International Journal of Scientific Research and Engineering Development-– Volume 4 Issue 6, Nov-Dec 2021
Available at www.ijsred.com

RESEARCH ARTICLE OPEN ACCESS

Analysis of Advanced Transformation Technique to Solve Multi-Objective Optimization Problems

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ABSTRACT

The present paper evaluates the advanced transformation technique proposed by Yesmin and Alim for solving multi-objective optimization problems. The technique has not been formulated appropriately. Unsuitable examples have been used for testing the technique. The results have also not been interpreted correctly. Improvements in the technique and its applications are suggested.

KEYWORDS

Sen's Multi-objective Programming, Harmonic Average Technique, Modified Harmonic Average Technique, Advanced Transformation Technique.

I. INTRODUCTION

Many a times the decision making process needs to achieve multiple objectives simultaneously. The problem becomes little difficult in presence of conflicts amongst objectives. For improving the decision making, Sen [1] proposed the multi-objective optimization (MOO) technique in year 1983. The technique is efficient in achieving multiple objectives at a time. Several new MOO techniques [4], [5], [6],]7], [8], [9], [10], [11] have been proposed during past four decades.Sen [3] observed few deficiencies in many MOO techniques and suggested the improving possibilities. Yesmin and Alim proposed an advanced transformation technique [2] for solving MOO problems. The MOO techniques used in the paper have been evaluated with respect to the formulation of multi-objective functions, examples used for testing these techniques and the interpretation of the results. The formulation of multi-objective function has not been done appropriately. The technique has been tested with unsuitable examples.The wrong conclusions have been drawn with incorrect results.

II. FORMULATION OF MULTI-OBJECTIVE FUNCTION

The multiple objectives are combined for formulation of multi-objective function as explained below:

Optimize $Z = [\text{Max. } Z_1, \text{Max. } Z_2 \ldots\ldots\ldots\text{Max. } Z_r, \text{Min.} Z_{r+1} \ldots\ldots \text{Min. } Z_s]$

Subject to:

$AX \leq / = / \geq b$

All the objection functions are optimized (max./min.) individually for the formulation of Multi-Objective function. The values of individual optima are given as:

$Z_{\text{optima}} = [\Theta_1, \Theta_2 \ldots\ldots\ldots\ldots\ldots \Theta_r, \Theta_{r+1} \ldots\ldots\ldots\ldots \Theta_s]$

The multi-objective functions of various techniques are formulated as detailed below.

II-A CHANDRA SEN'S MOO TECHNIQUE

Sen formulated the multi-objective function as mentioned below:

$$\text{Max. } Z = \sum_{i=1}^{r} \frac{z_i}{\theta_i} - \sum_{J=r+1}^{s} \frac{Z_j}{\theta_j}$$

Subject to common constraints mentioned as above.

II-B AVERAGING TECHNIQUES

Three multi-objective functions have been formulated as explained below:

(1) Harmonic Average Technique (HAT)

$$\text{Max. } Z = \frac{\sum_{i=1}^{r} Z_i}{HM_1} - \frac{\sum_{j=r+1}^{s} Z_j}{HM_2}$$

Where HM_1 = Harmonic mean of individual optima of maximization objective functions and HM_2 = Harmonic mean of individual optima of minimization objective functions.

(2) Modified Harmonic Average Technique (MHAT)

$$Max. Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{j=r+1}^{s} Z_j}{HM}$$

Where

HM= Average of harmonic means of minimum values of maximization and minimization objective functions.

(3) Advanced Transformation Technique (ATT)

$$Max. Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{j=r+1}^{s} Z_j}{\Theta m}$$

Where Θ_m= Minimum value of individual optima

It is clear from the above mentioned multi-objective functions that Senscalarized the objective functions by individual optima for making the multi-objective function dimension free.HoweverYesmin and Alim formulated the multi-objective functionby scalarizingthe objective functions with various forms of harmonic averages of individual optima. The problem of multi-dimension aggregation was noticed in formulation of multi-objective functions.These techniques have been tested using five examples [2] as mentioned below.

III. EXAMPLES

The first example in paper under review is as below:

$Max.\ Z_1 = 4X_1 + 2X_2 - X_1^2 - X_2^2 + 5$

$Max.\ Z_2 = 2X_1 + X_2 - X_1^2$

$Min.\ Z_3 = 6 - 6X_1 + 2X_1^2 - 2X_1X_2 + 2X_2^2$

$Min.\ Z_4 = 2X_1 + 3X_2 - 2X_1^2$

Subject to:

$X_1 + 4X_2 \leq 9$

$X_1 + X_2 \leq 3$

$3X_1 + 2X_2 \leq 8$ and $X_1, X_2 \geq 0$

Example 6a

$Maxi.\ Z_1 = -3X_1^2 - 3X_2^2 - 6X_1X_2 + 30X_1 + 30X_2 - 48$

$Maxi.\ Z_2 = -4X_1^2 - 4X_2^2 - 8X_1X_2 + 38X_1 + 38X_2 - 48$

$Mini.\ Z_3 = 2X_1^2 + 2X_2^2 + 4X_1X_2 - 20X_1 - 20X_2 + 32$

$Mini.\ Z_4 = 5X_1^2 + 5X_2^2 + 10X_1X_2 - 42X_1 - 42X_2 + 34$

Subject To:

$X_1 + 2X_2 \leq 7$

$5X_1 + 2X_2 \leq 11$

$3X_1 + 2X_2 \leq 10$ and $X_1, X_2 \geq 0$

Example 6b

$Max.\ Z_1 = -3X_2^2 + 36X_1 + 34X_2$

$Max.\ Z_2 = -2X_1^2 + 32X_1 + 32X_2$

$Min.\ Z_3 = -4X_1^2 - 36X_1 + 32X_2$

$Min.\ Z_4 = -2X_1^2 - 34X_1 - 30X_2$

$Min.\ Z_5 = 3X_2^2 + 6X_1X_2 - 32X_1 - 32X_2$

Subject To:

$X_1 + 2X_2 \leq 7$

$5X_1 + 2X_2 \leq 11$

$3X_1 + 2X_2 \leq 10$ and $X_1, X_2 \geq 0$

Example 6c

Max. $Z_1 = 0.5\ X_1 + 0.66\ X_2 + 0.833\ X_3$

Max. $Z_2 = 0.25\ X_1 + 0.33\ X_2 + 0.415\ X_3$

Min. $Z_3 = 0.2\ X_1 - 0.34\ X_2 - 0.3\ X_3$

Min. $Z_4 = 0.3\ X_1 - 0.32\ X_2 - 0.32\ X_3$

Subject To:

$3X_1 + 4X_2 + 2X_3 \leq 60$

$2X_1 + X_2 + 2X_3 \leq 40$

$X_1 + 3X_2 + 2X_3 \leq 80$and $X_1, X_2, X_3 \geq 0$

Example 6d

Max. $Z_1 = X_1$

Max. $Z_2 = 2 + X_1 + 2\ X_2$

Max. $Z_3 = 3 + X_2$

Min. $Z_4 = -\ 3\ X_2$

Min. $Z_5 = -X_1 - 3\ X_2$

Subject To:

$2X_1 + 3X_2 \leq 6$

$X_1 \leq 4$

$X_1 + 2X_2 \leq 2$ and $X_1, X_2 \geq 0$

IV. RESULTS AND INTERPRETATION

The solution of individual optimization is presented in table 1. The values of objective functions Z_1 and Z_2 are correct. However the optimal values of the minimization objective functions Z_3 and Z_4 are not minimum. The individual optimal value of Z_3 should be 16.125 instead of 15 as per the value of X_1 and X_2. Using these results the multi-objective functions

Table 1: Individual optimization

Item	Individual Optimization			
	Max. Z_1	Max. Z_2	Min. Z_3	Min. Z_4
X_1	2	0.875	0	0.3125
X_2	1	2.0313	2.25	2.1719
Z_1	10	7.6708	4.4375	5.779
Z_2	1	3.0156	2.25	2.6992
Z_3	0	6.9788	15 (16.125*)	12.3972
Z_4	-1	6.3127	6.75	6.9453

* correct value

have been formulated and optimized. The results are presented in table 2. The values of multi-objective function Z* have been estimated using all the four techniques. It is lowest in Sen's MOO technique and highest in the Advance Transformation technique. Yesmin and Alim declared the superiority of Advanced Transformation Technique on the basis of these results.

Table 2: Multi-objective optimization

Item	Sen's MOO	HAT	MHAT	ATT
X_1	2.18	2.11	2.16	2.29
X_2	0.73	0.52	0.26	0.55
Z*	1.5	2.35	2.85	4.3
Z_1	9.8947	9.7575	9.4268	9.7134
Z_2	0.3376	0.2879	-0.0856	-0.1141
Z_3	0.3078	0.5906	1.3832	0.8342
Z_4	-2.9548	-3.1242	-4.2312	-4.2582

HAT= Harmonic Average Technique, MHAT= Harmonic Average Technique Modified
ATT= Advanced Transformation Technique Z*= Multi-objective Function.

The basic purpose of multi-objective optimization is to achieve all the objectives simultaneously. Hence, the values of all the objective functions should be considered for comparing all the MOO techniques. The achievements of all the four objective functions have been estimated using the values of decision variable X_1 and X_2 obtained in multi-objective optimizations. The results mentioned in table 2 reveals that the achievements of Z_1, Z_2 and Z_3 are superior in Sen's MOO technique. Advanced transformation Technique has obtained the least value of a single objective function Z_4 only. The conclusion of superiority of the Advance Transformation technique over other techniques seems unjustified.Yemin and Alim have also solved four additional examples6a, 6b, 6c, and 6d. The solution of individual optimizations of all these examples is presented in table 3.

Table 3: Individual optimization of examples 6a, 6b, 6c and 6d.

Item	Individual Optimization												
	Example 6a			Example 6b			Example 6c				Example 6d		
	X_1	X_2	Z_i	X_1	X_2	Z_i	X_1	X_2	X_3	Z_i	X_1	X_2	Z_i
Z_1	1	3	24	1	3	111	0	6.67	16.67	18.28	2	0	2
Z_2	1	3	58	1	3	126	0	6.67	16.67	9.12	0	1	4
Z_3	1	3	16	1	3	128	0	6.67	16.67	7.27	0	1	4
Z_4	1	3	54	1	3	122	0	6.67	16.67	7.47	0	1	-3
Z_5				1	3	83					0	1	-3

It is clear from the values of decision variables in achievements of all the objective functions are same. This reveals the absence of conflicts amongst objective functions. The application of any MOO technique is not required. Therefore, it can be concluded that the examples used in the study are not suitable for testing the MOO techniques.

V. CONCLUSION

The above analysis reveals that the values of multi-objective functions should not be used in comparing MOO techniques. The achievements of all the objective functions should be considered for the evalution of the MOO techniques. The appropriate examples should be used for testing the proposed MOO techniques.

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