RESEARCH ARTICLE OPEN ACCESS

Practical application of Operation Research Using Simplex Method

Harshini.K*
*(Department of Mathematics, Bharathiyar Arts and Science College for Women, and Deviyakurichi
Email: harshinikrishnan7991@gmail.com)

----------------------------------**********************----------------------------------

Abstract:
Operations Research (OR) has benefited the mankind in nearly all aspects of its life. Health care which attracts a great attention in order to provide adequate health services to the people. Owing the scarce resources and increased population it has become quite difficult, especially those of developing countries to provide quality health services to their citizens. Operations Research can be most fruitful tool at this juncture. This paper describes the Operations Research applications in the field of medicine, which can be solved by using simplex method. Finally an example is illustrated to explicate the importance of the simplex method in the medical field.

***Keywords* —**OR in health care, simplex method

----------------------------------**********************----------------------------------

I. INTRODUCTION

Operation research deals with the real world and helps us to make better decisions. Operation research is also applied in the field of health care to improve its efficiency and effectiveness. Operation research identifies the problem which has to be solved, and construct the model, and derives the successive solution after analysing it.

In mathematical optimization, simplex method is one of the most popular algorithms in linear programming.

II. PRELIMINARIES

A. *Simplex method*

The simplex method provides an algorithm which consists in moving from one vertex of the region of feasible solutions to another vertex in such a way that the value of the objective function at the succeeding vertex is more(or less, as the case may be) than at the preceding vertex. This procedure of one vertex to another vertex is then repeated. Since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or indicates the existence of an unbounded solution.

B. *Basic solution*

For a system of m linear equations with n variables (m<n), if (n-m) variables are zero and then solve remaining m variables it is said to be a Basic solution.

The m variables are called Basic variables and they form the basic variables and they form the basic solution. The (n-m) variables which are put to zero are called as non –basic variables.

NON-DEGENERATE BASIC SOLUTION: If none of the basic variables is zero, then the basic solution is called nan-degenerate basic solution.

DEGENERATE BASIC SOLUTION: *If* one or more of the basic variables are zero, then the basic solution is called degenerate basic solution.

III. ***BASIC FEASIBLE SOLUTION***: A FEASIBLE SOLUTION WHICH IS ALSO BASIC IS CALLED A BASIC FEASIBLE SOLUTION.**ALGORITHM**

Algorithm for the LPP is as follows:

1. First predict that the given objective function is to be maximized or minimized. If it is to be minimized, then convert it into a problem of maximization, by

Minimize z = -Maximize (-z)

2. Check whether all b_i's are positive. If any of the b_i's is negative, in constraint multiply both sides by -1 for making its right hand side as positive.

3. By introducing slack or surplus variables, convert the inequality constraints into equal constraints and express the given LPP into its standard form.

4. Find an initial basic feasible solution(IBFS) and express the above information conveniently in the following simplex table.

		C_j	C_1	C_2	…	0	0	…
C_B	Y_B	X_B	x_1	x_2	…	S_1	S_2	…
C_{B_1}	S_1	b_1	a_{11}	a_{12}	…	1	0	…
C_{B_2}	S_2	b_2	a_{21}	a_{22}	…	0	1	…
⋮								
$(z_j - c_j)$		z	$(z_1 - c_1)$	$(z_2 - c_2)$		…	…	

[whereC_j – row denotes the coefficients of the variables in the Objective function,

C_B –column denotes the coefficients of the basic variables in the objective function,

Y_B –column denotes the basic variables,

X_B –column denotes the value of the basic variables.

The coefficients of the non-basic variables in the constraint equations constitute the body matrix while the coefficients of the basic variables constitute the unit matrix. The row $(z_j - c_j)$ denotes the next evaluations (or) index for each column]

5. Compute the net evaluation $(z_j - c_j)$ (j=1,2,…,n) by using the relation $z_j - c_j = C_B a_j - c_j$.

Examine the sign of $z_j - c_j$

a) if all $(z_j - c_j) \geq 0$ then the current basic feasible solution X_B is optimal.

b) if at least one $(z_j - c_j)$<0, then the current basic feasible solution is not optimal, go to the next step.

6. TO FIND THE ENTERING VARIABLE

The entering variable is the non-basic variable corresponding to the most negative value of $(z_j - c_j)$. Let it be x_r for some j=r. the entering variable column is known as the KEY COLUMN (or) PIVOT COLUMN which is shown marked with an arrow at the bottom. If more than one variable has the same most negative$(z_j - c_j)$, any of these variables may be selected arbitrarily as the entering variable.

7. TO FIND THE LEAVING VARIABLE

Compute the ratio $\theta = \min \left\{\frac{X_{b_i}}{a_{ir}}, a_{ir} > 0\right\}$ (ie)the ratio between the solution column and the entering variable column by considering only the positive denominators)

a)if all $a_{ir} \leq 0$, then there is an unbounded solution to the given LPP.

b)if at least one $a_{ir} > 0$, then the leaving variable is the basic variable corresponding to the minimum ratio θ. If $\theta = \frac{X_{Bk}}{a_{kr}}$, then the basic variable x_k leaves the basis. The leaving variable row is called the Key row (or) Pivot row (or) Pivot Equation and the element at the intersection of the pivot column and pivot row is called the Pivot element (or) Key element (or) Leading element.

8. Drop the leaving variable and introduce the entering variable along with its associated value under C_B column. Convert the pivot element to unity by dividing the pivot equation by the pivot element and all other elements in its column to zero by making use of

a) new pivot equation = old pivot equation + pivot element

b)new equation (all other rows including $(z_j - c_j)$ row) = old equation-(corresponding column coefficient×new pivot equation)

9. Go to step (5) and repeat the procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

NOTE 1:

1. If all $(z_j - c_j) \geq 0$, then the current basic feasible solution is optimal.
2. If atleast one $(z_j - c_j) < 0$, then the current basic feasible solution is not optimal.
3. The entering variable is the non-basic variable corresponding to the most negative value of $(z_j - c_j)$.

NOTE 2:

1. If all $(z_j - c_j) \leq 0$, then the current basic feasible solution is optimal.
2. If atleast one $(z_j - c_j) > 0$, then the current basic feasible solution is not optimal.
3. The entering variable is the non-basic variable corresponding to the most positive value of $(z_j - c_j)$.

NOTE 3:

For the maximization and minimization problems, the leaving variable is the basic variable corresponding to the minimum ratio θ.

IV. PROBLEM

Let's say a dress manufacturer wishes to maximize his production with the available sources that are given below

RESOURCE	SALWAR	SAREE	AVAILABLE
Raw materials	30	20	300
Machine	5	10	110
Unit Production	6	8	

Maximize Z=$6X_1$+$8X_2$(objective function)

Subject to: $30X_1+20X_2 \leq 300$

$5X_1+10X_2 \leq 110$

$X_1, X_2 \geq 0$(saree,salwar)

Now formulate the salwar and Saree in standard Form as:

Maximize: $Z=6X_1+8X_2+0S_1+0S_2$

Subject to: $30X_1+20X_2+S_1+0S_2=300$

$5X_1+10X_2+0S_1+S_2=110$

$X_1, X_2, S_1, S_2 \geq 0$

The simplex table:

c_j			6	8	0	0	
c_B	Y_B	X_B	X_1	X_2 ↓	S_1	S_2	θ
0	S_1	300	30	20	1	0	15
0	S_2 ←	110	5	⑩	0	1	11
	$z_j - c_j$	0	-6	-8	0	0	

X_2= Entering variable

S_2= Leaving Variable

Replace

X_2=$S_2/10$, and S_1= $S_1 - (X_2 \times 2)$

c_j			6	8	0	0	
c_B	Y_B	X_B	X_1 ↓	X_2	S_1	S_2	θ
0	S_1 ←	80	⑳	0	1	-2	4
8	X_2	11	0.5	1	0	0.1	22
	$z_j - c_j$		-2	0	0	0.8	

X_1= Entering variable

S_1= Leaving Variable

Replace

X_1=$S_1/20$,and

X_2=$X_2 - (X_1 \times 0.5)$

c_j			6	8	0	0
c_B	Y_B	X_B	X_1	X_2	S_1	S_2
6	X_1	4	1	0	0.05	-0.1
8	X_2	9	0	1	-0.025	0.15
	$z_j - c_j$		0	0	0.1	0.6

This Simplex Table represents the optimal solution to the LP Problem and is interpreted as

$$X_1 = 4, X_2 = 9,$$

$$S_1 = 0, S_2 = 0,$$

and production is 96.

V. CONCLUSION

Operation research are used to identifying problems in time, and helps us to make better and correct deicisions accordingly. Its is applicable in many aspects such as health care, medical, industries, marketing, research, etc.,

VI. REFERENCES

[1] *J.Reeb and S.Leavengood, 'Using the Simplex Method to Solve Linear Programming Maximization Problems'.*

[2] *Linda V.Green,"Using Operations Research to Reduce Delays for Healthcare", ISBN 978-1-877640-23-0.*

[3] *P.K.Gupta, Manmohan and KantiSwarup,Operations Research, 9th edition, 2001, Sultan Chand &Sons, Chennai.*

