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An Approach for Engineering Tuning of PI-Controller with Dynamic Object from Third Order

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Abstract:

An approach is proposed for engineering tuning of the PI-controller with a dynamic third order object. There is a proposal to solve the problem by solving the characteristic equation. As a result of the fourth row dynamic system analysis, the tuning parameters of the PI-controller are calculated. The transitional processes of the closed system (object-controller) are dealt with by assignment and disturbance. For the transitional process by assignment, overshoot $\sigma = 54{,}3\ \%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 0,% inaccuracy is observed in theory. Therefore, the proposed approach for engineering tuning for a PI-controller with a third order dynamic object is suitable for use in fourth order dynamic systems analysis.

Keywords **—PI-controller, tuning, dynamic system,third order object, transfer function**

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I. INTRODUCTION

Fourth order dynamic systems can be obtained in the following cases [1,2,3]:

- object three aperiodic links operating with I-controller;
- object three aperiodic links working with PI-controller;
- object three aperiodic links, working with PID-controller with perfect differentiation;
- objectfour aperiodic links operating with a P-controller;
- objectfour aperiodic links, working with a PD-controller with perfect differentiation.

The operation of the so-defined objects with stated linear controllers results in dynamic systems of fourth order. In the study of these systems - an analysis of the dynamic characteristics and determination of the desired tuning of the controller, two approaches are used. The first approach addresses the universal methodologies and guidelines developed for more sophisticated systems. A great deal of these methodologies are also applicable to lower order systems. The second approach explores the differential equations of the second or third order, which is considered to be easy as the equations are relatively well studied.

As a disadvantage of the first approach, it can be noted that it does not always give accurate results. In some cases, it can not be used. Its advantage is that it is easier and more suitable for engineering work.

As a disadvantage of the second approach (above all for third order systems) it can be noted the relative complexity of the research in engineering work. An important advantage here is higher accuracy.

The theoretical investigation of fourth order systems is generally not complicated. For conducting quick and accurate engineering calculations related to tuning the controllers, there are not always suitable nomograms and formulas from the first approach.

PROBLEMS WITH THE TUNING OF CONTROLLERS IN FOURTH ORDER SYSTEMS

Fourth and higher order dynamic systems are often used in industrial automation systems for a variety of production processes, but due to their complexity, few authors have attempted to do theoretical research on them [6,7]. The complexity is that the roots of the characteristic equation of the closed ACS (automatic control system) is four, and it is not clear how the fourth real root influences the stability of the system, and hence the indicators of quality of the transitional processes.

POSSIBLE OPTIONS FOR SOLUTION OF THE ASSIGNED TASK

In analyzingfourth order dynamic systems, the determination of dependencies between quality indicators and system parameters is considerably more complicated. One of the possible options for solving the task is through the use of Prof.Vishnegradski's diagram [1]. The diagram he suggests allows to judge not only sustainability but also some key quality indicators. In the study of dynamic systems of third order, he concluded that the nature of the transitional process can be determined without solving the characteristic equation of the system. For this purpose, it is sufficient for hyperbola built according to its parameters - X and Y to be supplemented with three auxiliary curves [1]. He has given an original word formulation of his criterion, which states: To be a dynamic third-order system sustainable, it is both necessary and sufficient to fulfill the following two conditions: 1. All the coefficients of the characteristic equation must be positive; 2. The average output minus the output of the final coefficients of the characteristic equation of the system must be positive. Failure to comply with these conditions will make the third order dynamic system unstable or at the limit of resistance.Other possible options for solving this task are by using Ziegler &Nicols first method, Koppelovich's nomograms and nomograms given in [2]. These are methods for determining the parameters for tuning of the controllers by known data for the transitional characteristic of the control object [3,5].

The purpose of this paper is to offer an engineering tuning for a proportional-integral PI-controller with a dynamic third order object by solving the characteristic equation of the closed system.

PROPOSAL FOR SOLVING THE PROBLEM BY SOLVING THE CHARACTERISTIC EQUATION

Figure 1 shows the structural diagram of a ACS comprising a second order object (three aperiodic units) and a PI-controller.

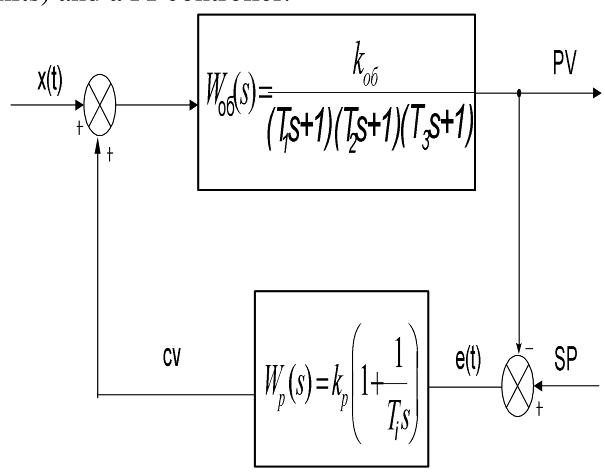

Fig.1. A system with a third order object and a PI-controller

where: SP –Set point ; PV – Process Variable ; e(t)=SP-PV – System error ; CV –Control Variable ; x(t) – Disturbance ; $W_{o6}(s)$ – Transfer function of control object ; k_{o6} – Static gain of the process ; T_1, T_2, T_3 – Time-constants of the process ; $W_p(s)$ – Transfer function of controller ; k_p - proportionality coefficient of the controller; T_i - Time constant of integrationof the controller.

The transfer function of the closed system (fig.1) regarding the assignment is the type

$$W_{sp}(s)=\frac{Y(s)}{Y_{sp}(s)}=\frac{W_{\text{об}}(s)W_p(s)}{1+W_{\text{об}}(s)\cdot W_p(s)}=$$

$$=\frac{\frac{k_{\text{об}}}{(T_1s+1)(T_2s+1)(T_3s+1)}\cdot k_p\left(\frac{T_is+1}{T_is}\right)}{1+\frac{k_{\text{об}}}{(T_1s+1)(T_2s+1)(T_3s+1)}\cdot k_p\left(\frac{T_is+1}{T_is}\right)}=$$

$$=\frac{T_is+1}{\frac{T_1T_2T_3T_i}{k_{\text{об}}k_p}s^4+\frac{T_i(T_1T_2+T_1T_3+T_2T_3)}{k_{\text{об}}k_p}s^3+\frac{T_i(T_1+T_2+T_3)}{k_{\text{об}}k_p}s^2+\frac{T_i(1+k_{\text{об}}k_p)}{k_{\text{об}}k_p}s+1}. \quad (1)$$

The transfer function of the closed system (fig.1) regarding the disturbance is the type

$$W_x(s)=\frac{Y(s)}{X(s)}=\frac{W_{\text{об}}(s)}{1+W_{\text{об}}(s)\cdot W_p(s)}=$$

$$=\frac{\frac{k_{\text{об}}}{(T_1s+1)(T_2s+1)(T_3s+1)}}{1+\frac{k_{\text{об}}}{(T_1s+1)(T_2s+1)(T_3s+1)}\cdot k_p\left(\frac{T_is+1}{T_is}\right)}=$$

$$=\frac{T_i}{k_p}\cdot\frac{s}{\frac{T_1T_2T_3T_i}{k_{\text{об}}k_p}s^4+\frac{T_i(T_1T_2+T_1T_3+T_2T_3)}{k_{\text{об}}k_p}s^3+\frac{T_i(T_1+T_2+T_3)}{k_{\text{об}}k_p}s^2+\frac{T_i(1+k_{\text{об}}k_p)}{k_{\text{об}}k_p}s+1}. \quad (2)$$

We propose that the analysis of the fourth order dynamic system be carried out with a successively connected oscillating and two aperiodic links, i.

$$W(s)=\frac{1}{T_o^2s^2+2\xi T_0s+1}\cdot\frac{1}{Ts+1}\cdot\frac{1}{Ts+1} \qquad (3)$$

Assuming that the time constant of the two aperiodic links is equal to the time constant of the oscillating link, i. $T=T_0$ is obtained

$$W(s)=\frac{1}{T_o^2s^2+2\xi T_0s+1}\cdot\frac{1}{T_0s+1}\cdot\frac{1}{T_0s+1} \qquad (4)$$

For the polynomial in the denominator of expression (4) the characteristic equation is obtained

$$(T_o^2s^2+2\xi T_0s+1)(T_0s+1)(T_0s+1)=T_o^4s^4+(2\xi T_0^3+2T_0^3)s^3+(4\xi T_0^2+2T_0^2)s^2+(2\xi T_0+2T_0)s+1. \quad (5)$$

If we equal the corresponding coefficients in front of s^3, s^2 etc.from the characteristic equation (5) to the coefficients of s^3, s^2 etc. of the polynomial in the denominator of expression (1), the transfer function of the closed system regarding the assignment will have the final appearance

$$W_{sp}(s)=k_{sp}\frac{T_is+1}{T_o^4s^4+(2\xi T_o^3+2T_o^3)s^3+(4\xi T_o^2+2T_o^2)s^2+(2\xi T_o+2T_o)s+1}, \quad (6)$$

where $k_{sp}=1$ is called a coefficent of the system assignment.

The transfer function of the closed disturbance system will have the final appearance

$$W_x(s)=k_x\frac{T_0s}{T_o^4s^4+(2\xi T_o^3+2T_o^3)s^3+(4\xi T_o^2+2T_o^2)s^2+(2\xi T_o+2T_o)s+1}, \quad (7)$$

where $k_x=\frac{T_i}{k_p}\cdot\frac{1}{T_0}=\frac{T_i}{k_p}\cdot\sqrt[4]{\frac{k_{\text{об}}k_p}{T_1T_2T_3T_i}}=\sqrt[4]{\frac{T_i^3k_{\text{об}}}{k_p^3T_1T_2T_3}}$ is called the system disturbance factor.

By comparing the coefficients in front of the corresponding degrees of s in the polynomials of expressions (1) and (6), dependencies between the parameters of the transition process and the parameters of the system can be determined. Equivalent time constant is

$$T_o=\sqrt[4]{\frac{T_1T_2T_3T_i}{k_{\text{об}}k_p}}. \qquad (8)$$

Similarly, the attenuation coefficient ξ is determined. For it three expressions of s^3,s^2 and s of (6) are obtained, ie.

The first expression that can be determined ξ is

$$2\xi T_0^3+2T_0^3=\frac{T_i(T_1T_2+T_1T_3+T_2T_3)}{k_{\text{об}}k_p}. \quad (9)$$

If we only express ξ we obtained

$$\xi=\frac{1}{2}\left[\frac{T_i(T_1T_2+T_1T_3+T_2T_3)}{T_0^3k_{\text{об}}k_p}-1\right]. \qquad (10)$$

The second expression from which can be determined ξ is

$$4\xi T_0^2+2T_0^2=\frac{T_i(T_1+T_2+T_3)}{k_{\text{об}}k_p}. \qquad (11)$$

If we express only ξ it is obtained

$$\xi=\frac{1}{4}\left[\frac{T_i(T_1+T_2+T_3)}{T_0^2k_{\text{об}}k_p}-0,5\right]. \qquad (12)$$

The third expression from which can be determined ξ is

$$2\xi T_0+2T_0=\frac{T_i(1+k_{\text{об}}k_p)}{k_{\text{об}}k_p}. \qquad (13)$$

If we express only ξ it is obtained

$$\xi = \frac{1}{2}\left[\frac{T_i\left(1 + k_{об} k_p\right)}{T_0 k_{об} k_p} - 1\right] \quad . (14)$$

If the expressions (9), (11) and (13) are divided into one another, it is obtained

$$2\xi + 1 = \frac{T_1 T_2 + T_1 T_3 + T_2 T_3}{\left(T_1 + T_2 + T_3\right)\left(1 + k_{об} k_p\right)} . \quad (15)$$

If the expressions (10), (12) and (14) are equal to one anotherand then simplified, an expression of the type (15) is obtained.

If an expression (8) is solved regarding the time constant of integration T_i, it is obtained

$$T_i = \frac{T_0^4 k_{об} k_p}{T_1 T_2 T_3} . \quad (16)$$

The proportionality coefficient of the controller k_p can be determined by an expression (15), ie.

$$k_p = k_{об}\frac{\left(T_1T_2 + T_1T_3 + T_2T_3\right) - \left(T_1 + T_2 + T_3\right)\left(2\xi + 1\right)}{\left(T_1 + T_2 + T_3\right)\left(2\xi + 1\right)} \quad (17)$$

Example: Transitional process of object is given with three aperiodic links. The following algorithm performs the following:

1. Take the transitional process of the object that is smooth and normalizing.
2. Since the object model is of third order – fig.2 (three consecutively connected aperiodic links with equal time constants) - the transitional characteristic is monotone with a transient delay, it is chosen to approximate the method of Ormans [4]. After the approximation, it is determined: $k_{об} = 1$, $T_1 = T_2 = T_3 = 19.5$ sec.
3. By the expressions (16) and (17), the tuning parameters of the PI-controller are calculated.

First, calculate the value of proportionalitycoefficientofthecontroller k_p byanexpression (17), ie.

$$k_p = k_{об}\frac{\left(T_1T_2 + T_1T_3 + T_2T_3\right) - \left(T_1 + T_2 + T_3\right)\left(2\xi + 1\right)}{\left(T_1 + T_2 + T_3\right)\left(2\xi + 1\right)} = \frac{1140,75 - 409,5}{409,5} = 1,786 .$$

Then calculate the value of T_i byanexpression (16), ie.

$$T_i = \frac{T_0^4 k_{об} k_p}{T_1 T_2 T_3} = \frac{19,5^4 . 1,786}{19,5^3} = 34,821 \quad \text{sec} .$$

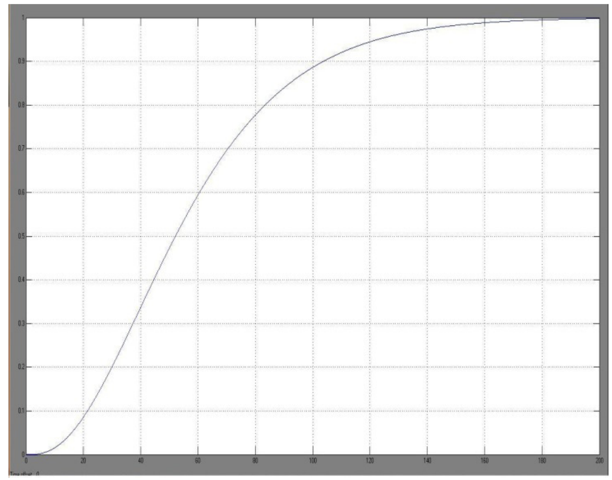

Fig.2.Transitional process of control object

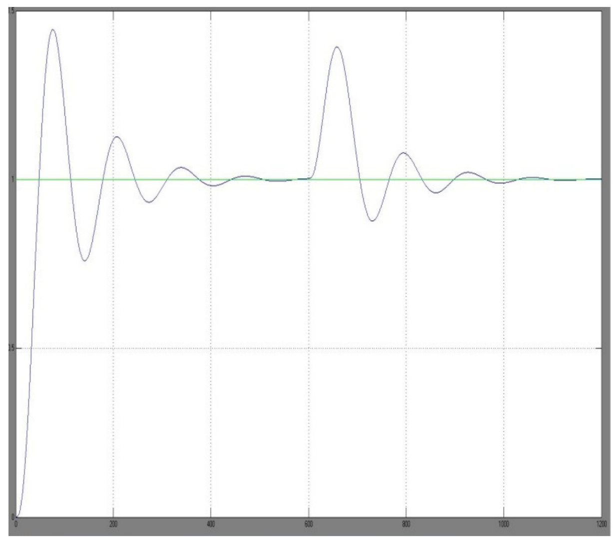

Fig.3.Transitional processes by assignment and by disturbance

The transitional processes of the closed system (fig.1) by assignment and by disturbance are shown in fig.3.For the transitional process by assignment, overshoot $\sigma = 54{,}3$ % occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 0 % inaccuracy is observed in theory. Therefore, the proposed sub-process for engineering tuning of a PI-controller with a dynamic object with three aperiodic links is suitable for use in the analysis of fourth order dynamic systems.

CONCLUSIONS

Anapproachisproposedforengineeringtuningofthe PI-controllerwithadynamicthirdorderobject (threeaperiodiclinkswithequaltimeconstants). Thereisaproposaltosolvetheproblembysolvingthecharacteristicequation. Asaresultoftheanalysisofthefourthorderdynamicsystem, thetuningparametersofthePI-controllerarecalculated.

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