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Application of Shehu transform To Mechanics, Newton's Law of Cooling and Electric circuit problems

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Abstract:

In this study, we will discuss the Shehu transform methodto solve the ordinary differential equation of constant coefficient and its application in different areas such as in physics followed by the application to Electric circuit damped and undamped motion and Newton's law of cooling.

Keyword: Differential equation, Shehu transform, Newton's law of cooling, Mechanics.

Introduction.

Many problems in engineering and science can be formulated in terms of differential equations. The ordinary differential equations arise in many areas of Mathematics, as well as in Sciences and Engineering. In order to solve the certain ordinary differential equations integral transforms are widely used. In this paper, we willbe discussed about some applications of first and second order linear differential equationsbased on Newton's law of cooling, damped and undamped motion and electric circuit using Shehu transform.

Shehu Transform

Definition: A new transform called the Shehu transform of the function $v(t)$belonging to a class A, where

$$A = \left\{ v(t): \exists\, N, \eta_1, \eta_2 > 0, |v(t)| < Ne^{\frac{|t|}{\eta_i}}, if\ t \in (-1)^i \times [0\ \infty) \right\}$$

Where $v(t)$defined by$\mathbb{S}[v(t)]$and is defined as:

$$\mathbb{S}[v(t)] = V(s,u) = \int_0^\infty e^{\left(\frac{-st}{u}\right)} v(t)dt \qquad (1.1)$$

And the inverse Shehu transform is defined as

$$\mathbb{S}^{-1}[V(s,u)] = v(t) \text{for } t \geq 0 \qquad (1.2)$$

Properties of the Shehu transform

1. Property 1. Linearity property of Shehu transform. Let the functions $\alpha v(t)$ and $\beta w(t)$ be in set A, then $(\alpha v(t) + \beta w(t)) \in A$, where α and β are nonzero arbitrary constants, and $\mathbb{S}[\alpha v(t) + \beta w(t)] = \alpha \mathbb{S}\,[v(t)] + \beta \mathbb{S}\,[w(t)]$

   Proof: Using the Definition (1.1) of Shehu transform, we get

$$\mathbb{S}\,[\alpha v(t) + \beta w(t)] = \int_0^\infty e^{\left(\frac{-st}{u}\right)} (\alpha v(t) + \beta w(t))dt \qquad 1.3$$

$$= \int_0^\infty e^{\left(\frac{-st}{u}\right)} \alpha v(t)dt + \int_0^\infty e^{\left(\frac{-st}{u}\right)} \beta w(t)dt$$

$$= \alpha \int_0^\infty e^{\left(\frac{-st}{u}\right)} v(t)dt + \beta \int_0^\infty e^{\left(\frac{-st}{u}\right)} w(t)dt$$

$$= \alpha\mathbb{S}[v(t)] + \beta\mathbb{S}[w(t)]$$

Property 2. Let the function $v(\beta t)$ be in set A, where β is an arbitrary constant. Then

$$\mathbb{S}[\beta v(t)] = \frac{u}{\beta}V\left(\frac{s}{\beta}, u\right)$$

Using the Definition 1.1 of Shehu transform, we deduce

$$\mathbb{S}[\beta v(t)] = \int_0^\infty e^{\left(\frac{-st}{u}\right)} v(\beta t)dt \qquad 1.4$$

Substituting $x = \beta t \Rightarrow t = \frac{x}{\beta}$ and $\frac{dt}{dx} = \frac{1}{\beta} \Rightarrow dt = \frac{dx}{\beta}$ in equation 1.4 yield

$$\mathbb{S}[\beta v(t)] = \frac{1}{\beta}\int_0^\infty e^{\left(\frac{-sx}{u\beta}\right)} v(x)dx$$

$$= \frac{1}{\beta}\int_0^\infty e^{\left(\frac{-st}{u\beta}\right)} v(t)dt$$

$$= \frac{u}{\beta}\int_0^\infty e^{\left(\frac{-st}{\beta}\right)} v(ut)dt$$

$$= \frac{u}{\beta}V\left(\frac{s}{\beta}, u\right)$$

Derivative of Shehu transform. If the function $v^{(n)}(t)$is the nth derivative of the function $v(t) \in A$ with respect to t, then its Shehu transform is defined by

$$\mathbb{S}\left[v^{(n)}(t)\right] = \frac{s^n}{u^n}V(s,u) - \sum_{k=0}^{n-1}\left(\frac{s}{u}\right)^{n-(k+1)} v^{(k)}(0) \qquad 1.5$$

When $n = 1$, we obtain the following derivatives with respect to t.

$$\mathbb{S}\left[v^{(1)}(t)\right] = \mathbb{S}[v'(t)] = \frac{s}{u}V(s,u) - v(0) \qquad 1.6$$

When $n = 1$, we obtain the following derivatives with respect to t.

$$\mathbb{S}\left[v^{(2)}(t)\right] = \mathbb{S}[v''(t)] = \frac{s^2}{u^2}V(s,u) - \frac{s}{u}v(0) - v'(0) \qquad 1.7$$

Assume that equation 1.5 true for $n = k$. Now we want to show that for $n = k + 1$

$$\mathbb{S}\left[v^{(k+1)}(t)\right] = \mathbb{S}[(v^{(k)}(t))'] = \frac{s}{u}S\left[v^{(k)}(t)\right] - v^{(k)}(0) \text{using equation 1.6}$$

$$= \frac{s}{u}\left[\frac{s^k}{u^k}\mathbb{S}[v(t)] - \sum_{i=0}^{k-1}\left(\frac{s}{u}\right)^{k-(i+1)} v^{(i)}(0)\right] v^{(k)}(0)$$

$$= \left(\frac{s}{u}\right)^{k+1}\mathbb{S}[v(t)] - \sum_{i=0}^{k}\left(\frac{s}{u}\right)^{k-i} v^{(i)}(0)$$

which implies that Eq (1.5) holds for n = k+1. By induction hypothesis the proof is complete

Property 3: Let the function $v(t) = 1$ be in set A. Then its Shehu transform is given by

$$\mathbb{S}[1] = \frac{u}{s}$$

Poof: Using equation 1.1

$$\mathbb{S}[1] = \int_0^\infty e^{\left(\frac{-st}{u}\right)}\, dt$$

$$= -\frac{u}{s}\lim_{\eta\to\infty}\left[e^{\left(\frac{-s\eta}{u}\right)}\right]_0^\infty = \frac{u}{s}$$

Property 4: Let the function $v(t) = sin(\alpha t)$be in set A. Then its Shehu transform is given by

$$\mathbb{S}[\, sin(\alpha t)] = \frac{\alpha u^2}{s^2 + \alpha^2 u^2}$$

Property 5: Let the function $v(t) = cos(\alpha t)$be in set A. Then its Shehu transform is given by

$$\mathbb{S}[cos(\alpha t)] = \frac{us}{s^2 + \alpha^2 u^2}$$

Property 6: Let the function $v(t) = texp(\alpha t)$and $v(t) = texp(\alpha t)$ be in set A. Then its Shehu transform is given by$\frac{u^2}{(s-\alpha u)^2}$ and $\frac{u}{(s-\alpha u}$ respectively.

Shehu Transform for handling Newton's law of cooling problem

Newton's law of cooling states that the temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself.

If $T_s$be the temperature of the surroundings and T that of the body at anytime t, then

$$\frac{dT}{dt} = -k(T - T_s)$$

Solution using Shehu transform

$$\frac{dT}{dt} = -k(T - T_s)1.8$$

$$\frac{dT}{dt} + kT = kT_s \qquad 1.9$$

Taking Shehu transform both sides of Equation 1.9

$$\mathbb{S}\left[\frac{dT}{dt} + kT = kT_s\right]$$

$$\mathbb{S}\left[\frac{dT}{dt}\right] + k\mathbb{S}[T] = kT_s\,\mathbb{S}[1]$$

$$\frac{s}{u}V(s,u) - T(0) + kV(s,u) = kT_s\frac{s}{u}$$

$$\frac{s}{u}V(s,u) + kV(s,u) = T(0) + kT_s\frac{s}{u}$$

$$V(s,u) = \frac{T(0) + kT_s\frac{s}{u}}{\frac{s}{u} + k}$$

$$= \frac{T(0)}{\frac{s}{u}+k} + \frac{kT_s\frac{s}{u}}{\frac{s}{u}+k} = T(0)\frac{u}{s+uk} + T_s\frac{u}{s} - T_s\frac{u}{s+uk}$$

Applying the inverse transform gives the solution:

$$T(0) + e^{-kt} - T_s e^{-kt} = T(0) + (1 - T_s)e^{-kt}$$

Example: If the temperature of the air is 30°C and the substance cools from 100^0C to $70°C$ in 15 minutes. Find when the temperature will be $40°C$

Solution: Given T_s(Temperature of surrounding medium)= 30^0C, initial temperature at time $t = 0$ is 100^0C and after $t = 15$ min is 70^0C.

Now, from Newton's law of cooling we have

$$T'(t) = -k(T - 30)$$

The above equation can be rewritten as:

$$\frac{dT}{dt} + kT = 30k$$

Take the Shehu Transform both sides:

$$\mathbb{S}\left[\frac{dT}{dt} + kT = 30k\right]$$

$$\Rightarrow \frac{s}{u}T(u,s) - T(0) + kT(s,u) = 30k\frac{u}{s}$$

$$\Rightarrow T(s,u)\left[\frac{s+uk}{u}\right] = 30k\frac{u}{s} + 100$$

$$\Rightarrow T(s,u) = 30k\frac{u^2}{s(s+uk)} + 100\frac{u}{s+uk}$$

$$\Rightarrow T(s,u) = \frac{30u}{s} + 70\frac{u}{s+uk}$$

Apply the invers Shehu transform both sides, then we get

$$\Rightarrow T(t) = 30 + 70e^{-kt}$$

Now, using the condition at time $t = 15$, i.e $T(15) = 70$

$$\Rightarrow 70 = 30 + 70e^{-15k}$$

$$\Rightarrow \frac{4}{7} = e^{-15k}$$

$$e^{-k} = \left(\frac{4}{7}\right)^{\frac{1}{15}} \Rightarrow k = \frac{1}{15}\log\left(\frac{4}{7}\right) = 0.037308$$

Now substituting the value of k

Thus $T(t) = 30 + 700e^{-0.037308t}$

We observe that this solution furnishes no finite solution to $T(t) = 30$ Since, $\lim_{t\to\infty} T(t) = 30$. The temperature variation is shown graphically in the figure below. We observe that the limiting temperature is 30°C.

$\gg t = [0:10:60];$

$>> \ T = 30 + 70 * exp(-0.037308 * t);$

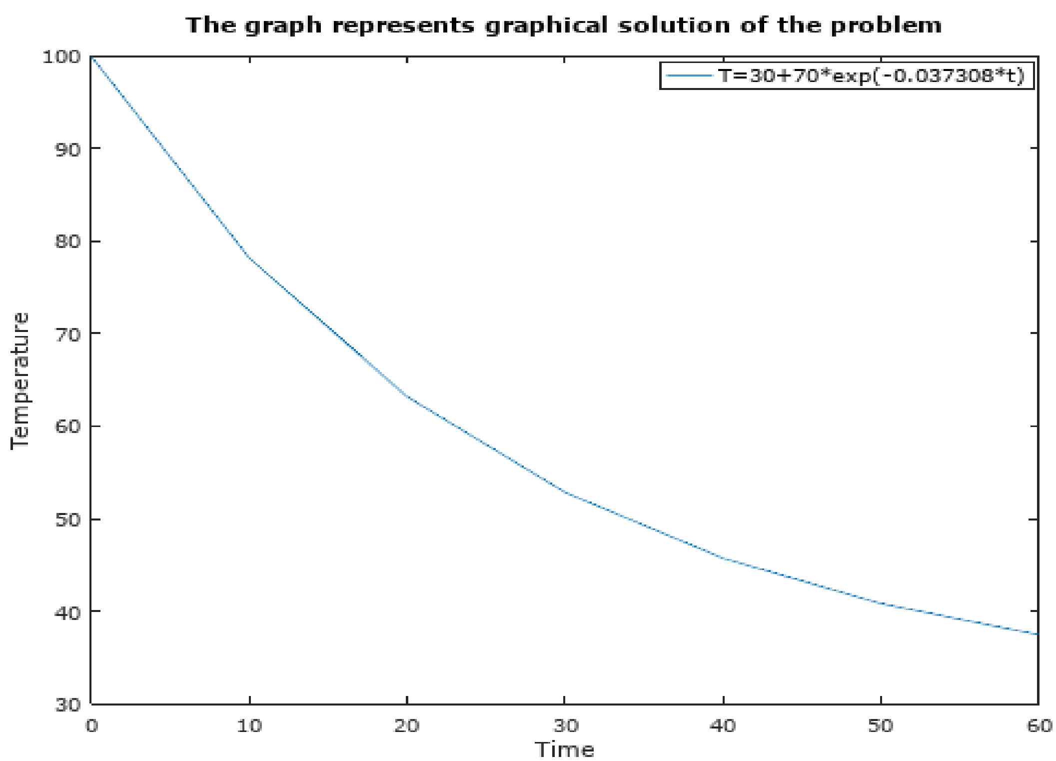
$>> \ plot(t, T, '-');$

$>> \ xlabel('\text{time}');$

$>> \ ylabel('Temperature');$

$>> \ legend('T = 30 + 70 * exp(-0.037308 * t)');$

>> title('The graph represents graphical solution of the problem');



We are required to find $t$when $T = 40^0c$

$$40 = 30 + 70e^{-kt} \Longrightarrow 10 = 70e^{-kt} \Longrightarrow e^{-kt} = \frac{1}{7}$$

$$\Rightarrow \left(\frac{4}{7}\right)^{\frac{t}{15}} = \frac{1}{7}$$

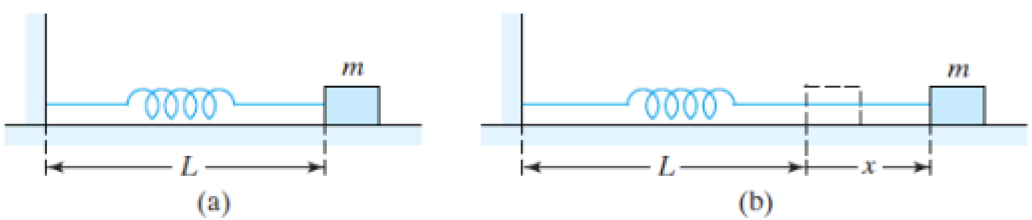
Now, taking both sides natural logarithmic, we obtained:

$$\frac{t}{15}\ln\left(\frac{4}{7}\right) = \ln\left(\frac{1}{7}\right)$$

$$\Rightarrow t = 15\frac{\ln\left(\frac{1}{7}\right)}{\ln\left(\frac{4}{7}\right)}\text{min} = 52.158\,\text{min}$$

Application of Shehu transform on Damped Oscillations

Consider a particle of mass m that moves along the x −axis, with position $x(t)$ at time t. A spring exerts a force $-kx$ on the object, pulling it towards the equilibrium position $x = 0$. The object also experiences a damping (or frictional) force that is taken to be proportional to the object's velocity $\frac{dx}{dt}$. This is the homogeneous linear constant-coefficient differential equation



$$M\frac{d^2x}{dt^2} = -k\frac{dx}{dt} - px$$

$$\Rightarrow M\frac{d^2x}{dt^2} + k\frac{dx}{dt} + px = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = 0$$

Where, $a = \frac{k}{M} > 0, b = \frac{p}{M} > 0$

If we apply Shehu transform we can have

$$\Rightarrow \frac{s^2}{u^2}V(s,u) - \frac{s}{u}v(0) - v'(0) + a\left[\frac{s}{u}V(s,u) - v(0)\right] + bV(s,u) = 0$$

$$\Rightarrow \frac{s^2}{u^2}V(s,u) + a\frac{s}{u}V(s,u) + bV(s,u) = \frac{s}{u}v(0) + v'(0) + av(0)$$

$$\Rightarrow V(s,u)\left[\frac{s^2}{u^2} + a\frac{s}{u} + b\right] = v(0)\left[\frac{s+au}{u}\right] + v'(0)$$

$$\Rightarrow V(s,u)\left[\frac{s^2 + asu + bu^2}{u^2}\right] = v(0)\left[\frac{s+au}{u}\right] + v'(0)$$

$$\Rightarrow V(s,u) = \frac{u^2}{s^2 + asu + bu^2}\left[v(0)\frac{s+au}{u} + v'(0)\right]$$

$$= \frac{v'(0)u^2}{s^2 + asu + bu^2} + v(0)\left[\frac{su + au^2}{s^2 + asu + bu^2}\right]$$

$$= \frac{v'(0)}{\left(\frac{s}{u}\right)^2 + a\frac{s}{u} + b} + \frac{v(0)\frac{s}{u}}{\left(\frac{s}{u}\right)^2 + a\frac{s}{u} + b} + \frac{av(0)}{\left(\frac{s}{u}\right)^2 + a\frac{s}{u} + b}$$

Now, if we see the roots of$\left(\frac{s}{u}\right)^2 + a\frac{s}{u} + b$

$$r_1, r_2 = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$$

Case 1:

$a^2 - 4b > 0$has two distinct real roots

$$V(s,u) = \frac{v'(0)}{\left(\frac{s}{u} - r_1\right)\left(\frac{s}{u} - r_2\right)} + \frac{v(0)\frac{s}{u}}{\left(\frac{s}{u} - r_1\right)\left(\frac{s}{u} - r_2\right)} + \frac{av(0)}{\left(\frac{s}{u} - r_1\right)\left(\frac{s}{u} - r_2\right)}$$

When we apply partial decomposition, we can have

$$\frac{-v'(0)}{r_2-r_1}\left[\frac{u}{s-r_1u}\right]+\frac{v'(0)}{r_2-r_1}\left[\frac{u}{s-r_2u}\right]-\frac{v(0)r_1}{r_2-r_1}\left[\frac{u}{s-r_1u}\right]$$

$$+\frac{v(0)r_2}{r_2-r_1}\left[\frac{u}{s-r_2u}\right]-\frac{av(0)}{r_2-r_1}\left[\frac{u}{s-r_2u}\right]+\frac{av(0)}{r_2-r_1}\left[\frac{u}{s-r_1u}\right]$$

$$\frac{u}{s-r_1u}\left[\frac{-v'(0)}{r_2-r_1}-\frac{v(0)r_1}{r_2-r_1}+\frac{av(0)}{r_2-r_1}\right]+\frac{u}{s-r_2u}\left[\frac{v'(0)}{r_2-r_1}-\frac{av(0)}{r_2-r_1}+\frac{v(0)r_2}{r_2-r_1}\right]$$

$$c_1=\frac{-v'(0)}{r_2-r_1}-\frac{v(0)r_1}{r_2-r_1}+\frac{av(0)}{r_2-r_1}\text{ and }c_2=\frac{v'(0)}{r_2-r_1}-\frac{av(0)}{r_2-r_1}+\frac{v(0)r_2}{r_2-r_1}$$

$$\Rightarrow V(s,u)=\frac{c_1u}{s-r_1u}+\frac{c_2u}{s-r_2u}$$

Applying the inverse transform gives the solution

$$x(t)=c_1e^{r_1t}+c_2e^{r_2t}$$

Case 2:

$a^2-4b<\ 0$ has complex roots

$$r_1,r_2=\frac{-a\pm i\sqrt{4b-a^2}}{2}$$

From the above

$$x(t)=c_1e^{r_1t}+c_2e^{r_2t}$$

$$\Rightarrow x(t)=e^{\frac{-at}{2}}[c_1\cos(\omega t)+c_2i\sin(\omega t)]$$

Case 3:

$a^2 - 4b = 0$ has one root $r_1, r_2 = \frac{-a}{2}$

$$V(s,u) = \frac{v'(0)}{\left(\frac{s}{u}+\frac{a}{2}\right)^2} + \frac{v(0)\frac{s}{u}}{\left(\frac{s}{u}+\frac{a}{2}\right)^2} + \frac{av(0)}{\left(\frac{s}{u}+\frac{a}{2}\right)^2}$$

$$\Longrightarrow V(s,u) = \frac{u^2 v'(0)}{\left(s+\frac{a}{2}u\right)^2} + \frac{av(0)u^2 + v(0)us}{\left(s+\frac{a}{2}u\right)^2}$$

$$\Longrightarrow V(s,u) = \frac{u^2 v'(0)}{\left(s+\frac{a}{2}u\right)^2} + \frac{v(0)u}{\left(s+\frac{a}{2}u\right)}$$

Applying the inverse transform gives the solution

$$x(t) = c_1 e^{\frac{-at}{2}} + tc_2 e^{\frac{-at}{2}}$$

Shehu Transform for handling Free Undamped Motion problem

Consider a linear second order DE $m\frac{d^2x}{dt^2} = -k(x+s) + mg = -kx + mg - ks$ 1.10

Since, $mg - ks = 0$ from newton second law Therefore equation 1.10 is equal to:

$$m\frac{d^2x}{dt^2} = -kx \quad 1.11$$

DE of Free Undamped Motion: By dividing (1.11) by the mass m we obtain the second order differential equation$\frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$, this can be written as:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0, \text{where}\omega^2 = \frac{k}{m} \quad 1.12$$

Equation (1.12) is said to describe simple harmonic motion or free undamped motion.

In this section, we present Shehu Transform for handling free undamped motion problem given by (1.12)

Taking Shehu transform both sides of Equation 1.12

$$\mathbb{S}\left[\frac{d^2x}{dt^2}+\omega^2x=0\right]$$

$$\mathbb{S}\left[\frac{d^2x}{dt^2}\right]+\omega^2\mathbb{S}[x]=0$$

$$\Rightarrow\frac{s^2}{u^2}V(s,u)-\frac{s}{u}v(0)-v'(0)+\omega^2V(s,u)=0$$

$$\Rightarrow V(s,u)\left[\frac{s^2}{u^2}+\omega^2\right]=\frac{s}{u}v(0)+v'(0)$$

$$\Rightarrow\frac{v(0)su}{s^2+\omega^2u^2}+\frac{v'(0)u^2}{s^2+\omega^2u^2}=\frac{v(0)su}{s^2+\omega^2u^2}+\frac{v'(0)}{\omega}\frac{\omega u^2}{s^2+\omega^2u^2}$$

Applying the inverse transform gives the solution

$$v(0)\cos(\omega t)+\frac{v'(0)}{\omega}sin(\omega t)$$

$$\Rightarrow c_1\cos(\omega t)+c_2\sin(\omega t),\text{where}c_1=v(0),c_2=\frac{v'(0)}{\omega}$$

Example: A spring with a mass of 2 kg has natural length 0.5 m. a force of 25.6 N is required to maintain it stretched to length of 0.7 m. If the spring is stretched to a length of 0.7m and then released with initial velocity 0, find the position of the mass at any time t.

Solution: From hook's law the force required to stretched the spring is

$$k(0.2) = 25.6$$

So $k = \frac{25.6}{0.2} = 128$. Using this value of the spring constant k, together with $m = 2$

We have,

$$2\frac{d^2x}{dt^2} + 128x = 0$$

This also equal to

$$\frac{d^2x}{dt^2} + 64x = 0$$

Now taking the Shehu Transform both sides

$$\mathbb{S}\left[\frac{d^2x}{dt^2} + 64x = 0\right]$$

$$\Rightarrow \frac{s^2}{u^2}X(s,u) - \frac{s}{u}x(0) - x'(0) + 64X(s,u) = 0$$

$$\Rightarrow \frac{s^2}{u^2}X(s,u) + 64X(s,u) = \frac{s}{u}x(0) + x'(0)$$

$$\Rightarrow X(s,u)\left[\frac{s^2 + 64u^2}{u^2}\right] = \frac{s}{u}x(0) + x'(0)$$

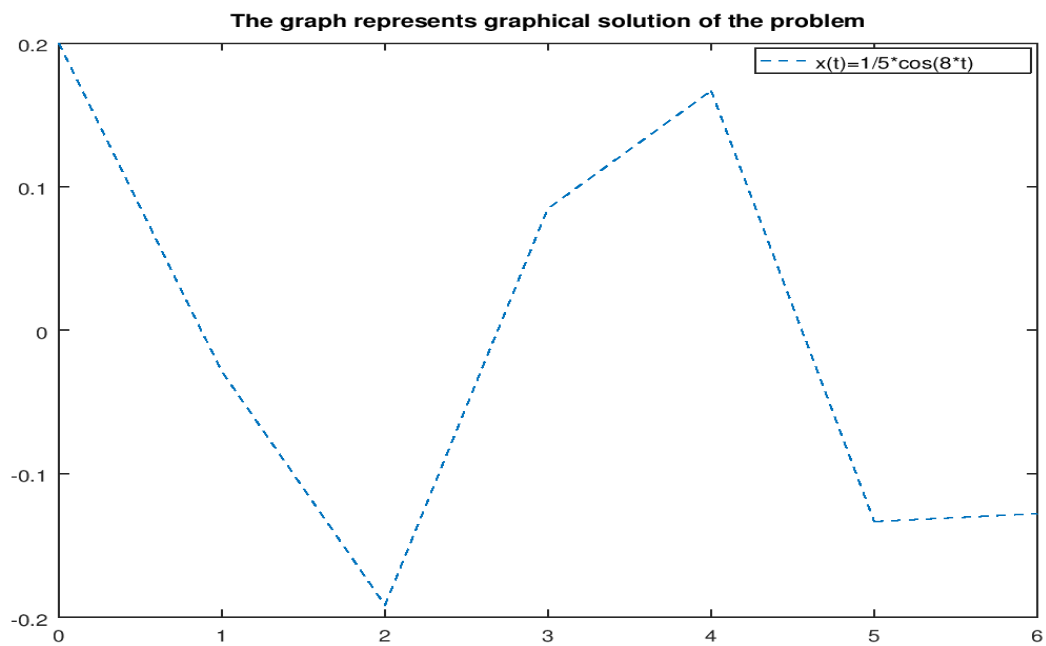
$$\Rightarrow X(s,u) = x(0)\frac{us}{s^2 + (8u)^2} + \frac{1}{8}x'(0)\frac{8u^2}{s^2 + (8u)^2}$$

Now taking the inverse of Shehu Transform, we get

$$x(t) = x(0)\cos(8t) + \frac{1}{8}x'(0)\sin(8t)$$

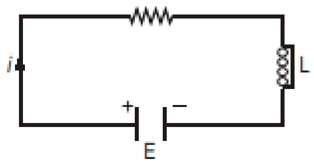
We are given initial condition that $x(0) = 0.2$ and the initial velocity given as $x'(0) = 0$, therefore our solution is converted to:

$$x(t) = \frac{1}{5}\cos(8t)$$



Shehu Transform for handlingan Electric circuit in series containing resistance and self-inductance (RL series circuit) problem

In this section, we present Shehu Transform for handling**RL series circuit**.



$$Ri + L\frac{di}{dt} = E \qquad \text{Or}$$

$$\frac{di}{dt} + ai = b \qquad i(0) = 0 \qquad 1.13$$

Where $a = \frac{R}{L}, b = \frac{E}{L}$

Taking Shehu transform on both sides of (1.13), we have

$$\mathbb{S}\left[\frac{di}{dt} + ai = b\right]$$

Now applying the properties of Shehu transform, we have

$$\mathbb{S}\left[\frac{di}{dt}\right] + a\mathbb{S}[i(t)] = b\mathbb{S}[1]$$

$$\Rightarrow \frac{s}{u}I(s,u) - i(0) + aI(s,u) = \frac{bu}{s}$$

$$\Rightarrow I(s,u)\left[\frac{s}{u} + a\right] = \frac{bu}{s} + 0, \text{since } i(0) = 0$$

$$\Rightarrow I(s,u) = \frac{bu^2}{s(s+au)}$$

This also can be rewritten as:

$$\Rightarrow I(s,u) = \frac{bu}{as} - \frac{bu}{as + a^2u}$$

$$= \frac{b}{a}\left[\frac{u}{s} - \frac{u}{s-(-au)}\right] \quad 1.14$$

Operating inverse Shehu transform on both sides of (1.14)

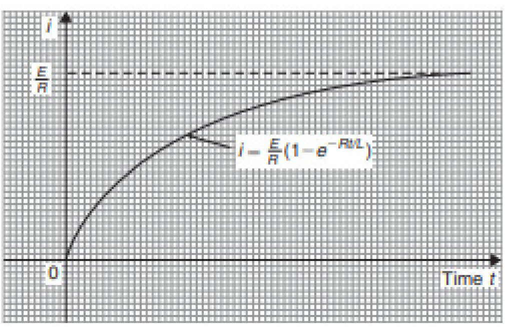
$$\Rightarrow i(t) = \frac{b}{a}\mathbb{S}^{-1}\left[\frac{u}{s}\right] - \frac{b}{a}\mathbb{S}^{-1}\left[\frac{u}{s-(-au)}\right]$$

$$\Rightarrow i(t) = \frac{b}{a} - \frac{b}{a}e^{-at} \quad 1.15$$

Since, $a = \frac{R}{L}, b = \frac{E}{L} \Rightarrow \frac{b}{a} = \frac{E}{R}$. Therefore equation 1.15 is equal to:

$$\Rightarrow i(t) = \frac{E}{R} - \frac{E}{R}e^{\left(-\frac{R}{L}\right)t} = \frac{E}{R}\left(1 - e^{\left(-\frac{R}{L}\right)t}\right)$$

This equation gives the required amount of current in the circuit at any time t as shown in figure below



Conclusion

In this paper, the Shehu Transform Method was proposed for solving ordinary differential equation occurred in Engineering and Physics problems. We successfully found an exact solution in all the examples.

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