

**Comments on Using Interactive Techniques and New Geometric Average Techniques to Solve MOLFP**

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**Abstract**

In the study [1] of Basia & Shorish, the interactive and geometric average techniques have been used to solve multi-objective linear fractional programming problems. This is a great contribution in the methodology of multi-objective optimization. However, a minor deficiency in the formulation of multi objective function was noticed. The examples used were not appropriate and the solutions have not been interpreted correctly.

**Keywords:** Linear Programming, Multi-objective Optimization. Interactive Techniques, Geometrical Average Techniques,

**1. Introduction**

The purpose of multi objective optimization is to search an appropriate solution that achieves all the objectives under consideration. An efficient multi objective optimization (MOO) method was introduced in year 1983 [2]. The MOO method was successfully applied in resource use panning in agriculture [3]----[11]. A number of new methods of MOO have been proposed during past three decades [12]-----[19]. The methodological weaknesses in these techniques have been discussed and improved MOO techniques have also been proposed in the recent studies [20]----[23]. The study under review is of similar category and deficiencies are discussed below.

**2. Inefficient Multi-Objective Function**

The multi-objective function is formulated by scalarizing the objective functions using various methods. The values of individual optima of various objectives are combined for formulation of multi-objective function. It seems inappropriate when objectives are non commensurate. The approach is also found less efficient in high deviations in the coefficients of decision variables of different objective functions. These problems have been extensively analyzed in the recent studies [ 20]---[23] and improved methods have been suggested. The multi-objective functions have been formulated as described below:

$$\text{Maximize } Z = \frac{\sum_{j=1}^r Z_j}{|\Theta_1|} - \frac{\sum_{j=r+1}^s Z_j}{|\Theta_2|}$$

**Subject to:**

$$AX = b \quad \text{and } X \geq 0$$

$$\Theta_j \neq 0 \quad \text{for } j=1, 2, \dots, s.$$

**Where,**

$\Theta_1$  is the scalarizing factor for Short-Hierarchical, Geometric Arithmetic, New Geometric Arithmetic, Interactive Arithmetic and New Interactive Arithmetic Averages of optimal values of the maximizing objectives.

$\Theta_2$  is the scalarizing factor for Short-Hierarchical, Geometric Arithmetic, New Geometric Arithmetic, Interactive Arithmetic and New Interactive Arithmetic Averages of optimal values of the minimization objectives.

### 3. Unsuitable Examples

Similar to several studies [12]---[19], the examples used by Basia & Shorish were not suitable. Example 1 has been taken from the studies [12] and [13] and the second example has been used in the studies [12]---[14], [18]. Both the examples have been reproduced here :

#### Example 1

$$\text{Max. } Z_1 = (5X_1 + 3X_2)/(X_1 + X_2 + 1)$$

$$\text{Max. } Z_2 = (9X_1 + 5X_2)/(3X_1 + 3X_2 + 3)$$

$$\text{Max. } Z_3 = (3X_1 - 4X_2)/(X_1 + X_2 + 1)$$

$$\text{Max. } Z_4 = (3X_1 + 2X_2)/(2X_1 + 2X_2 + 2)$$

**Subject to:**

$$2X_1 + 4X_2 \geq 8$$

$$X_1 + X_2 \leq 3$$

$$X_1 + 2X_2 \leq 10$$

$$2X_1 + X_2 \leq 5$$

$$X_1 \leq 2$$

$$X_1, X_2 \geq 0$$

and

#### Example 2

$$\text{Max. } Z_1 = (3X_1 - 2X_2)/(X_1 + X_2 + 1)$$

$$\text{Max. } Z_2 = (9X_1 + 3X_2)/(X_1 + X_2 + 1)$$

$$\text{Max. } Z_3 = (3X_1 - 5X_2)/(2X_1 + 2X_2 + 2)$$

$$\text{Min. } Z_4 = (-6X_1 + 2X_2)/(2X_1 + 2X_2 + 2)$$

$$\text{Min. } Z_5 = (-3X_1 - X_2) / (X_1 + X_2 + 1)$$

**Subject to:**

- $X_1 + X_2 \leq 2$
- $9X_1 + X_2 \leq 9$
- $X_1, X_2 \geq 0$

The presence of conflicts amongst objectives has been tested through individual optimization. The results of individual optimization of examples 1 and 2 are presented in tables 1 and 2 respectively, It is clear in the results that all the objectives have been optimized with the single solution in both the examples. There is no conflicts amongst objectives of both the examples and hence, the examples are not suitable for the application of MOO techniques.

**4. Misinterpretation of the Results**

The solutions of the above mentioned examples have been mentioned in the table 1 and 2. The examples have been solved using Short-Hierarchical, Geometric Arithmetic Average, New Geometric Arithmetic Average, Interactive Arithmetic Average and New Interactive Arithmetic Average techniques.

Table 1: Individual and Multi Objective Optimization

Item	Individual Optimization				Multi Objective Optimization			
	Max. Z <sub>1</sub>	Max. Z <sub>2</sub>	Min. Z <sub>3</sub>	Min. Z <sub>3</sub>	GAA	NGAA	IAA	NIAA
X <sub>1</sub> , X <sub>2</sub>	2, 1	2, 1	2, 1	2, 1	2, 1	2, 1	2, 1	2, 1
Z <sub>1</sub>	3.25	3.25	3.25	3.25	3.25	3.25	3.25	3.25
Z <sub>2</sub>	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Z <sub>3</sub>	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Z <sub>4</sub>	1	1	1	1	1	1	1	1
Z*					15.86	31.71	274.55	549.10

Z\* = Multi Objective Function, GAA= Geometric Arithmetic Average, NGAA=New Geometric Arithmetic Average, IAA= Interactive Arithmetic Average, NIAA= New Interactive Arithmetic Average

Table 2: Individual and Multi Objective Optimization

Item	Individual Optimization					Multi Objective Optimization				
	Max. Z <sub>1</sub>	Max. Z <sub>2</sub>	Max. Z <sub>3</sub>	Min. Z <sub>3</sub>	Min. Z <sub>5</sub>	SH	GAA	NGAA	IAA	NIAA
X <sub>1</sub> , X <sub>2</sub>	1, 0	1, 0	1, 0	1, 0	1, 0	1, 0	1, 0	1, 0	1, 0	1, 0
Z <sub>1</sub>	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
Z <sub>2</sub>	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
Z <sub>3</sub>	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75	0.75
Z <sub>4</sub>	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5
Z <sub>5</sub>	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5	-1.5
Z*						6.75	9.14	22.85	107.71	269.27

Z\* = Multi Objective Function, SH= Short Hierarchical, GAA= Geometric Arithmetic Average, NGAA=New Geometric Arithmetic Average, IAA= Interactive Arithmetic Average, NIAA= New Interactive Arithmetic Average.

These techniques have optimized both the examples with the same solutions of their individual optimizations. The values of decision variables  $X_1$  and  $X_2$  are 2, 1 for example 1 and 1, 0 for example 2. However, the values of the multi-objective function  $Z^*$  are different for both the examples in all the MOO techniques used. The superiority of the multi objective optimization technique was measured by the values of multi objective function  $Z^*$  which is not appropriate. The multi objective functions are formulated using different approaches. Hence, the values of multi objective functions are non comparable. However, the solutions in terms of values of basic objective functions along with the values of the decision variables should be compared for analyzing the suitability of the MOO techniques. It is very clear by the individual optimization that all the four objectives of example 1 and five objectives of example 2 are optimal at the same values of decision variables. This clearly indicates the absence of conflicts amongst the objectives of both the examples and hence the application of any MOO technique is not required. Further, the MOO techniques have been evaluated on the basis of the values of multi-objective function which is not logical.

## **5. Conclusion**

The above analysis is very clear in identifying the deficiencies in the methodologies of MOO techniques. Imperfections in the examples and misinterpretation of the results have also been observed. The multi-objective optimization with conflicting objectives can be solved using improved MOO techniques [20]-----[23].

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