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On Some Properties of Fuzzy and Crisp Polynomials

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Abstract:

The concept of fuzzy number has led to what has come to be called as fuzzy arithmetics. Arithmethic operations on fuzzy number are the basic information in fuzzy mathematics. In this paper, we are dealing with crisp polynomials and fuzzy polynomials in the form $\boldsymbol{f(x) = Ax + B}$, where $\boldsymbol{A, B \in R}$ and $\boldsymbol{\tilde{f}(\tilde{X}) = \sum_i A_i\tilde{X}_i = A_1\tilde{X}_1 + A_0}$ respectively. We extended this crisp coefficient and variables in fuzzy coefficient and fuzzy variables which leads to a new approach of defining fuzzy polynomials. Then, we will compare the results of multiplications and addition operations on triangular and trapezoidal fuzzy number between analytical calculations and $\boldsymbol{\alpha}$-cut method. Besides that, we also give a numerical examples and graphical representations supporting the example given.

Keywords **—fuzzy polynomials; crisp polynomials; triangular fuzzy number; trapezoidal fuzzy number**

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I. INTRODUCTION

Zadeh [1]was the first one who introduced the word fuzzy through his paper *"Fuzzy Set"*. The word "*fuzzy*" is used to generalize the mathematical concept of set to one of fuzzy set or fuzzy subset, where in a fuzzy set a membership function is defined for each element of the referential set. The membership function takes its values in the interval $[0,1] \in R^+$. One of the interesting characteristics of this fuzzy system is that they are capable to handle in the same framework numeric and linguistics information. This characteristic helps so much in handling expert control tasks. Besides, fuzzy numbers also developed in terms of their arithmetic operations, and hence focusing more on extension principle [4]. When calculating fuzzy numbers, the results will be strongly depending on the membership function defined, as in our paper, we are mainly based on membership function of triangular and trapezoidal fuzzy number.

II. PRELIMINARY NOTES

We consider a situation where a certain value isto be uncertain. Suppose that the uncertain value belongs to the set of real number R, (referential set). For most situation we have, Kaufmann, A. and Gupta, M.M. [2] mentioned that it is possible to locate the value inside an interval confidence, $R = [a, b]$ inside a closed interval of R. We are certain that the value is greater than or equal to a and smaller than or equal to b. In this case, we use the symbol of $A = [a, b]$ .Below are some operations

for the interval confidence, as in [3],assumingthat $A = [a, b]$ and $B = [c, d]$ are two intervals in R.

i. Addition/Subtraction:$A\ (\pm) B = [a \pm c, b \pm d]$

ii. Multiplication: $A\ (*) B = [a * c, b * d]$

Definition 1 *Characteristic function : It is a function on a crisp set assigns a value of either 1 or 0 to each individual in the universal set, thereby* *discriminating between members and non-members of the crisp set under consideration.*

Let E be a referential set (for example, R and Z). A characteristic function, or also known as membership function that takes values in an interval $[0,1]$ will defined a fuzzy subset of A. In this study, we will define our membership function of the terms in our polynomials based on triangular and trapezoidal fuzzy number.

Definition 2 *Triangular fuzzy number : If* $A = (a, b, c)$, *the membership functions of* A *is given by*

$$\mu_A(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x > c \end{cases}$$

Definition 3 *Trapezoidal fuzzy number : If* $A = (a, b, c, d)$, *the membership functions of* A *is given by*

$$\mu_A(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & x > d \end{cases}$$

Definition 4 α*-cut interval of triangular fuzzy number interval* A *is given by*

$$A_\alpha = [a^{(\alpha)}, c^{(\alpha)}]$$
$$= [(b-a)\alpha + a] + [-(c-b)\alpha + c]$$

Below are some operations on the fuzzy number :

i. Addition/Subtraction :

$$A_\alpha(\pm)B_\alpha = [a^{(\alpha)}, b^{(\alpha)}] \pm [c^{(\alpha)}, d^{(\alpha)}]$$
$$= [a^{(\alpha)} \pm c^{(\alpha)}, b^{(\alpha)} \pm d^{(\alpha)}]$$

ii. Miltiplication

$$A_\alpha(.)B_\alpha = [a^{(\alpha)}, b^{(\alpha)}] * [c^{(\alpha)}, d^{(\alpha)}]$$
$$= [a^{(\alpha)} * c^{(\alpha)}, b^{(\alpha)} * d^{(\alpha)}]$$

III. RESULT AND DISCUSSION

A crisp polynomials is given by

$$f(x) = 10x + 7. \quad (1)$$

When $x = 5$, we have

$$f(5) = 10(5) + 7$$
$$f(5) = 50 + 7$$
$$f(5) = 5 \quad (2)$$

By using the same polynomials in (1), we will extend it to fuzzy polynomials in the form of

$$\tilde{f}(\tilde{X}) = \sum_i A_i \tilde{X}_i = A_1\tilde{X}_1 + A_0,$$

such that $A_1 = [9,10,11]$ and $A_0 = [6,7,8]$.

Now, we have

$$\tilde{f}(\tilde{5}) = \tilde{f}([4,5,6])$$
$$\tilde{f}([4,5,6]) = [9,10,11] * [4,5,6] + [6,7,8] (3)$$

From Equation (3), multiplication process are as follows:

Let $A = [9,10,11]$ and $B = [4,5,6]$.

By Definition 2, we have

$$\mu_A(x) = \begin{cases} 0, & x < 9 \\ \dfrac{x-9}{1}, & 9 \le x \le 10 \\ \dfrac{11-x}{1}, & 10 \le x \le 11 \\ 0, & x > 11 \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < 4 \\ \dfrac{x-4}{1}, & 4 \le x \le 5 \\ \dfrac{6-x}{1}, & 5 \le x \le 6 \\ 0, & x > 6 \end{cases}$$

respectively. While α-cut interval for A and B are given by

$$A_\alpha = [(\alpha + 9), (11 - \alpha)]$$

and

$$B_\alpha = [(\alpha + 4), (6 - \alpha)]$$

respectively.

Thus, we have,

$$A_\alpha(\cdot)B_\alpha = [(\alpha+9),(11-\alpha)]\cdot[(\alpha+4),(6-\alpha)]$$
$$= [(\alpha+9)\cdot(\alpha+4)\,,(11-\alpha)(6-\alpha)]$$
$$= [\alpha^2+13\alpha+36\,,\alpha^2-17\alpha+66]$$

When $\alpha = 0$, then $A_\alpha(\cdot)B_\alpha = [36,66]$ while $\alpha = 1$, then $A_\alpha(\cdot)B_\alpha = [50,50]$. Overall, we have $A_\alpha(\cdot)B_\alpha = [36,50,66]$.

We represent the results graphically as shown below :

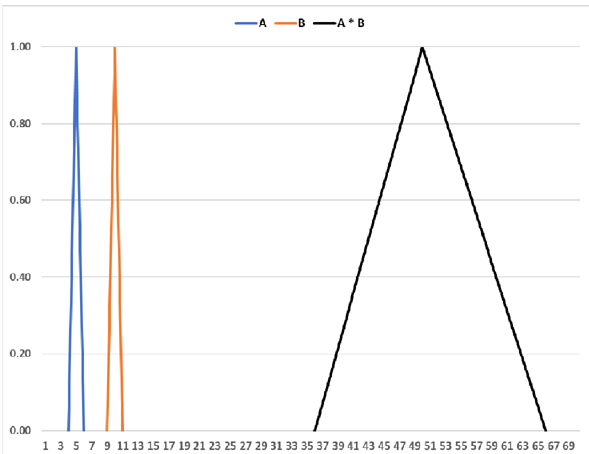

Fig. 1 Multiplication of fuzzy number A and B

Now, continue solving (3), we have

$$\tilde{f}([4,5,6]) = [36,50,66] + [6,7,8] \qquad (4)$$

We repeat the same process, where we let $C = [36,50,66]$ and $D = [6,7,8]$. By Definition 2, we have

$$\mu_C(x) = \begin{cases} 0, & x < 36 \\ \dfrac{x-36}{14}, & 36 \le x \le 50 \\ \dfrac{66-x}{16}, & 50 \le x \le 66 \\ 0, & x > 66 \end{cases}$$

and

$$\mu_D(x) = \begin{cases} 0, & x < 6 \\ \dfrac{x-6}{1}, & 6 \le x \le 7 \\ \dfrac{8-x}{1}, & 7 \le x \le 8 \\ 0, & x > 8 \end{cases}$$

respectively. Thus, α-cut interval for C and D are given by $C_\alpha = [(14\alpha+36),(66-16\alpha)]$ and $D_\alpha = [(\alpha+6),(8-\alpha)]$ respectively.

Then, we have

$$C_\alpha(+)D_\alpha = [(14\alpha+36),(66-16\alpha)]$$
$$+ [(\alpha+6),(8-\alpha)]$$
$$= [(15\alpha+42),(74-17\alpha)]$$

When $\alpha = 0$, then $C_\alpha(+)D_\alpha = [42,74]$ while $\alpha = 1$, then $C_\alpha(+)D_\alpha = [57,57]$. Therefore, from Equation (4), we have $C(+)D = [42,57,74]$.

We represent the results graphically as shown below.

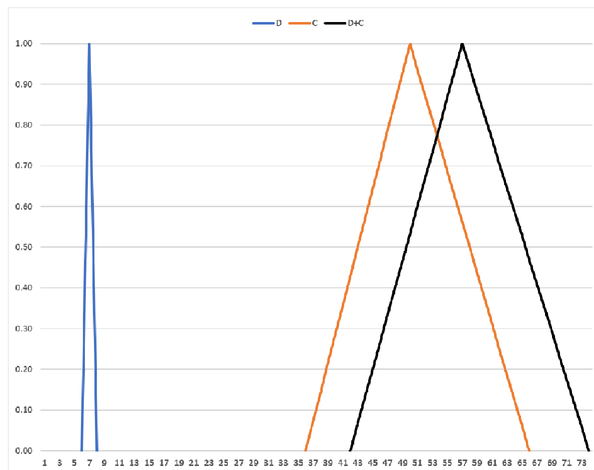

Fig.2 Addition of fuzzy number C and D

Comparing the results that we get from Equation (2), the results from triangular fuzzy shows that the middle value holds the same value as in crisp polynomials. The same process is repeated for trapezoidal fuzzy number. By Definition 3, we have as follows :

$$\tilde{f}(\tilde{X}) = \sum_i A_i\tilde{X}_i = A_1\tilde{X}_1 + A_0$$

such that $A_1 = [8,9,10,11]$ and $A_0 = [5,6,7,8]$ and $(x) = [3,4,5,6]$. We have

$$\tilde{f}([3,4,5,6]) = [8,9,10,11] * [3,4,5,6] + [5,6,7,8] \qquad (5)$$

Let $A = [8,9,10,11]$ and B= $[3,4,5,6]$. Thus, the membership function of A and B are as follows,

$$\mu_A(x)=\begin{cases}0, & x<8\\ \dfrac{x-8}{1}, & 8\le x\le 9\\ 1, & 9\le x\le 10\\ \dfrac{11-x}{1}, & 10\le x\le 11\\ 0, & x>11\end{cases}$$

and

$$\mu_B(x)=\begin{cases}0, & x<3\\ \dfrac{x-3}{1}, & 3\le x\le 4\\ 1, & 4\le x\le 5\\ \dfrac{6-x}{1}, & 5\le x\le 6\\ 0, & x>6\end{cases}$$

with α-cut interval for A is $A_\alpha=[(\alpha+8),(11-\alpha)]$ and B is $B_\alpha=[(\alpha+3),(6-\alpha)]$ respectively.

Thus,

$$A_\alpha(\cdot)B_\alpha=[(\alpha+8),(11-\alpha)]\cdot[(\alpha+3),(6-\alpha)]$$
$$=[(\alpha+9)\cdot(\alpha+4),(11-\alpha)(6-\alpha)]$$
$$=[\alpha^2+11\alpha+24,\alpha^2-17\alpha+66].$$

When $\alpha=0$, then $A_\alpha(\cdot)B_\alpha=[24,66]$ while $\alpha=1$, then $A_\alpha(\cdot)B_\alpha=[36,50]$. The approximated value of $A(\cdot)B=[24,36,50,66]$.We represent the results graphically as shown below.

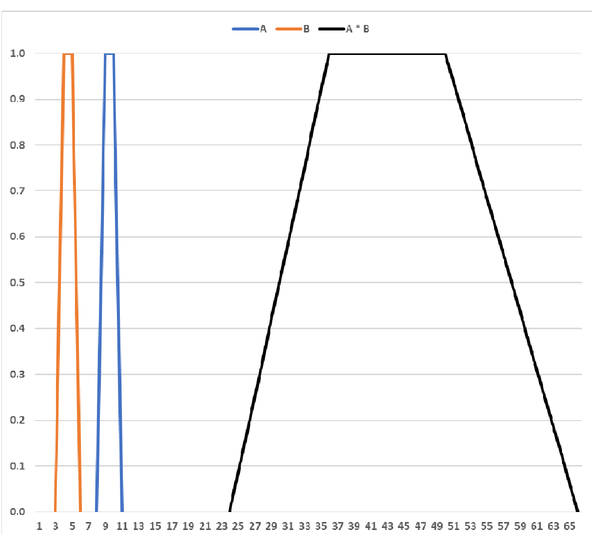

Fig. 3 Multiplication of fuzzy number A and B

Next, we let $C=[24,36,50,66]$ and $D=[5,6,7,8]$, repeat the same procedure as shown above to have approximate value of $C+D=[29,42,57,74]$. We represent the results graphically as shown below.

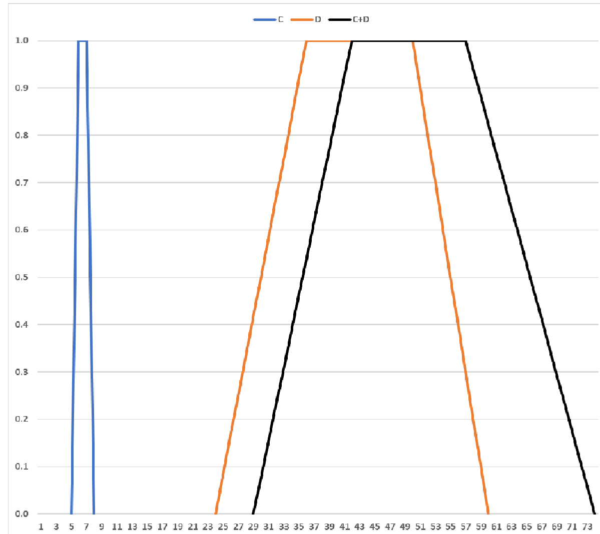

Fig. 4 Addition of fuzzy number C and D

Thus, we can say that the third value in the interval gives the same value as in crisp polynomials. By repeating the same procedure as above, we can used it to test on another different polynomials, for example, $f(x)=x^2+6x-10$ and so on.

IV. CONCLUSIONS

Crisp polynomials have been used in all branches of advanced mathematics. Extending crisp polynomials to fuzzy polynomials has now become important, and hence fuzzy arithmetic operations also play an important role as it is being used in different field of sciences and engineering. Here, we are using fuzzy arithmetic operation on triangular and trapezoidal fuzzy number between analytical and approximation method, and thus compared the results on our polynomials. We also represent these results graphically.

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REFERENCES

[1] Zadeh, L. A., "Fuzzy Set", Information and Control Vol 8 (1965), 338-353.
[2] J.Kaufman, A. and Gupta, M. M., "Introduction to Fuzzy Arithmetic Theoryand Applications", International Thomson Computer Press, 1991
[3] Dubois, D and Prade, H. "Operations on Fuzzy Numbers", International Journal of Systems Science, Vol. 9 No. 6, (1978) 613-626
[4] Rojers, F. and Jun, Y., "Fuzzy Nonlinear Optimization for Linear Fuzzy Real Number System", International Mathematical Forum, 4(12) (2009), 587-596
[5] Ali, M. D., Sultana, A. and Khodadad Khan, A.F.M., "Comparison of Fuzzy Multiplication Operation on Triangular Fuzzy Number", IOSR Journal of Mathematics}, Vol.12, Issue 4 Ver.I (2016), 35-41.
[6] Barhoi, A., "A comparative study of Fuzzy Polynomials and Crisp Polynomials", International Journal of Computational and Applied Mathematics, Vol.12, No.3 (2017), 657-662.