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# Relation Between Bertrand's Postulates and Twin Prime Conjecture

Navaraj Panday and Prashant Shah(Mathematics Initiatives in Nepal)

Author's email: navaraj.research@gmail.com
Co-Author's email: prashant.research21@gmail.com

## Abstract:

This paper starts by considering that twin primes satisfy the bertrand's postulateand a constant is introduced to represent the twin primes using an already existing method. The constants representing the prime numbers and the natural numbers is also introduced. After that, the authors show the relationship between these constants which reveals that despite the fact that twin primes are subsets of primes, the prime representing constants can be formed without the twin primes.

## 1. Introduction:

This paper is inspired by the paper 'A prime representing constant' [1], and throughout this paper we will reference this paper as the original paper. In the original paper, the authors present a constant and a recursive relation that generates sequences of prime. In this paper, we are going to introduce a constant that generates twine prime set S as mentioned above.

Twin primes are the primes that have a difference of 2, for e.g. 3 and 5 or 11 and 13. The concentration of this type of primes is currently unknown and finding if there are infinite twin prime is a famous question in number theory.

Bertrand's postulate states that for any integer more than 3, there exists a prime between an arbitrary integer n and 2n-2. It can be better formulated with another explanation: for every prime number $p_n$, the next term $p_{n+1}$ is less than $2p_n$.[4]

$$p_n < p_{n+1} < 2p_n$$

Bertrand's postulate, by definition, is applicable to primes but it can be expanded to any set of numbers. We do the same to twin primes so that they can me modeled in the formula

The method used in this paper to represent the twin primes:

$$S_{n+1} = \lfloor S_n \rfloor (S_n - \lfloor S_n \rfloor + 1) \qquad (1)$$

Such that $\lfloor S_n \rfloor = P_n$.

The main assumption behind this formation is that the twin primes satisfy Bertrand's postulates. Here, $S_n$ is a twin prime equivalent constant.

  

**2. Main work:**

Let $p_n$ denote the nth member of the set S, then either $p_n + 2$ or $p_n - 2$ is another member of the set S. The above relation cane be shown as $Either\ p_{n+1} - p_n = 2\ or\ p_n - p_{n-1} = 2$. If $p_n$ denotes the nth member of the set S then there exist ;

$$c_{n+1} = \lfloor c_n \rfloor (c_n - \lfloor c_n \rfloor + 1) \qquad (1)$$

Such that $\lfloor c_n \rfloor = p_n$.

Here, $c_1 = 3.840103153843755166 \ldots$

The proof of the existence of this constant is same as in the original paper as the requirement for the existence of this constant is if the series satisfies Bertrand's postulates, which we have assumed that it does. Although the proof is not included in this paper, we are presenting the basis of $c_n$, which is also derivable from the proof included in the original paper.

$$c_n = (p_n - 1\ ) + \frac{p_{n+1} - 1}{p_n} + \frac{p_{n+2} - 1}{p_n . p_{n+1}} + \frac{p_{n+3} - 1}{p_n . p_{n+1} . p_{n+2}} + \cdots$$

Then for $c_1$,

$$c_1 = (p_1 - 1\ ) + \frac{p_2 - 1}{p_1} + \frac{p_3 - 1}{p_1 . p_2} + \frac{p_4 - 1}{p_1 . p_2 . p_3} + \cdots$$

$$c_1 = (3 - 1) + \frac{5 - 1}{3} + \frac{7 - 1}{3.5} + \frac{11 - 1}{3.5.7} + \cdots$$

As seen from (1),

If $c_n$ is irrational then $c_{n+1}$ will be irrational too, which means there will be infinite of these constants, and as each of these constants directly corresponds to a member in the set S, that means Set S will have infinite elements. So, if we can prove $c_1$ as irrational then it proves that there are infinite element in the set S.

Now, we introduce two more constants, both of which is a direct result of the original paper. These constants represent the prime numbers and the natural number, and both have been proved to be irrational. The constant that represents the natural number is the famous Euler number 'e'

e = $(2 - 1) + \frac{3-1}{2} + \frac{4-1}{2.3} + \frac{5-1}{2.3.4} + \cdots$ = 2.718281828459045 . . .

It can be also be proved as this:

$=(2 - 1) + \frac{3-1}{2} + \frac{4-1}{2.3} + \frac{5-1}{2.3.4} + \cdots$

$=(2 - 1) + \frac{2}{2} + \frac{3}{2.3} + \frac{4}{2.3.4} + \cdots$

$=\sum_1^{\infty} \frac{1}{(n-1)!}$

=e

And, we have a constant for prime number, p = $(2-1)+\frac{3-1}{2}+\frac{5-1}{2.3}+\frac{7-1}{2.3.5}+\cdots = 2.920050977316\ldots$

Let $p_n$ denote the $n^{th}$ term of the sequence of the prime representing constant p, such that $p_1 = 2-1$, $p_2 = \frac{3-1}{2}$, $p_3 = \frac{5-1}{2.3}$, etc. And, $k_s$ denote the terms in $e$ that isn't in $p$, for e.g. $k_1 = \frac{4-1}{2.3}, k_2 = \frac{6-1}{2.3.4.5}$, and $c_s$ denote the $s^{th}$ composite number.

Now,

e = $p_1 + p_2 + k_1 + \frac{p_3}{4} + k_2 + \frac{p_4}{4.6} + \ldots + k_s + \frac{p_n}{\prod_s c_s} + \ldots$

e = $\sum_1^n \frac{p_n}{\prod_1^s c_s} + \sum_1^s k_s$

We can write p in terms of $p_n$ as:

p = $\sum_1^n p_n$

As we can clearly see from the definition that , p is greater than e. So, their difference will be:

p – e = $\sum_1^n p_n - (\sum_1^n \frac{p_n}{\prod_1^s c_s} + \sum_1^s k_s)$ → eq. 2

$=\sum_1^n p_n - \sum_1^n \frac{p_n}{\prod_1^s c_s} - \sum_1^s k_s$

$=\sum_1^n p_n \left(1 - \frac{1}{\prod_1^s c_s}\right) - \sum_1^s k_s$

$=\sum_1^n p_n \left(\frac{\prod_1^s c_s - 1}{\prod_1^s c_s}\right) - \sum_1^s k_s$

Here, As n approaches infinity, s approaches infinity too, that means when lim s -->∞,

p – e approaches $\sum_1^n p_n - \sum_1^s k_s$

- ⇨ $\lim_{n,s\to\infty}$ p – e $= \sum_1^n p_n - \sum_1^s k_s$
- ⇨ p – e = p $- \sum_1^s k_s$

That gives us e = $\sum_1^s k_s$

It shows us that 'e' can be made without using any terms from p, even though the numbers representing p are the subset of the numbers representing e.

Now, we do the same with 'p' and 'c'. Where p represents the prime number constant and c represents the twin prime constant.

We use p', k', and c' replacing p, k and c respectively from the equation 2.

  

p = $\sum_1^n \frac{p'_n}{\prod_1^s c'_s} + \sum_1^s k'_s \rightarrow$ eq. 3

c = $\sum_1^n p'_n$

Now, the same relation as in eq. 2 can be shown here as:

⇨ $c - p = \sum_1^n p'_n - (\sum_1^n \frac{p_n}{\prod_1^s c'_s} + \sum_1^s k'_s)$

⇨ $c - p = \sum_1^n p'_n (1 - \frac{p_n}{\prod_1^s c'_s}) - \sum_1^s k'_s)$

⇨ $c - p = \sum_1^n p'_n \left(\frac{\prod_1^s c'_s - 1}{\prod_1^s c'_s}\right) - \sum_1^s k'_s$

We don't know if n goes to infinity but we do know that s goes to infinity. That is to say that we don't know if there are infinite twin prime but we know that there are infinite primes that are not twin prime.

As 's' approaches infinity, c – p approaches $\sum_1^n p'_n - \sum_1^s k'_s$

⇨ $\lim_{s\to\infty} c - p = \sum_1^n p'_n - \sum_1^s k'_s$

This gives p as $\sum_1^s k'_s$. It shows us that 'p' can be made without using any terms from 'c', even though the numbers representing 'c'(twin primes) are the subset of the numbers representing 'p'(primes).

Conclusion:

From the results above, it can be concluded that a constant representing a set can be made without using its one subsetprovided that they both satisfy Bertrand's postulate.Further research on this attribute may provide insight on why exactly this happens and its further application.

**References**

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