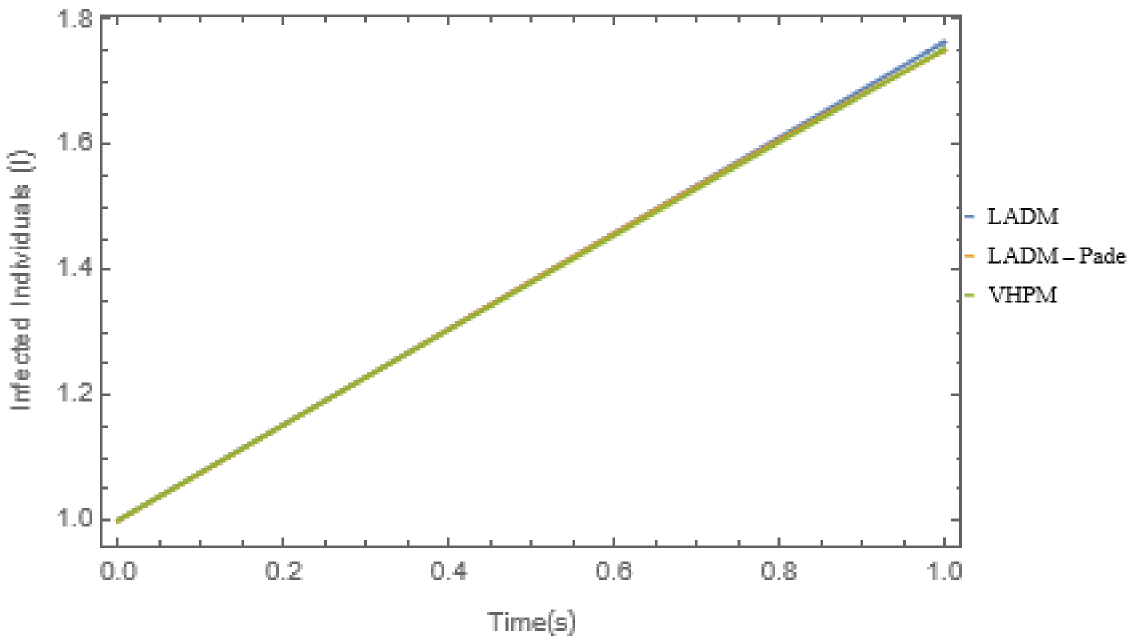

Figure 3. Plot showing Infected Individuals with Time

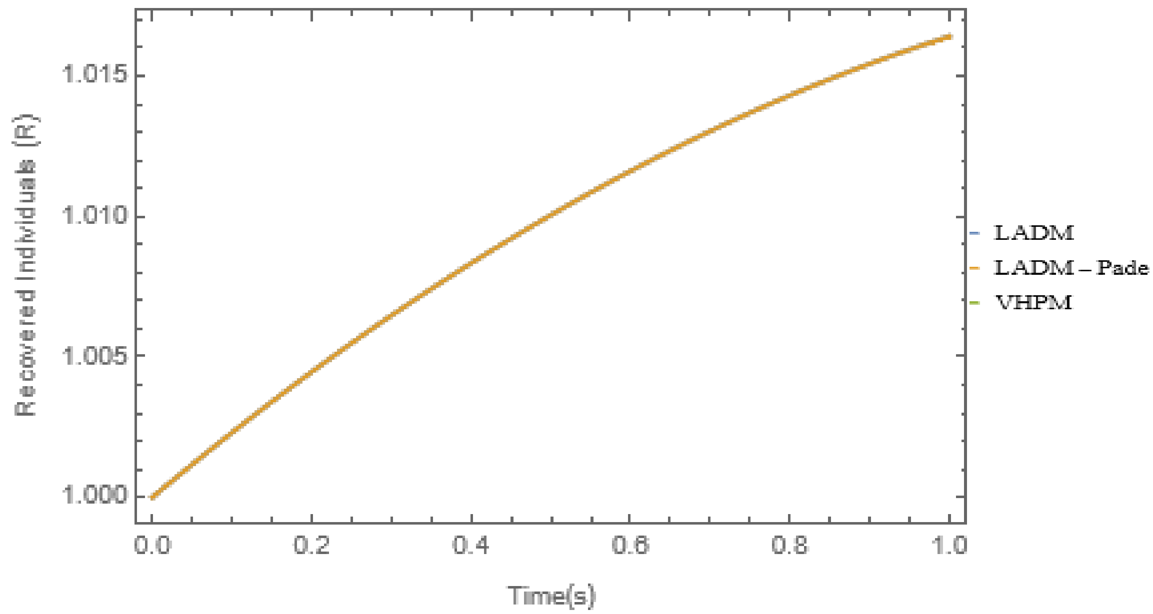

Figure 4. Behaviour of Recovered Individuals with Time

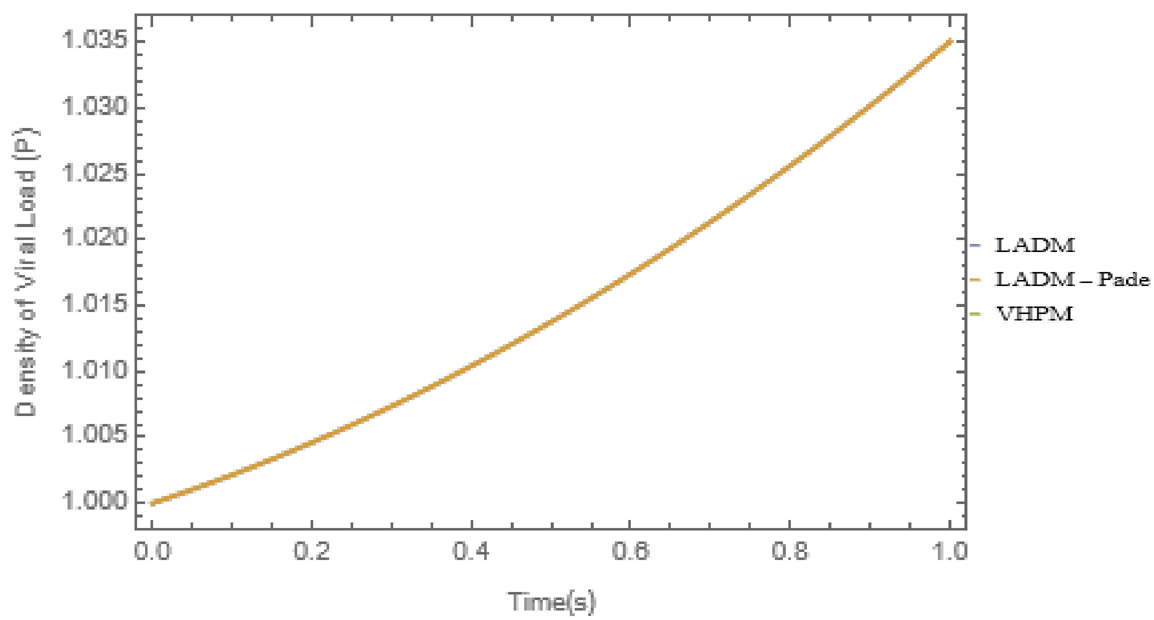

Figure 5. Behaviour of Density of Viral load with Time

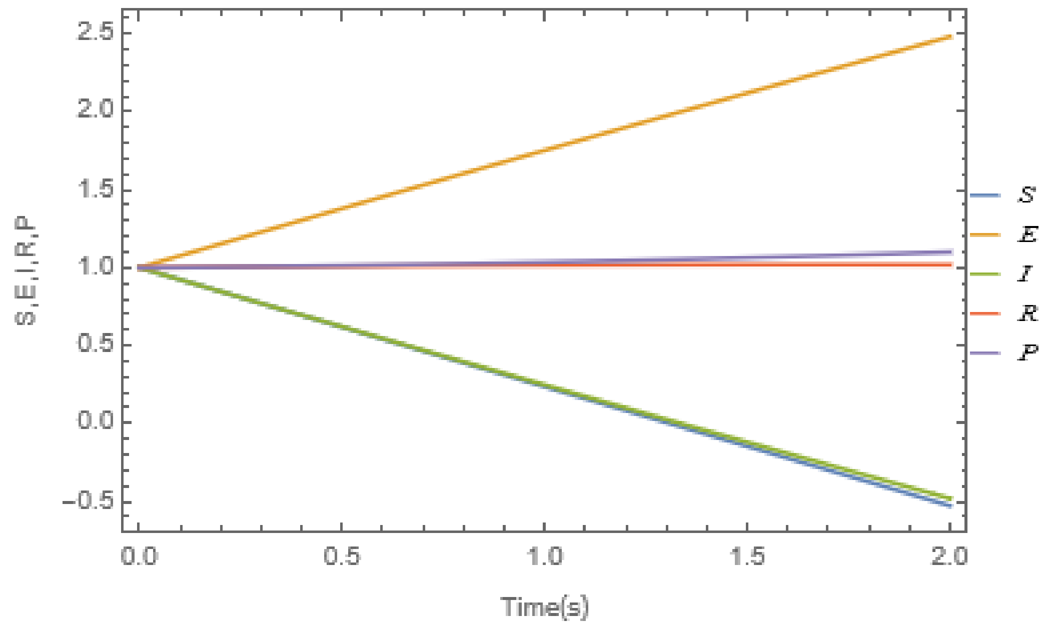

Figure 6. Behaviour of S, E, I, R, P Individuals with Time

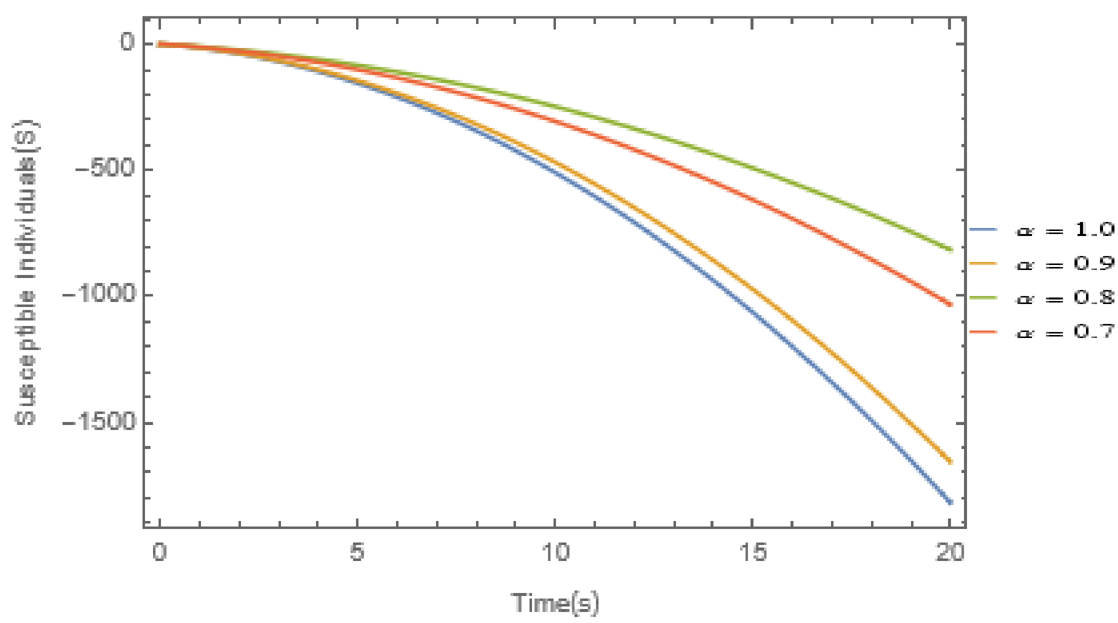

Figure 7. Variation of Susceptible population, S for various values of α and time

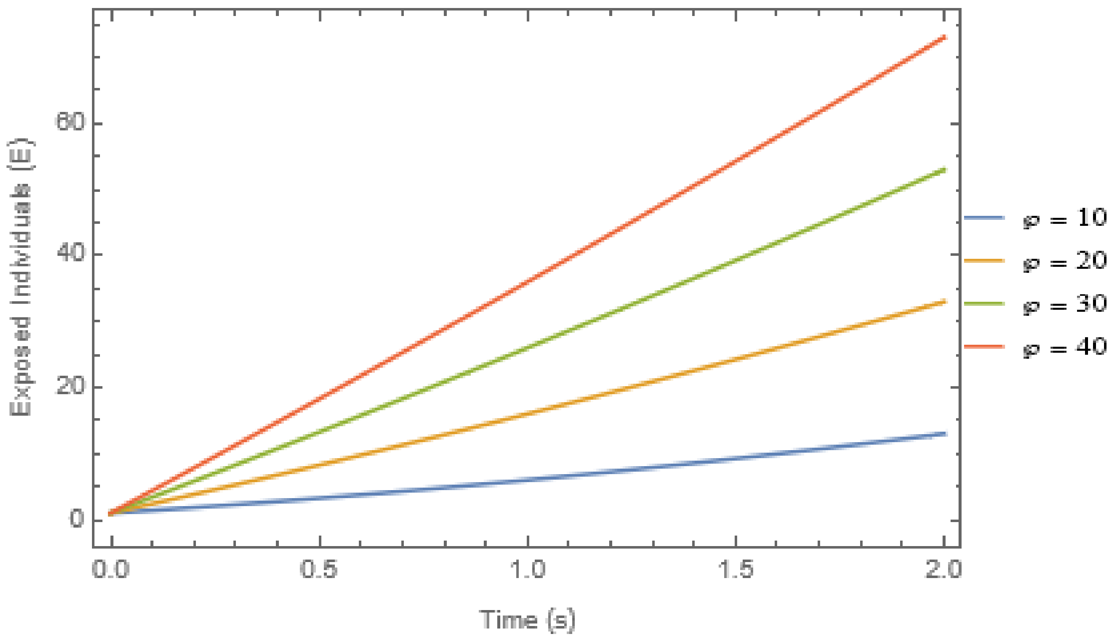

Figure 8. Behaviour of Exposed population, E for various values of φ and time

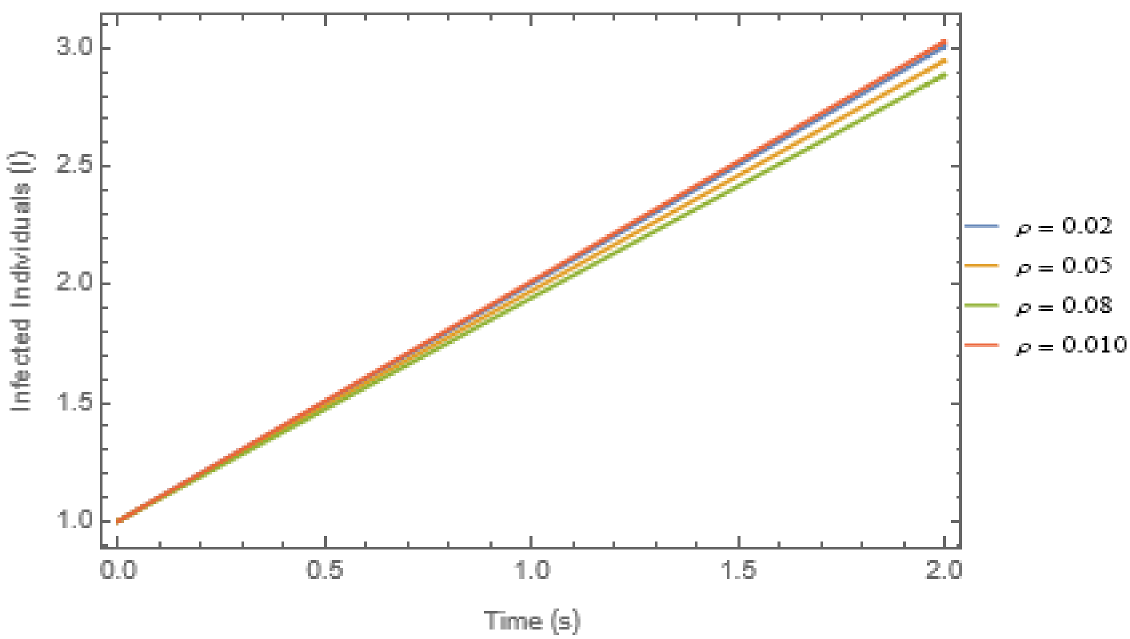

Figure 9. Variation of Infected Individuals, I for various values of ρ, with time

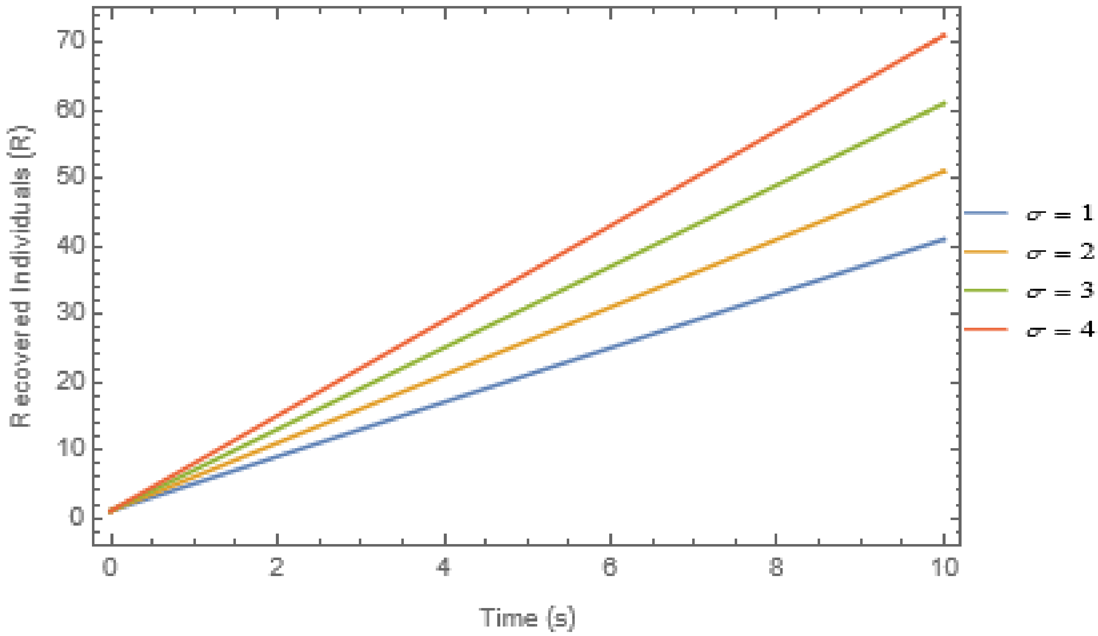

Figure 10. Effect of recovery rate on Recovered Individuals, R with time

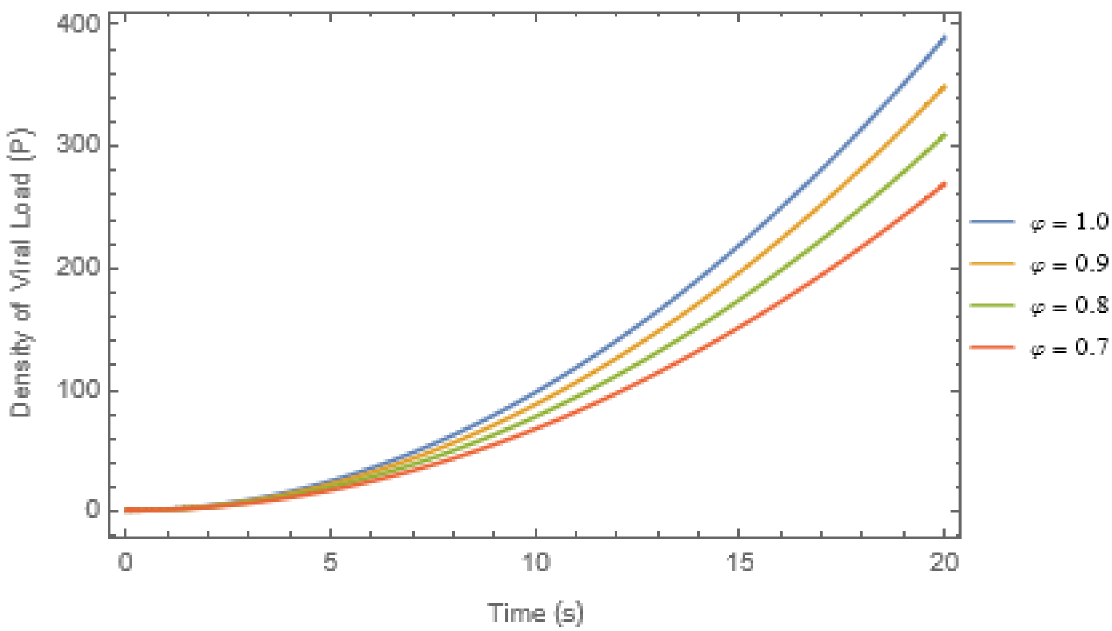

Figure 11. Behaviour of Density of viral load, P with regards to φ and time

VII CONCLUDING REMARKS AND OBSERVATIONS.

In this present article, the dynamics of Hepatitis E virus (HEV) model has been comprehensively investigated analytically using a hybrid semi-analytical iterative scheme, Laplace Adomian decomposition method(LADM). The study examined in detail the analyticalsolution of the model parameters and the result is compared with variational Homotopy perturbation with results displayed in tables and figures. The result obtained reveal, the model parameterssignificantly impacted the model. The proposed method is reliable, efficient, and computationally convenient and the result is a benchmark for future research. All numerical computations are carried out using Mathematica13.

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