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On Non-homogeneous Second Degree Equation with Two Unknowns $y^2 = 80x^2 - 31$

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Abstract:

The non-homogeneous second degree equation with two unknowns given by $y^2 = 80x^2 - 31$ is studied to obtain solutions in integers. Some observations between the solutions are presented. Various choices of hyperbolas ,parabolas and Pythagorean triangle are formulated through the linear combinations of integer solutions to the considered equation in title.

Keywords **— Binary quadratic, Non-homogeneous quadratic, Negative Pell, hyperbola, Integer solutions.**

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I. INTRODUCTION

It is well-known that the non-homogeneous second degree equation with two unknowns $y^2 = Dx^2 - N$ is called negative Pell equation ,where, D is a given positive square-free integer and N is a non-zero integer .The above equation seems to have integer solutions for particular values of D and N. For simplicity and clear understanding ,the readers may refer [1- 15]. These results have motivated us for determining integer solutions to other choices of non-homogeneous binary quadratic equations. In this communication ,the non-homogeneous binary quadratic equation $y^2 = 80x^2 - 31$ is considered for determining many non-zero distinct solutions in integers. Some observations between the solutions are presented. Various choices of hyperbolas ,parabolas and Pythagorean triangle are formulated through the linear combinations of integer solutions to the considered equation in title.

II. METHOD OF ANALYSIS

The non-homogeneous second degree equation with two unknowns to be solved is

$$y^2 = 80x^2 - 31 \quad (1)$$

whose smallest positive integer solution is $x_0 = 1, y_0 = 7$

.Employing the solutions of the Pell equation $y^2 = 80x^2 + 1$,the other integer solutions of (1) are given by

$$2\sqrt{80}x_{n+1} = \sqrt{80}f_n + 7g_n$$
$$2y_{n+1} = 7f_n + \sqrt{80}g_n$$

where

$$f_n = (9+\sqrt{80})^{n+1} + (9-\sqrt{80})^{n+1}$$
$$g_n = (9+\sqrt{80})^{n+1} - (9-\sqrt{80})^{n+1}$$

The solutions x_{n+1} & y_{n+1} satisfy the following relations respectively:

$$x_{n+3} - 18x_{n+2} + x_{n+1} = 0$$
$$y_{n+3} - 18y_{n+2} + y_{n+1} = 0$$

Some numerical examples of x & y satisfying (1) are given in the Table1 below:

Table I:Examples

n	x_n	y_n
0	1	7
1	16	143
2	287	2567
3	5150	46063
4	92413	826567

Observations:

1. Both x_{2n} & y_n values are odd where as values of x_{2n+1} are even
2. Expressions representing perfect squares.

❖ $$\frac{286x_{2n+2} - 14x_{2n+3} + 62}{31}$$
❖ $$\frac{2567x_{2n+2} - 7x_{2n+4} + 558}{279}$$
❖ $$\frac{160x_{2n+2} - 14y_{2n+2} + 62}{31}$$
❖ $$\frac{2y_{2n+3} - 32y_{2n+2} + 62}{31}$$

Remarkable observations:

I. Examples of hyperbolas

3. Expressions representing multiple of cubical integers

❖ $286x_{3n+3} - 14x_{3n+4} + 858x_{n+1} - 42x_{n+2}$
❖ $(2567x_{3n+3} - 7x_{3n+5} + 7701x_{n+1} - 21x_{n+3})$
❖ $(160x_{3n+3} - 14y_{3n+3} + 480x_{n+1} - 42y_{n+1})$
❖ $(2y_{3n+4} - 32y_{3n+3} + 6y_{n+2} - 96y_{n+1})$

4. Relations among the solutions

❖ $y_{n+1} + 9x_{n+1} = x_{n+2}$
❖ $y_{n+2} + x_{n+1} = 9x_{n+2}$
❖ $y_{n+3} = 161x_{n+2} - 9x_{n+1}$
❖ $18y_{n+1} = x_{n+3} - 161x_{n+1}$
❖ $2y_{n+2} = x_{n+3} - x_{n+1}$
❖ $18y_{n+3} = 161x_{n+3} - x_{n+1}$
❖ $x_{n+2} = 9x_{n+1} + y_{n+1}$
❖ $x_{n+3} = 161x_{n+1} + 18y_{n+1}$
❖ $y_{n+2} = 80x_{n+1} + 9y_{n+1}$
❖ $y_{n+3} = 1440x_{n+1} + 161y_{n+1}$
❖ $80x_{n+2} = 9y_{n+2} - y_{n+1}$
❖ $80x_{n+3} = 161y_{n+2} - 9y_{n+1}$

Table II :Hyperbolas

S.NO	(X,Y)	Hyperbola
1	$(-14x_{n+2}+286x_{n+1},-32x_{n+1}+2x_{n+2})$	$X^2-80Y^2=3844$
2	$(-7x_{n+3}+2567x_{n+1},-287x_{n+1}+x_{n+3})$	$X^2-80Y^2=311364$
3	$(160x_{n+1}-14y_{n+1},2y_{n+1}-14x_{n+1})$	$X^2-80X^2=3844$
4	$(2y_{n+2}-32y_{n+1},286y_{n+1}-14y_{n+2})$	$80X^2-Y^2=307520$

II. Examples of parabolas

Table III: Parabolas

S.NO	(X,Y)	Parabola
1	$(286x_{2n+2}-14x_{2n+3}+62,2x_{n+2}-32x_{n+1})$	$80Y^2=31X-3844$
2	$(2567x_{2n+2}-7x_{2n+4}+558,x_{n+3}-287x_{n+1})$	$80Y^2=279X-311364$
3	$(160x_{2n+2}-14y_{2n+2}+62,2y_{n+1}-14x_{n+1})$	$80Y^2=31X-3844$
4	$(2y_{2n+3}-32y_{2n+2}+62,286y_{n+1}-14y_{n+2})$	$Y^2=2480X-307520$

III. Let $p=x_{n+1}+y_{n+1},q=x_{n+1.}$ Note that $p>q>0$. Treat p, q as the generators of the Pythagorean triangle T(α, β, γ), where

$\alpha=2pq,\beta=p^2-q^2,\gamma=p^2+q^2$

Let A,P represent the area and perimeter of of the above pythagorean triangle.
Then the following interesting relations are observed.

a) $\alpha-40\beta+39\gamma=31$

b) $41\alpha-\gamma-\frac{160A}{P}=31$

c) $\frac{2A}{P}=x_{n+1}y_{n+1}$

d) $-40\gamma+41\beta-\frac{4A}{p}=-31$

IV. $9\left(P^4_{\frac{y_{2n}-1}{2}}\right)^2\left(t_{3,\frac{x_{2n}-1}{2}}\right)^2=45\left(P^3_{x_{2n}-1}\right)^2\left(t_{3,\frac{y_{2n}-1}{2}}\right)^2-31\left(t_{3,\frac{y_{2n}-1}{2}}\right)^2\left(t_{3,\frac{x_{2n}-1}{2}}\right)^2$

V. $9\left(P^4_{\frac{y_{2n}-1}{2}}\right)^2\left(P^5_{\frac{x_{2n}-1}{2}}\right)^2=80\left(t_{3,\frac{y_{2n}-1}{2}}\right)^2\left(t_{3,\frac{x_{2n}-1}{2}}\right)^2\left(t_{3,x_{2n}-1}\right)^2-31\left(t_{3,\frac{y_{2n}-1}{2}}\right)^2\left(P^5_{\frac{x_{2n}-1}{2}}\right)^2$

III. CONCLUSIONS

In this paper, an attempt has been made to determine non-zero distinct integer solutions for the non-homogeneous second degree equation $y^2=80x^2-31$. As the second degree equations are many ,the researchers may search for their integer solutions along with suitable properties..

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