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Bipolar Pythagorean Fuzzy Regular Generalized Closed Sets

K.Vishalakshi*, S.Maragathavalli**

*(Research Scholar, Government Arts College, Udumalpet)

** (Assistant Professor, Government Arts College, Udumalpet)

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Abstract:

In this paper, we introduce a new class of generalization of regular closed sets in Bipolar Pythagorean Fuzzy Topological Spaces.We study the concept of Bipolar Pythagorean Fuzzy Regular Generalized closed sets and Bipolar Pythagorean Fuzzy Regular Generalized open sets.

Keywords — Bipolar Pythagorean Fuzzy sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagoren Fuzzy Regular Generalized Closed sets,BipolarPythagorean Fuzzy Regular Generalized Open sets.

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I. INTRODUCTION

Fuzzy set is introduced by Zadeh [1].After that Atanassov [2] introduced the notion of intuitionistic fuzzy sets and Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. Yager [4] proposed another class of nonstandard fuzzy sets, called Pythagorean fuzzy sets and Murat Olgun, Mehmet Ünver, Seyhmus Yardimci[5] introduced the notion of Pythagorean fuzzy topological spaces. Zhang [6] introduced the extension of fuzzy set with bipolarity, called, Bipolar value fuzzy sets. Bosc and Pivert[12]said that "Bipolarity"refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired or considered as acceptable. Negative statement corresponds to what is impossible, rejected or forbidden. Negative preferences to constraints, since they specify which values or objects have to be rejected, while positive preferences corresponds to wishes, as they specify which objects aremore desirable than others, without rejecting those that do not meet the wishes. In bipolar valued fuzzy set interval of membership value is [-1,1]. The positive membership degrees represents the possibilities of something to be happened whereas the negative membership degrees represents the impossibilities. Azhzgappan and Kamaraj[12] investigated Bipolar Fuzzy Topological Spaces. Kim et al[8]constucted bipolar fuzzy set and preserving mappings between them and studied it in the sense of a topological universe. Mohana and Jasnsi[13] has introduced the Bipolar Pythagorean π generalized Pre closed sets in Topological spaces.

In this paper, we introduce Bipolar Pythagorean Fuzzy Regular Generalized Closed sets, Bipolar Pythagorean Fuzzy Regular Generalized Open sets and discussed its properties.

II. PRELIMINARIES

Definition 2.1: Let X be the non empty universe of discourse. A fuzzy set A in X, $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A; $\mu_A(x) \in [0,1]$ is the membership of $x \in X$.

Definition 2.2: Let X be the non empty universe of discourse. An Intuitionistic fuzzy set(IFS) A in X is given by A=$\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A,

respectively, and $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$ for each $x \in X$.
The degree of indeterminacy
$I_A = 1 - (\mu_A(x) - \nu_A(x))$ for each $x \in X$.

Definition 2.3: Let X be the non empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by P=$\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set P, respectively, and $0 \leqslant \mu_P^2(x) + \nu_P^2(x) \leqslant 1$ for each $x \in X$.
The degree of indeterminacy
$I_P = \sqrt{1 - \mu_P^2(x) - \nu_P^2(x)}$ for each $x \in X$.

Definition 2.4: Let X be a non empty set. A Bipolar PythagoreanFuzzy Set $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-) : x \in X\}$ where $\mu_A^+ : X \to [0,1], \nu_A^+ : X \to [0,1], \mu_A^- : X \to [-1,0], \nu_A^- : X \to [-1,0]$ are the mappings such that $0 \leqslant (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leqslant 1$ and $-1 \leqslant (\mu_A^-(x))^2 + (\nu_A^-(x))^2 \leqslant 0$ where
$\mu_A^+(x)$ denote the positive membership degree.
$\nu_A^+(x)$ denote the positive non membership degree.
$\mu_A^-(x)$ denote the negative membership degree.
$\nu_A^-(x)$ denote the negative non membership degree.

Definition 2.5:Let
$A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ and
$B = \{\langle x, \mu_B^+(x), \nu_B^+(x), \mu_B^-(x), \nu_B^-(x)\rangle : x \in X\}$ be two Bipolar Pythagorean Fuzzy sets over X. Then,

(i) The Bipolar Pythagorean fuzzy Complement of A is defined by
$A^c = \{\langle x, \nu_A^+(x), \mu_A^+(x), \nu_A^-(x), \mu_A^-(x)\rangle : x \in X\}$,

(ii) The Bipolar Pythagorean fuzzy intersection of A and B is defined by
$$A \cap B = \{\langle x, \min\{\mu_A^+(x), \mu_B^+(x)\}, \max\{\nu_A^+(x), \nu_B^+(x)\}, max\{\mu_A^-(x), \mu_B^-(x)\}, min\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$$

(iii) The Bipolar Pythagorean fuzzy union of A and B is defined by
$$A \cup B = \{\langle x, \max\{\mu_A^+(x), \mu_B^+(x)\}, \min\{\nu_A^+(x), \nu_B^+(x)\}, min\{\mu_A^-(x), \mu_B^-(x)\}, max\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$$

(iv) A is a Bipolar Pythagorean subset of B and write $A \subseteq B$ if $\mu_A^+(x) \leqslant \mu_B^+(x)$, $\nu_A^+(x) \geqslant \nu_B^+(x)$, $\mu_A^-(x) \geqslant \mu_B^-(x), \nu_A^-(x)\nu_B^-(x)\rangle$ $for\ each\ x \in X$.

(v) $0_X = \{\langle x, 0,1,0,-1\rangle : x \in X\}$and
$1_X = \{\langle x, 1,0,-1,0\rangle : x \in X\}$.

Definition2.6:Bipolar Pythagorean Fuzzy Topological Spaces: Let $X \neq \emptyset$ be a set and τ_p be a family of Bipolar Pythagorean fuzzy subsets of X. If
$T_1 0_p, 1_p \in \tau_p$.
T_2For any $P_1, P_2 \in \tau_p$, we have $P_1 \cap P_2 \in \tau_p$.
T_3 $\cup P_i \in \tau_p$ for arbitrary family$\{P_i \ \ i \in J\} \subseteq \tau_p$.
Then τ_p is called Bipolar Pythagorean Fuzzy Topology on X and the pair (X, τ_p) is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of τ_p is called Bipolar Pythagorean fuzzy open set(BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set(BPFCS).

Definition 2.7:Let (X, τ_p) be a BPFTS and $P = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ be a BPFS over X. Then the Bipolar Pythagorean Fuzzy Interior, Bipolar Pythagorean Fuzzy Closure of P are defined by:
a) BPF$int(P)$ =∪{G / G is a BPFOS in (X, τ_p) and G ⊆ P}.
b) BPF$cl(P)$ =∩{K / K is a BPFCS in (X, τ_p) and P⊆ K}.
It is clear that
a) BPF$int(P)$ is the biggest Bipolar Pythagorean Fuzzy Open set contained in P.
b) BPF$cl(P)$ is the smallest Bipolar Pythagorean Fuzzy Closed set containing P.

Proposition 2.8: Let (X, τ_p) be a BPFTS and A, B be two Bipolar Pythagorean Fuzzy sets in (X, τ_p). Then Bipolar Pythagorean Fuzzy Interior holds the following properties:
a) $int(A) \subseteq A$
b) $A \subseteq B \Longrightarrow int(A) \subseteq int(B)$
c) $int(int(A)) = int(A)$

d) $int(A \cap B) = int(A) \cap int(B)$
e) $int(0_A) = 0_A$
f) $int(1_A) = 1_A$
g) $int(A \cup B) \supseteq int(A) \cup int(B)$

Proposition 2.9: Let (X, τ_p) be a BPFTS and A, B be two Bipolar Pythagorean Fuzzy sets in (X, τ_p). Then Bipolar Pythagorean Fuzzy Closure holds the following properties:
a) $A \subseteq cl(A)$
b) $A \subseteq B \Longrightarrow cl(A) \subseteq cl(B)$
c) $cl(cl(A)) = cl(A)$
d) $cl(A \cup B) = cl(A) \cup cl(B)$
e) $cl(0_A) = 0_A$
f) $cl(1_A) = 1_A$
g) $cl(A \cap B) \subseteq cl(A) \cap cl(B)$

Definition2.10:If BPFS $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ in a BPTS (X, τ_p) is said to be
(a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if $int(cl(A)) \subseteq A$.
(b) Bipolar Pythagorean Fuzzy Semi open set (BPFSOS) if $A \subseteq cl(int(A))$.
(c) Bipolar Pythagorean Fuzzy Preclosed set (BPFPCS) if $cl(int(A)) \subseteq A$.
(d) Bipolar Pythagorean Fuzzy Preopen set (BPFPOS) if $A \subseteq int(cl(A))$.
(e) Bipolar Pythagorean Fuzzy α closed set (BPFαCS) if $cl(int(cl(A)) \subseteq A$.
(f) Bipolar Pythagorean Fuzzy α open set (BPFαOS) if $A \subseteq int(cl(int(A))$.
(g) Bipolar Pythagorean Fuzzy γ closed set (BPFγCS) if $A \subseteq int(cl(A) \cup cl(int(A))$.
(h) Bipolar Pythagorean Fuzzy γ open set (BPFγOS) if $cl(int(A) \cup int(cl(A)) \subseteq A$.
(i) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if $A = cl(int(A))$.
(j) Bipolar Pythagorean Fuzzy regular open set (BPFROS) if $A = int(cl(A))$.
(k) If BPF set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Generalized closed set (BPFGCS), if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPFOS in (X, τ_p).
(l) If BPF set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Generalized open set (BPFGCS), if A^c is a BPFGCS in (X, τ_p).

III. BIPOLAR PYTHAGOREAN FUZZY REGULAR GENERALIZED CLOSED SETS

Definition 3.1: A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space (X, τ_p) is called Bipolar Pythagorean Regular Generalized closed (BPFRGCS in short), if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPF regular Open in (X, τ_p).

In this paper we denote, Bipolar Pythagorean Fuzzy Regular Generalized Closed sets shortly as BPFRGCS and Bipolar Pythagorean Fuzzy Regular Generalized Open sets as BPFRGOS.

Example 3.2: Let X={a,b} and τ_p={$0_p, T, 1_p$} be a BPFT on X, where T=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)). Then, the BPFS A=(x, (0.5, 0.6), (0.4, 0.6), (-0.6, -0.7), (-0.5, -0.7)) is a BPFRGCS in (X, τ_p).

Proposition 3.3: Every BPFCS is BPFRGCS in (X, τ_p) and every BPFGCS is BPFRGCS in (X, τ_p), but not conversely.

Example 3.4: Let X={a,b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.5, 0.7), (0.5, 0.3), (-0.6, -0.8), (-0.6, -0.4)). Here A is the BPFS then A = (x, (0.4, 0.3), (0.6, 0.7), (-0.5, -0.4), (-0.7, -0.8)) is a BPFRGCS, but is not a BPFCS in (X, τ_p). Since $cl(A) = T^c \nsubseteq U$, whenever $A \subseteq U$.

Example 3.5: Let X={a,b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.4, 0.7), (0.6, 0.8), (-0.5, -0.8), (-0.7, -0.4)). Here A is the BPFS then A = (x, (0.5, 0.2), (0.7, 0.8), (-0.4, -0.3), (-0.8, -0.9)) is a BPFRGCS, but is not a BPFGCS in (X, τ_p). Since $cl(A) \nsubseteq U$, whenever $A \subseteq U$.

Theorem 3.6:If A and B are BPFRGCS, then the disjunction of A and B is also BPFRGCS in (X, τ_p).

Proof: Let $A \cup B \subseteq U$, where U is BPF regular open in (X, τ_p). Then $A \subseteq U$ and $B \subseteq U$, where U is BPF regular open, which implies $cl(A) \subseteq U$ and $cl(B) \subseteq U$, this implies $cl(A \cup B) \subseteq U$, since $cl(A \cup B) = cl(A) \cup cl(B)$.

Remark 3.7: The conjunction of two BPFRGCS is not BPFRGCS in (X, τ_p), as shown in the following Example 3.8.

Example 3.8: Let X={a,b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.4, 0.3), (0.6, 0.7), (-0.5, -0.4), (-0.7, -0.8)). The BPFS A =(x, (0.3, 0.9), (0.7, 0.1), (-0.4, -0.9), (-0.8, -0.2)) and B=(x, (0.7, 0.2), (0.3, 0.8), (-0.8, -0.3), (-0.4, -0.9)) is a BPFRGCS.Here $A \cap B$ is not a BPFRGCS in (X, τ_p). Since $A \cap B \subseteq U$, but $cl(A \cap B) \subseteq T^c \nsubseteq U$.

Theorem 3.9: If A is BPFRGCS and $A \subseteq B \subseteq cl(A)$, then B is BPFGRCS in (X, τ_p).

Proof : Let U be Bipolar Pythagorean Fuzzy regular open set such that $B \subseteq U$.Since $A \subseteq B, A \subseteq U$ and A is BPFRGCS,then $cl(A) \subseteq U$, but $cl(B) \subseteq cl(A)$ which implies that $cl(B) \subseteq U$. Therefore, B is BPFRGCS.

Theorem 3.10: A BPF set A is BPFRGopen iff $G \subseteq int(A)$ whenever G is BPFregular closed and $G \subseteq A$.

Proof: Let A be a BPFRGopen set. Let G be BPFregular closed set such that $G \subseteq A$. Then G^c is BPFregular open set and $A^c \subseteq G^c$. Since A^c is BPFRGclosed, $(int(A))^c = cl(A^c) \subseteq G^c$ which implies $G \subseteq int(A)$.

Conversely, suppose A is a BPFset such that $G \subseteq int(A)$ whenever G is BPFregular closed and $G \subseteq A$. We claim that A^c is BPFRGCS. Let $A^c \subseteq U$ and $U^c \subseteq A$. Hence by assumption we have $U^c \subseteq int(A) \Rightarrow (int(A))^c \subseteq U \Rightarrow cl(A^c) \subseteq U$. Therefore, $A^c$is BPFRGCS in (X, τ_p).

Theorem 3.11: Every BPFRCS is BPFRGCS in (X, τ_p), but not conversely true.

Proof: Let U be a BPFregular open set in (X, τ_p) such that $A \subseteq U$. Since every BPFRCS is BPFCS, $cl(A) = A$. By hypothesis, $cl(A) = A \subseteq U$. Hence $cl(A) \subseteq U$, whenever $A \subseteq U$. Thus A is BPFRGCS in (X, τ_p).

Example 3.12: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.6, 0.7), (0.4, 0.2), (-0.7, -0.8), (-0.5, -0.3)) and T_2 = (x, (0.1, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.9, -0.9)). Here A is the BPFS then A = (x, (0.2, 0.2), (0.6, 0.7), (-0.1, -0.2), (-0.7, -0.7)) is a BPFRGCS, Since $cl(A) = T_1 \subseteq U$, whenever $A \subseteq U$,but $cl(int(A)) = T_1^c \neq A$, A is not BPFRCS in (X, τ_p).

Theorem 3.13: Every BPFGCS in BPFRGCS in (X, τ_p), but not conversely true.

Proof : Let U be a BPFregular open set in (X, τ_p). Since A is BPFGCS in (X, τ_p), and every BPFROS is BPFOS in (X, τ_p). By hypothesis, $cl(A) \subseteq U$. Hence A is BPFRGCS in (X, τ_p).

Example 3.14: Let X={a,b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.8, 0.7), (0.2, 0.8), (-0.7, -0.7), (-0.3, -0.9)). Here A is the BPFS then A = (x, (0.3, 0.5), (0.3, 0.3), (-0.4, -0.5), (-0.3, -0.4)) is a BPFRGCS, but not BPFGCS. Since $A \subseteq 1_p$, we have $cl(A) = 1_p \subseteq 1_p$, but A is not BPFGCS in (X, τ_p). Since $A \subseteq U$ and U is BPFOS, we have $cl(A) = 1_p \nsubseteq U$.

Theorem 3.15:Every BPFαCSis BPFRGCS in (X, τ_p), but the converse is not true.

Proof: Let U be a BPFregular open set in (X, τ_p) such that $A \subseteq U$. Since A is BPF α closed set, $cl(int(cl(A))) \subseteq A$. By hypothesis, $\alpha cl(A) \subseteq A \subseteq U$. Hence $\alpha cl(A) \subseteq U$ and A is BPFRGCS in (X, τ_p).

Example 3.16:Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.6, 0.7), (0.4, 0.2), (-0.7, -0.8), (-0.5, -0.3)) and T_2 = (x, (0.1, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.9, -0.9)). Here A is the BPFS then A = (x, (0.2, 0.2), (0.6, 0.7), (-0.1, -0.2), (-0.7, -0.7)) is a BPFRGCS, but is not a BPFαCS in (X, τ_p).Since $cl(A) \subseteq T_1^c \subseteq U$,A is BPFRGCS, but $cl(int(cl(A))) \subseteq T_1^c \nsubseteq A$, Therefore, A is not BPFαCS in (X, τ_p).

Proposition 3.17: Every BPFRGCS and BPFPCS in (X, τ_p) are independent of each other.

Example 3.18: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.7, 0.8), (0.4, 0.2), (-0.7, -0.8), (-0.5, -0.3)) and T_2 = (x, (0.1, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.9, -0.9)). Here A is the BPFS then A = (x, (0.2, 0.2), (0.7, 0.7), (-0.1, -0.2), (-0.7, -0.7)) is a BPFRGCS, but is not a BPFPCS in (X, τ_p). Since $cl(int(A)) = T_1^c \nsubseteq A$.

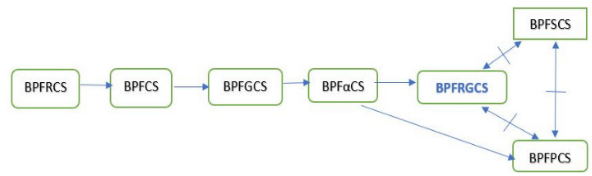
Example 3.19: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.2, 0.3), (0.7, 0.7), (-0.3, -0.3), (-0.7, -0.8)) and T_2 = (x, (0.8, 0.7), (0.2, 0.2), (-0.8, -0.8), (-0.1, -0.2)). Here A is the BPFS then A = (x, (0.1, 0.2), (0.7, 0.8), (-0.1, -0.2), (-0.8, -0.9)) is a BPFPCS, but is not a BPFRGCS in (X, τ_p). Since $cl(A) \nsubseteq T_1$,whenever $A \subseteq U$

Proposition 3.20: Every BPFRGCS and BPFSCS in (X, τ_p) are independent of each other.

Example 3.21:Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.6, 0.7), (0.4, 0.2), (-0.7, -0.8), (-0.5, -0.3)) and T_2 = (x, (0.1, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.9, -0.9)). Here A is the BPFS then A = (x, (0.2, 0.2)(0.6, 0.7), (-0.1, -0.2), (-0.7, -0.7)) is a BPFRGCS, but is not a BPFSCS in (X, τ_p). Since $int(cl(A)) = T_2 \nsubseteq A$.

Example 3.22: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.2), (0.5, 0.8), (-0.5, -0.3), (-0.6, -0.8)) and T_2 = (x, (0.2, 0.2), (0.8, 0.8), (-0.3, -0.2), (-0.9, -0.8)). Here A is the BPFS then A = (x, (0.5, 0.2), (0.5, 0.8), (-0.6, -0.2), (-0.6, -0.8)) is a BPFSCS, but is not a BPFRGCS in (X, τ_p). Since $cl(A) \nsubseteq U$, whenever $A \subseteq U$.

Figure 1: Relation between BPFRGCS and other existed BPFSs



IV. BIPOLAR PYTHAGOREAN FUZZY REGULAR GENERALIZED OPEN SETS

Definition 4.1: A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space (X, τ_p) is called Bipolar Pythagorean Regular Generalized Open (BPFRGOS in short), if $int(A) \supseteq U$ whenever $A \supseteq U$ and U is BPFR-closed in (X, τ_p). Alternatively, a BPFset A is said to be Bipolar Pythagorean Fuzzy Regular Generalized Open Set(BPFRGOS), if its complement A^c is BPFRGCS in (X, τ_p).

Example 4.2: Let X={a,b} and τ_p=$\{0_p, T, 1_p\}$ be a BPFT on X, where T=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)). Then the BPFS A=(x, (0.4, 0.6), (0.5, 0.6), (-0.5, -0.7), (-0.6, -0.7)) is a BPFRGOS in (X, τ_p). Since $int(A) \supseteq U$ whenever $A \supseteq U$ and U is BPFR-closed in (X, τ_p).

Theorem 4.3: A subset A of (X, τ_p) is BPFRGOS iff $B \subseteq int(A)$ whenever B is BPFRGCS in (X, τ_p) and $B \subseteq A$.

Proof:Necessity: Let A be a BPFRGCS in (X, τ_p). Let B be a BPFRGCS in (X, τ_p) and $B \subseteq A$. Then,B^c is BPFRGOS in(X, τ_p) such that $A^c \subseteq B^c$. Since A^c is BPFGRCS, then we have $cl(A^c) \subseteq int(A)$.

Sufficieny: Let $B \subseteq int(A)$ whenever B is BPFRGCS in (X, τ_p) and $B \subseteq A$. Then $A^c \subseteq B^c$ and B^c is BPFRGOS.Byhypothesis, $(int(A))^c \subseteq B^c \Rightarrow cl(A^c) \subseteq B^c A^c$ is BPFRGCS in (X, τ_p). Therefore, A is BPFRGOS in (X, τ_p).

Definition 4.4: For any BPFset A in (X, τ_p) in BPFTspace

a) BPFRG $int(P) = \cup\{$G / G is a BPFGROS in (X, τ_p) and $G \subseteq P\}$.

b) BPFRG $cl(P) = \cap\{$K / K is a BPFGRCS in (X, τ_p) and $P \subseteq K\}$.

Definition 4.5: Every BPFOS is BPFRGOS in (X, τ_p) and every BPFGCS is BPFRGCS in (X, τ_p) and every BPFGOS is BPFRGOS in (X, τ_p), but the converse is not true as shown in the following Example 4.6 and 4.7.

Example 4.6: Let X={a,b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.7, 0.8), (0.3, 0.3), (-0.6, -0.8), (-0.4, -0.4)). Here A is the BPFS then A = (x, (0.8, 0.9)(0.2, 0.2), (-0.8, -0.8), (-0.1, -0.2)) is a BPFRGOS, but is not a BPFOS in (X, τ_p). Since $int(A) = T \neq A$.

Example 4.7: Let X={a,b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.8, 0.7), (0.2, 0.8), (-0.7, -0.7), (-0.3, -0.9)). Here A is the BPFS then A = (x, (0.3, 0.5), (0.3, 0.3), (-0.4, -0.5), (-0.3, -0.4)) is a BPFRGOS, but not BPFGOS. Since $int(A) \supseteq T \not\supseteq T^c$, whereas $A \supseteq T^c$.

Theorem 4.8: Let (X, τ_p) be a BPFTS.If A is BPFS in (X, τ_p), then for every $A \in BPFRGOS$ and every $B \in X$, $int(A) \subseteq B \subseteq A$. B is BPFRGCS in (X, τ_p).

Proof: By hypothesis, $int(A) \subseteq B \subseteq A$. By taking complement on bothsides, we get $(int(A))^c \supseteq B^c \supseteq A \Rightarrow A^c \subseteq B^c \subseteq (int(A))^c$. Let $B^c \subseteq$ BPFRGCS which implies $cl(B^c) \subseteq cl(A^c) \subseteq T$. Hence B^c is BPFRGCS in (X, τ_p). Therefore, B is BPFRGOS in (X, τ_p).

Theorem 4.9: If A and B are two BPFGROS in (X, τ_p), then the conjunction of A and B is also BPFRGOS in (X, τ_p).

Proof: Let A and B be two BPFRGOS in (X, τ_p). Then A^c and B^c are BPFRGCS in (X, τ_p). By Theorem 3.6, $A^c \cup B^c$ is BPFRGCS in (X, τ_p). $(A \cap B)^c$ is BPFRGCS in (X, τ_p). Therefore,$A \cup B$ is BPFRGOS in (X, τ_p).

Remark 4.10: The conjunction of any two BPFRGOS is not BPFRGOS in (X, τ_p) as shown in the following example .

Example 4.11: Let X={a,b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.4, 0.3), (0.6, 0.7), (-0.5, -0.4), (-0.7, -0.8)). Then, the BPFS A =(x, (0.7, 0.1), (0.3, 0.9), (-0.8, -0.2), (-0.4, -0.9)) and B=(x, (0.3, 0.8), (0.7, 0.2), (-0.4, -0.9), (-0.8, -0.3)) is a BPFRGOS in (X, τ_p), but $A \cup B$ is not a BPFRGOS in (X, τ_p).

Theorem 4.12 : Every BPFGOS in BPFRGOS in (X, τ_p), but the converse is not true.

Proof : Let U^c be a BPFregular closed set in (X, τ_p). Since A is BPFGOS in(X, τ_p), and every BPFRCS is BPFCS in (X, τ_p).By hypothesis, $int(A) \supseteq U$. Hence A is BPFRGOS in (X, τ_p).

Example 4.13 : Let X={a,b} and $\tau_p = \{0_p, T, 1_p\}$ be a BPFT on (X, τ_p), where T = (x, (0.8, 0.7), (0.2, 0.8), (-0.7, -0.7), (-0.3, -0.9)). Here A is the BPFS then A = (x, (0.3, 0.3), (0.3, 0.5), (-0.3, -0.4), (-0.4, -0.5)) is a BPFRGCS, but not BPFGCS. Since $A \supseteq U^c$, we have $int(A) = A \supseteq U^c$, but A is not BPFGOS in (X, τ_p).

***Theorem 4.14*:** Every BPF α OS is BPFRGOS in (X, τ_p), but the converse is not true.

Proof: Let U be a BPFregular closed set in (X, τ_p) such that $A \supseteq U$. Since A is BPF α closed set, $int(cl(int(A))) \supseteq A$. By hypothesis, $\alpha int(A) \supseteq A \supseteq U$. Hence $\alpha int(A) \supseteq U$ and A is BPFRGCS in (X, τ_p).

Example 4.15: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.6, 0.7), (0.4, 0.2), (-0.7, -0.8), (-0.5, -0.3)) and T_2 = (x, (0.1, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.9, -0.9)). Here A is the BPFS then A = (x, (0.6, 0.7), (0.2, 0.2), (-0.7, -0.7), (-0.1, -0.2)) is a BPFRGOS, but is not a BPFαOS in (X, τ_p).

Theorem 4.16: Every BPFROS is BPFRGOS in (X, τ_p), but the converse is not true.

Proof: Let U be a BPFregular closed set in (X, τ_p) such that $A \supseteq U$. Since every BPFROS is BPFOS, $int(A) = A$. By hypothesis, $int(A) = A \not\supseteq U$. Hence $int(A) \supseteq U$, whenever $A \supseteq U$. Therefore, A is BPFRGOS in (X, τ_p).

Example 4.17: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.6, 0.7), (0.4, 0.2), (-0.7, -0.8), (-0.5, -0.3)) and T_2 = (x, (0.1, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.9, -0.9)). Here A is the BPFS then A = (x, (0.6, 0.7), (0.2, 0.2), (-0.7, -0.7), (-0.1, -0.2)) is a BPFRGOS, since $int(A) = T_1 \subseteq U$, whenever $A \supseteq U$,but $int(cl(A)) = T_1^c \neq A$, A is not BPFROS in (X, τ_p).

Proposition 4.18: Every BPFRGOS and BPFSOS in (X, τ_p) are independent of each other.

Example 4.19: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.6, 0.7), (0.4, 0.2), (-0.7, -0.8), (-0.5, -0.3)) and T_2 = (x, (0.1, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.9, -0.9)). Here A is the BPFS then A = (x, (0.6, 0.7), (0.2, 0.2), (-0.7, -0.8), (-0.1, -0.2)) is a BPFRGOS, but is not a BPFSOS in (X, τ_p).

Example 4.20: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.5, 0.2), (0.5, 0.8), (-0.5, -0.3), (-0.6, -0.8)) and T_2 = (x, (0.2, 0.2), (0.8, 0.8), (-0.3, -0.2), (-0.9, -0.8)). Here A is the BPFS then A = (x, (0.5, 0.8), (0.5, 0.2), (-0.6, -0.8), (-0.6, -0.2)) is a BPFSOS, but is not a BPFRGOS in (X, τ_p).

Proposition 4.21: Every BPFRGOS and BPFPOS in (X, τ_p) are independent of each other.

Example 4.22: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.7, 0.8), (0.4, 0.2), (-0.7, -0.8), (-0.5, -0.3)) and T_2 = (x, (0.1, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.9, -0.9)). Here A is the BPFS then A = (x, (0.2, 0.2), (0.7, 0.7), (-0.1, -0.2), (-0.7, -0.7)) is a BPFRGOS, but is not a BPFPOS in (X, τ_p). Since $int(cl(A)) = T_1 \not\supseteq A$.

Example 4.23: Let X={a,b} and $\tau_p = \{0_p, T_1, T_2, 1_p\}$ be a BPFT on (X, τ_p), where T_1 = (x, (0.2, 0.3), (0.7, 0.7), (-0.3, -0.3), (-0.7, -0.8)) and T_2 = (x, (0.8, 0.7), (0.2, 0.2), (-0.8, -0.8), (-0.1, -0.2)). Here A is the BPFS then A = (x, (0.1, 0.2), (0.7, 0.8), (-0.1, -0.2), (-0.8, -0.9)) is a BPFPOS, but is not a BPFRGOS in (X, τ_p).

V. CONCLUSION

In this paper we have introduced Bipolar Pythagorean Fuzzy Regular generalized closed sets and studied some of its basic properties. The Relationship between BPFRGCS and other existed BPFs has been studied.

REFERENCES

[1] L. A. Zadeh Fuzzy sets, Information and Control 8(1965), 338-353.
[2] K.T.Atanassov, Intuitionisticfuzzysets, FuzzySetsSyst20(1986), 87-96.
[3] Coker.D, An introduction to intuitionistic topological spaces, Busefal, 81(2000), 51-56.
[4] J.Chen,S.Li,S.Ma and X.Wang,m-Polar Fuzzy Sets:An Extension of Bipolar Fuzzy Sets,The Scientific World Journal, 2014. http://dx.doi.org/10.1155/2014/416530.
[5] Yager R Pythagorean fuzzy subsets in Proceedings of the joint IFSA world

congress NAFIPS annual meeting(2013), 57-61.
[6] T.Fukutake, R.K.Saraf, M.Caldas, M.Mishra, Mappings via Fgp BPFclosed
Sets, Bull. of Fukuaka Univ. of Edu., 52 (2003),11-20.
[7] K.J.Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-Algebras,Bull.Malays.
[8] C.L.Chang, Fuzzy Topological Spaces, J.Math.Anal.Appl.,24(1968), 182- 190.
[9] Xindong peng, Yong Yan, Some Results for Pythagorean Fuzzy Sets, PFinternational Journal of BPFintelligent Systems,30(2015),1133-1160.
[10] K.Sakthivel, Intuitionistic Fuzzy Alpha Generalized Continuous Mappingsand intuitionistic Fuzzy Generalized Irresolute Mappings, Applied Mathematical Sciences, 4 (2010),1831-1842.
[11] Kim, J. H., P. K. Lim, J. G. Lee, and K. Hur, Bipolar Fuzzy Intuitionistic topologicalspaces, Infinite Study, 2018.
[12] M.Azhagappan and M.Kamaraj, Notes on Bipolar valiued fuzzy RW-closed and bipolar valued fuzzy RW-open sets in bipolar valued fuzzy
[14] K.Mohana, R.Jansi, Bipolar Pythagorean π generalized Pre closed setsin Topological Spaces, International journal of analytical and experimental modal analysis, Jan 2020, 164-171.
topological spaces, International Journal of Mathematical Archieve 7(3), (2016) 30-36
[13] P.Bosc, O.Pivert, On a fuzzy bipolar relational algebra, InformationSciences, 219(2013)1-16.

