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DOUBLE POWER OF 2 DECOMPOSITION [DPo2D] OF SOME TREES

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ABSTRACT

Let G be a simple, connected graph with p vertices and q edges. If $G_1,G_2,\ldots,G_n$ are connected edge-disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1,G_2,\ldots,G_n\}$ is said to be a Decomposition of G. In this paper we introduce a new concept called Double power of 2 Decomposition of graphs. A graph G is said to have Double Power of 2 Decomposition if G can be decomposed into subgraphs $\{2G_1, 2G_2, \ldots, 2G_n\}$ such that each G_{2^i} is connected and $|E(G_i)| = 2^i$, for $1 \le i \le n$. Clearly, $q = 4[2^n - 1]$. In this paper, we investigate the properties of some trees which accept Double Power of 2 Decomposition.

Keywords:**Decomposition of graph, Double Power of 2 Decomposition of graphs.**

1. INTRODUCTION

Let G be a simple, connected graph with p vertices and q edges. If $G_1,G_2,\ldots,G_n$ are connected edge-disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1,G_2,\ldots,G_n\}$ is said to be a Decomposition of G. Different type of decomposition of G have been studied in the literature by imposing suitable conditions on the subgraphs G_i. In this paper we introduce a new concept called Double power of 2 Decomposition of graphs. A graph G is said to have Double Power of 2 Decomposition if G can be decomposed into subgraphs $\{2G_1, 2G_2, \ldots, 2G_n\}$ such that each G_{2^i} is connected and $|E(G_i)| = 2^i$, for $1 \le i \le n$. Clearly, $q = 4[2^n - 1]$. In this paper, we investigate the properties of some trees which accept Double Power of 2 Decomposition.

2. PRELIMINARIES

Definition 2.1. Let G be a simple graph of order p and size q. If $G_1, G_2, \ldots, G_n$ are edge-disjoint subgraphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1, G_2, \ldots, G_n\}$ is said to be a Decomposition of G.

Definition 2.2. A graph G is said to have Power of 2 Decomposition if G can be decomposed into n subgraphs $\{G_1, G_2, \ldots, G_n\}$ such that each G_i is connected and $|E(G_i)| = 2^i$, for $1 \leq i \leq n$. Clearly $q = 2[2^n - 1]$ is the sum of 2, 2^2, $2^3, \ldots, 2^n$. Thus we denote the Power of 2 Decomposition as $\{G_2, G_4, G_8, \ldots, G_{2^n}\}$.

Theorem 2.3. A graph G admit Power of 2 Decomposition $\{G_2, G_4, G_8, \ldots, G_{2^n}\}$ if and only if $q = 2[2^n - 1]$ for each $n \in \mathbb{N}$.

Definition 2.4. A Pineapple graph, denoted by K_n^m, is a graph obtained by appending m pendant edges to a vertex of a complete graph K_n, where $m \geq 1$ and $n \geq 3$.

Definition 2.5. The corona of two graphs G_1 and G_2 denoted as $G_1 \odot G_2$, is defined to be the graph obtained by taking one copy of G_1 of order p_1 and p_1 copies of G_2 and joining i^{th} vertex of G_1 with an edge to every vertex in the ith copy of G_2.

3. DOUBLE POWER OF 2 DECOMPOSITION OF GRAPHS

Definition 3.1. A graph G is said to have Double Power of 2 Decomposition [DPo2D] if G can be decomposed into 2n subgraphs $\{2G_1, 2G_2, \ldots, 2G_n\}$ such that each G_i is connected and $|E(G_i)| = 2^i$, $1 \leq i \leq n$.

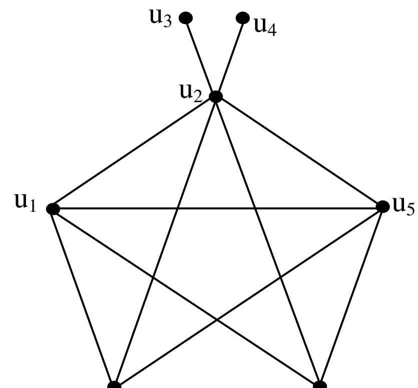
Clearly, $q = 4[2^n - 1]$. We denote the Double Power of 2 Decomposition [DPo2D] as $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$.

4. DOUBLE POWER OF 2 DECOMPOSITION OF TREES

Definition 4.1. A Double Power of 2 Decomposition $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$ in which each G_2^i, $1 \leq i \leq n$ is a tree of size 2^i is said to be a Double Power of 2 Tree Decomposition denoted by DPo2TD.

Definition 4.2. A Double Power of 2 Decomposition $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$ in which each G_2^i, $1 \leq i \leq n$ is a star of size 2^i is said to be a Double Power of 2 Star Decomposition denoted by DPo2SD.

Example 4.3. Consider the pineapple graph K_5^2. The graph K_5^2 is given in the following figure.

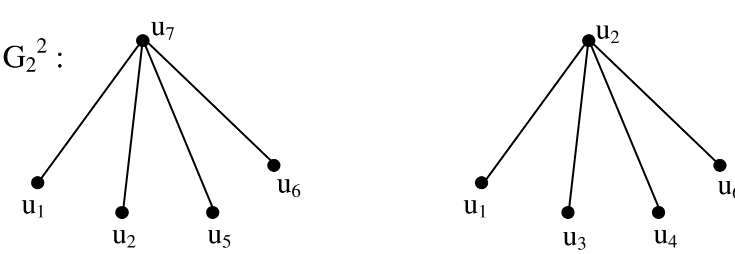


Pineapple Graph K_5^2

The pineapple graph K_5^2 admit Double Power of 2 Decomposition. The DPo2D of K_5^2 is given in the following figure.

G_2 :

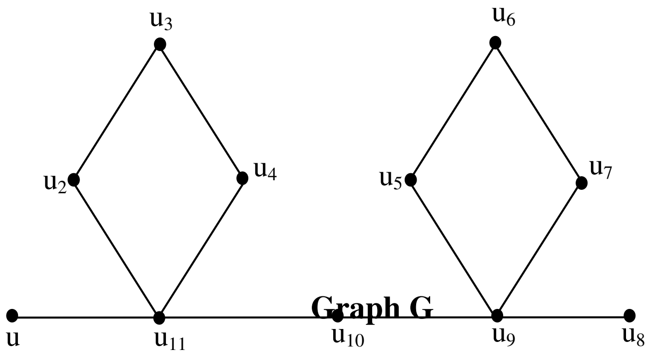
G_2^2 :



Here all the subgraphs G_i's are stars. Hence K_5^2 admit DPo2SD.

Definition 4.4. A Double Power of 2 Decomposition $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$ in which each G_2^i, $1 \le i \le n$ is a path of size 2^i is said to be Double Power of 2 Path Decomposition denoted by DPo2PD.

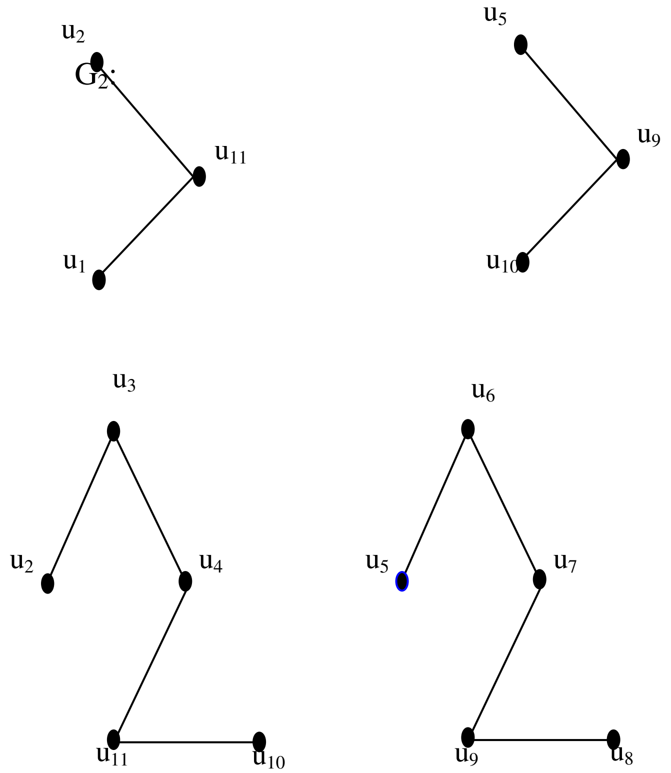
Example 4.5. Consider the graph G given in figure



Graph G

The graph G admit Double Power of 2 Decomposition. The DPo2D of G is given in figure.

G_2:



$G_2^2 G_2^2$

Here all the subgraphs G_i's are paths. Hence G admit DPo2PD.

Theorem 4.6. If a tree T has DPo2D $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$ then T has $2^{n+2} – 3$ vertices.

Proof . Let G be a tree T with p vertices. Then $|E(G)| = p – 1$. Suppose G has DPo2D $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$, then $|E(G)| = q = 4[2^n – 1]$. Hence $p – 1 = 4[2^n – 1]$. This implies $p = 2^{n+2} – 3$.

Theorem 4.7. Let $K_{1, m – 1}$ be a star. Then $K_{1, m-1}$ admit DPo2SD if and only if it has $2^{n+2} – 3$ vertices.

Proof . Let $G = K_{1, m-1}$ be a star with m vertices and m – 1 edges. Assume that G admit DPo2SD, then $q = 4 [2^n – 1]$. Hence $m – 1 = 4[2^n – 1]$. This implies $m = 2^{n+2} – 3$.

Conversely, let $K_{1, m – 1}$ be a star with $2^{n+2} – 3$ vertices. Then $q = 2^{n+2} – 3 – 1 = 2^{n+2} – 4 = 4[2^n – 1]$. Since G is a star, we can divide the edges of G into stars $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$. Hence G admit DPo2SD.

Theorem 4.8. Let P_m be a path. Then P_m admit DPo2PD if and only if it has $2^{n+2} – 3$ vertices.

Proof . Let $G = P_m$ be a path with m vertices and m – 1 edges. Assume that G admit DPo2PD. Then $q = 4[2^n- 1]$. Hence $m – 1 = 4[2^n – 1]$. This implies $m = 2^{n+2} – 3$.

Conversely, let P_m be a path with $2^{n+2} – 3$ vertices. Then $q = 2^{n+2} – 3 – 1 = 2^{n+2} – 4 = 4[2^n – 1]$. Since G is a path, we can divide the edges of G into subpaths $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$. Hence G admit DPo2PD.

Theorem 4.9. Let G = $P_m \odot K_1$. Then G does not admit DPo2D $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$ forany integer m.

Proof . Let G = $P_m \odot K_1$. Then G has 2m vertices and 2m - 1 edges. Hence G contains odd number of edges. But if G admit DPo2D $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$, then G should contain even number of edges. Hence G = $P_m \odot K_1$ does not admit DPo2D $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$ for any integer m.

Theorem 4.10. Let G = $C_m \odot K_1$. Then G does not admit DPo2TD $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$ for any integer m.

Proof . Let G = $C_m \odot K_1$. Then G has 2m vertices and 2m edges. Suppose G admit DPo2TD $\{2G_2, 2G_2^2, \ldots, 2G_2^n\}$, then q = $2^{n+2} - 3$. Hence 2m = $2^{n+2} - 3$. This implies $m = \frac{2^{n+2} - 3}{2}$, which is not an integer. Hence G does not admit DPo2TD.

Corollary 4.11. Let G = $C_m \odot K_1$. Then G does not admit DPo2SD for any integer m.

Corollary 4.12. Let G = $C_m \odot K_1$. Then G does not admit DPo2PD for any integer m.

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