International Journal of Scientific Research and Engineering Development-– Volume 3 Issues 1 Jan- Feb 2020

Available at www.ijsred.com

RESEARCH ARTICLE OPEN ACCESS

A Study on Exact Solution of an Integrable Generalized Hirota-Satsuma Equation of (2+1)-Dimensions Via Exp(-Φ(ξ))-Expansion Method

M. Amanat Ullah*, M. Belal Hossen**, Sagira Husna***

*(Department of Computer Science & Engineering, Uttara University, Bangladesh
Email: aman221_du@yahoo.com)

** (Department of Computer Science & Engineering, Uttara University, Bangladesh
Email: belalshimul14@gmail.com)

*** (Department of Mathematics, Rajshahi University, Bangladesh
Email: shshapla20@gmail.com)

--************************----------------------------------

Abstract:

In this paper, we have studied (2+1) Dimensional integrable generalized Hirota- Satsuma (gHS) equation to create abundant exact solution using exp(-Φ(ξ))-expansion method. As a result, we get some exact solutions in-terms of trigonometric, hyperbolic and logarithmic functions with various structures. Finally, we presented some of the obtained solution with graphical illustration via symbolic computation.

Keywords **—**Hirota- Satsuma equation,exp(-Φ(ξ))-expansion method, traveling wave, exact solution.

--************************----------------------------------

I. INTRODUCTION

Mathematical modeling of dynamical processes in a great variety of natural phenomena leads in general to NPDEs. There is a particular class of solutions for these nonlinear equations that are of considerable interest. They are the traveling wave solutions such a wave is special solution of the governing equations, that may be localized or periodic, which does not change its shape and which propagates at constant speed. In the case of linear equations the profile is usually arbitrary. In contrast a nonlinear equation will normally determine a restricted class of profiles, as the result of a balance between nonlinearity and dissipation. In recent years, the exact solutions of NPDEs have been investigated by many researchers (see for example [1-21]) who are interested in nonlinear physical phenomena which exist in all fields including either the scientific works or engineering fields.

The main purpose of this paper is to demonstrate the exp(-Φ(ξ))-expansion method for finding the exact traveling wave solutions of an integrable generalized Hirota- Satsuma equation [13].

II. METHODOLOGY

AIn this section, we will discussed about exp(-Φ(ξ))-expansion method step by step.

Step 1. Firstly, we consider aNLPDE with three independent variables x, y and t as

$$\rho(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, ...) = 0, \qquad (2.1)$$

where $u(\xi) = u(x, y, t)$ is an unknown function, ρ is a polynomial of $u(x, y, t)$ and involving the nonlinear terms and highest order partial derivatives.

Step 2. Combining the independent variables x, y and t into one variables, $\xi = x + y - wt$, we suppose that,

$$u = u(x, y, t), \xi = x + y - wt, u = u(\xi), u(x, y, t) = u(\xi). \quad (2.2)$$

The traveling wave transformation Eq. (2.2) permits us to reduce Eq. (2.1) to the following ordinary differential equation:

$$\wp(u, u', u'', u''' \ldots\ldots\ldots) = 0, \quad (2.3)$$

where $\wp$ is a polynomial in $u(\xi)$ and its derivatives whereas $u'(\xi) = \frac{du}{d\xi}, u''(\xi) = \frac{d^2u}{d\xi^2}, u'''(\xi) = \frac{d^3u}{d\xi^3}$ and so on.

Step 3. For this method, suppose the solutions of Eq. (2.3) are as follows:

$$u(\xi) = \sum_{j=0}^{m} B_j (\exp(-\phi(\xi)))^j. \quad (2.4)$$

where B_j $(0 \le j \le m)$ are constants to be determined, such that $B_m \neq 0$, and $\phi = \phi(\xi)$ satisfies the following ODE.

$$\phi'(\xi) = \exp(-\phi(\xi)) + \mu \exp(\phi(\xi)) + \lambda \quad (2.5)$$

Eq. (2.5) gives the following solutions:

When $\lambda^2 - 4\mu > 0, \mu \neq 0$,

$$\phi(\xi) = \ln\left\{\frac{1}{2\mu}\left(-\sqrt{(\lambda^2 - 4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}(\xi + \kappa)\right) - \lambda\right)\right\}, \quad (2.6)$$

$$\phi(\xi) = \ln\left\{\frac{1}{2\mu}\left(-\sqrt{(\lambda^2 - 4\mu)}\coth\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}(\xi + \kappa)\right) - \lambda\right)\right\}, \quad (2.7)$$

When $\lambda^2 - 4\mu < 0, \mu \neq 0$,

$$\phi(\xi) = \ln\left\{\frac{1}{2\mu}\left(-\sqrt{(4\mu - \lambda^2)}\tan\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\xi + \kappa)\right) - \lambda\right)\right\}, \quad (2.8)$$

$$\phi(\xi) = \ln\left\{\frac{1}{2\mu}\left(-\sqrt{(4\mu - \lambda^2)}\cot\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\xi + \kappa)\right) - \lambda\right)\right\}, \quad (2.9)$$

When $\lambda^2 - 4\mu > 0, \mu = 0, \lambda \neq 0$,

$$\phi(\xi) = \ln\left(\frac{\lambda}{\exp(\lambda(\xi + \kappa)) - 1}\right), \quad (2.10)$$

When $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0$,

$$\phi(\xi) = \ln\left(\frac{2(\lambda(\xi + \kappa) + 2)}{\lambda^2(\xi + \kappa)}\right), \quad (2.11)$$

When $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0$,

$$\phi(\xi) = \ln(\xi + \kappa), \quad (2.12)$$

When κ is an arbitrary constant and B_m, w, λ, μ are constants to be determined later, $B_m \neq 0$. For the value of m, we consider the homogeneous balance between highest order derivativesand non-linear terms appearing in Eq. (2.3).

Step 4. Inserting Eq. (2.4) into Eq. (2.3) along with Eq. (3) and Eq. (4), then we get a polynomial ofexp(-Φ(ξ)). Then setting all the coefficients of same power of exp(-Φ(ξ)) to be zero, we obtain a set of

PDEs with respect to B_m, w, λ and μ. By solving the PDEs using symbolic calculation Maple, we get some explicit expression of B_m, w, λ and μ. Substituting the values of B_m, w, λ, μ into Eq. (2.4) along with general solutions of Eq. (2.5) completes the determination of the solution of Eq. (2.1).

III. Application

AIn this section, we will apply the exp(-Φ(ξ))-expansion method to construct abundant exact solution of an integrable (2+1)-dimensional generalized Hirota- Satsuma equation[13],

$$3(u_x u_t)_x + u_{xxxt} + u_{yt} + u_{xx} = 0, \tag{3.1}$$

where $u(x, y, t)$ is the amplitude of the relevant wave mode.

Consider the traveling wave transformation

$$u = u(x, y, t), \xi = x + y - wt, u = u(\xi), u(x, y, t) = u(\xi). \tag{3.2}$$

Inserting Eq. (3.2) into Eq. (3.1), then we get the following ordinary differential equation:

$$(1-w)u'' - 6wu'u'' - wu'''' = 0 \ . \tag{3.3}$$

Integrating with respect to ξ, we obtain the following ODE

$$V + (1-w)u' - 3w(u')^2 - wu''' = 0 \tag{3.4}$$

Balancing the highest order derivative u''' and non-linear term $(u')^2$, we get $2(m+1) = m+3$, which gives $m = 1$. Hence Eq. (2.4) becomes

$$u(\xi) = a_0 + a_1 \exp(-\phi(\xi)). \tag{3.5}$$

where a_0 and a_1 are constants to be determined later such that $a_1 \neq 0$ while λ and μ are arbitrary constants.

$$u' = -a_1\left(e^{-\phi(\xi)}\lambda + e^{-2\phi(\xi)} + u\right) \tag{3.6}$$

$$u'' = a_1\left(e^{-\phi(\xi)}\lambda^2 + 3e^{-2\phi(\xi)}\lambda + 2e^{-\phi(\xi)}\mu + \lambda\mu + 2e^{-3\phi(\xi)}\right) \tag{3.7}$$

$$u''' = -a_1\begin{pmatrix} e^{-\phi(\xi)}\lambda^3 + 7e^{-2\phi(\xi)}\lambda^2 + 8e^{-\phi(\xi)}\lambda\mu + \lambda^2 u + \\ 8e^{-2\phi(\xi)}u + 12e^{-3\phi(\xi)}\lambda + 2u^2 + 6e^{-4\phi(\xi)} \end{pmatrix} \tag{3.8}$$

Substituting u, u', u'' and u''' into Eq. (3.4) and then equating the coefficients of exp(-Φ(ξ))to zero, we get a set of equations as follows:

$$-3wa_1^2 + 6wa_1 = 0 \tag{3.9}$$

$$-6\lambda wa_1^2 + 12\lambda wa_1 = 0 \tag{3.10}$$

$$-3\lambda^2 wa_1^2 + 7\lambda^2 wa_1 - 6\mu wa_1^2 + 8\mu wa_1 + wa_1 - a_1 = 0 \tag{3.11}$$

$$\lambda^3 wa_1 - 6\lambda\mu wa_1^2 + 8\lambda\mu wa_1 + \lambda wa_1 - \lambda a_1 = 0 \tag{3.12}$$

$$\lambda^2 \mu wa_1 - 3\mu^2 wa_1^2 + 2\mu^2 wa_1 + \mu wa_1 - \mu a_1 + V = 0 \tag{3.13}$$

Solving Eq. (3.9) –Eq. (3.13) by Maple, we get

$$V = 0, w = \frac{1}{\lambda^2 - 4\mu + 1}, a_0 = a_0, a_1 = 2. \tag{3.14}$$

Where λ and μ are arbitrary constants.

Inserting Eq. (2.6)-Eq. (2.12) with Eq. (3.14) into Eq. (3.5) respectively, we get the following seven traveling wave solutions of the gHS equation.

When $\lambda^2-4\mu>0, \mu\neq 0,$

$$u_1=a_0+\frac{4\mu}{-\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}(\xi+k)\right)-\lambda}, \quad (3.15)$$

$$u_2=a_0+\frac{4\mu}{-\sqrt{\lambda^2-4\mu}\coth\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}(\xi+k)\right)-\lambda}, \quad (3.16)$$

$\xi=-\frac{t}{\lambda^2-4\mu+1}+x+y,$ where k is an arbitrary constant.

When $\lambda^2-4\mu<0, \mu\neq 0,$

$$u_3=a_0+\frac{4\mu}{\sqrt{-\lambda^2+4\mu}\tan\left(\frac{1}{2}\sqrt{-\lambda^2+4\mu}(\xi+k)\right)-\lambda}, \quad (3.17)$$

$$u_4=a_0+\frac{4\mu}{\sqrt{-\lambda^2+4\mu}\cot\left(\frac{1}{2}\sqrt{-\lambda^2+4\mu}(\xi+k)\right)-\lambda}, \quad (3.18)$$

$\xi=-\frac{t}{\lambda^2-4\mu+1}+x+y,$ where k is an arbitrary constant.

When $\lambda^2-4\mu=0, \mu=0, \lambda\neq 0$

$$u_5=a_0+\frac{2\lambda}{e^{\lambda(\xi+k)}-1}, \quad (3.19)$$

$\xi=-\frac{t}{\lambda^2+1}+x+y,$ where k is an arbitrary constant.

When $\lambda^2-4\mu>0, \mu\neq 0, \lambda\neq 0$

$$u_6=a_0-\frac{\lambda\left(-\frac{t}{\lambda^2-4\mu+1}+x+y+k\right)^2}{\lambda\left(-\frac{t}{\lambda^2-4\mu+1}+x+y+k\right)+2}, \quad (3.20)$$

where k is an arbitrary constant.

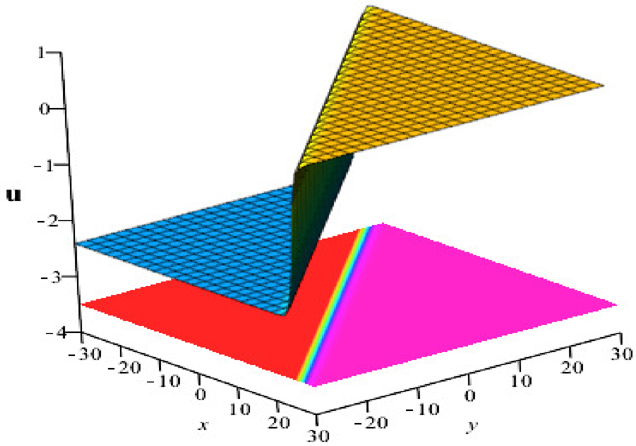
When $\lambda^2-4\mu=0, \mu=0, \lambda=0$

$$u_7=a_0+\frac{2}{-\frac{t}{\lambda^2-4\mu+1}+x+y+k}, \quad (3.21)$$

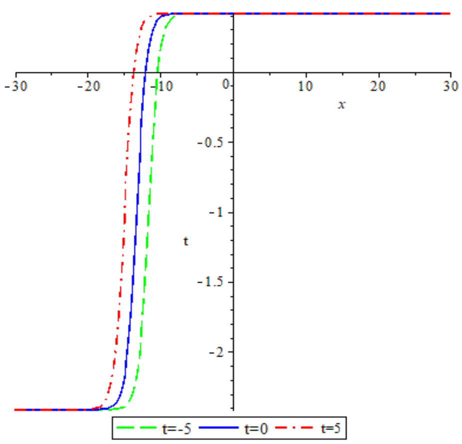
where k is an arbitrary constant.

IV. GRAPHICAL REPRESENTATION

In this subsection, we will discuss the graphical representation of some obtained solution of an integrable (2+1)-D generalized Hirota- Satsuma equation. By applying the exp(-Φ(ξ))-expansion method to above equation, we get some solutions which are $u_1(x,y,t), u_2(x,y,t), u_3(x,y,t),$ $u_4(x,y,t),$ $u_5(x,y,t),$ $u_6(x,y,t)$ and $u_7(x,y,t)$. Here, we plotted three types solution among seven, first is exact kinky type solitary wave, second is periodic kinky type solitary wave and third is periodic line breather wave solution, which are display in Fig.1-Fig.3, respectively.



(a)



(b)

Fig.1. Kink type soliton profile of Eq. (3.1) for $\lambda = 2, \mu = 0.5, a_0 = 1, E = 2$ of u_1. (a) Top shows the 3D plot and bottom is corresponding density plot. (b) Shows the 2D plot for different value of t.

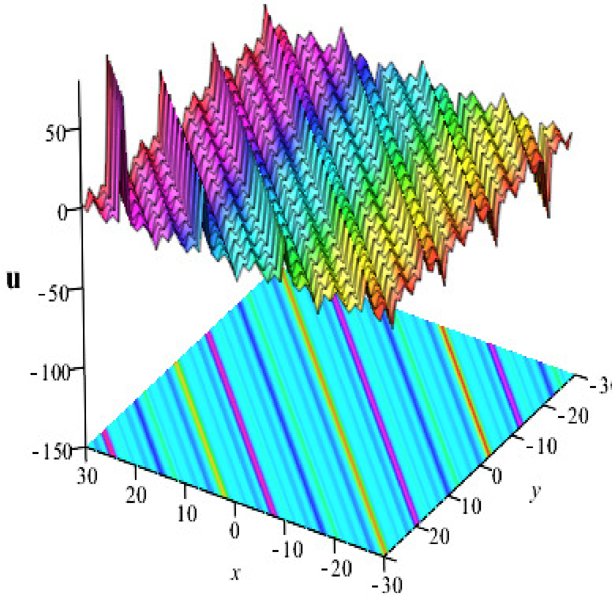
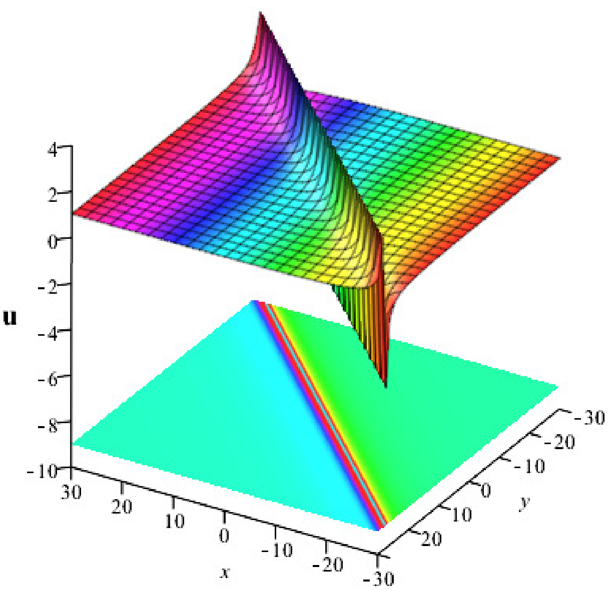
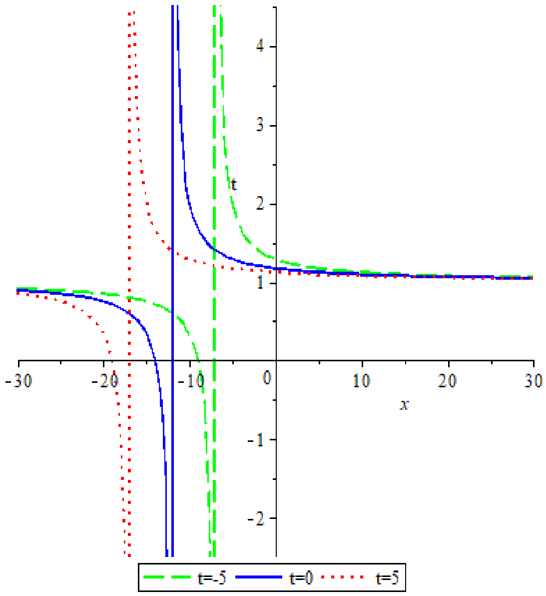


Fig.2. Periodic wave profile of Eq. (3.1) for $\lambda = 0.5, \mu = 1, a_0 = 1, E = 2$ of u_4, top shows the 3D plot and bottom is corresponding density plot.



(a)



(b)

Fig.3. Periodic line breather wave profile of Eq. (3.1) for $\lambda = 0, \mu = 0, a_0 = 1, E = 2$ of u_7 . (a) Top shows the 3D plot and bottom is corresponding density plot. (b) Shows the 2D plot for different value of t.

V. CONCLUSIONS

In summary, based on the exp(-Φ(ξ))-expansion method and by a symbolic computation, we obtain some exact solutions of an integrable (2+1)-dimensional generalized Hirota- Satsuma equation such as trigonometric, hyperbolic and breather type solution. Some of them are graphically illustrated which are shown in Fig.1 to Fig.3. We hope that presented results are very much helpful in the field of science and engineering.

REFERENCES

[1] M. Wang, X. Li, J. Zhang. The (G'/G) -expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics. Phys. Lett. A. 372 (2008) 417-423.

[2] S. T. Mohyud-din, A. Ali, M. A. Iqbal, Traveling wave solutions of Zakharov–Kuznetsov□modified equal□width and Burger's equations via exp(-Φ(ξ)) -expansion method, Asia Pac. J. Comput. Engin. 2 (1) (2015).

[3] M. B. Hossen, H. O. Roshid, M. Z. Ali, Characteristics of the solitary waves and rogue waves with interaction phenomena in a (2 + 1)-dimensional Breaking Soliton equation, Phys. Lett. A 382 (19) (2018) 1268-1274.

[4] M. B. Hossen, H. O. Roshid, M. Z. Ali, Multi-soliton, breathers, lumps and interaction solution to the (2+1)-dimensional asymmetric Nizhnik-Novikov-Veselov equation, Heliyon 5(10) (2019) e02548.

[5] A. Khater, W. Malfliet, D. Callebaut, E. Kamel E, The tanh method, a simple transformation and exact analytical solutions for nonlinear reaction–diffusion equations, Chaos. Soliton. Fract. 14 (2002), 513–522.

[6] Y. Yu, Supersymmetric Sawada – Kotera - Ramani equation: bilinear approach, Commun. Theor. Phys. 49 (3) (2008), 685–688.

[7] H.O. Roshid, M.N. Alam, M.F. Hoque, M.A. Akbar, A new extended (G'/G) -Expansion method to find exact traveling wave solutions of nonlinear evolution equations, Mathematics and Statistics 1 (2013) 162–166.

[8] M. Osman, Nonlinear interaction of solitary waves described by multi-rational wave solutions of the (2+1)-dimensional Kadomtsev–Petviashvili equation with variable coefficients, Nonlinear Dyn. 87 (2) (2017) 1209–1216.

[9] H. T. Chen, H. Q. Zhang, New multiple soliton-like solutions to the Generalized (2 + 1)-dimensional KP equation. Appl. Math. Comput. 157 (2004) 765-773.

[10] W. X. Ma, Complexiton solutions to the Korteweg-de Vries equation. Phys. Lett. A 301 (2002) 35-44.

[11] H. O. Roshid, M. N. Alam, Multi-Soliton Solutions to Nonlinear Hirota-Ramani Equation, Appl. Math. Inf. Sci. 11 (3) (2017) 723-727.

[12] M. B. Hossen, H. O. Roshid, M. Z. Ali, Modified Double Sub-equation Method for FindingComplexiton Solutions to the (1 + 1) Dimensional Nonlinear Evolution Equations, Int. J. Appl. Comput. Math 3 (3) (2017) 1–19.

[13] W. X. Ma, L. Jie, C. Khalique, A Study on Lump Solutions to a Generalized Hirota-Satsuma-Ito Equation in (2+1)-Dimensions, Complexity, Volume 2018, Article ID 9059858, 7 pages.

[14] J. Ji, A new expansion and new families of exact traveling solutions to Hirota equation, Appl. Math. Comput. 204(20) (2008), 881-883.

[15] K. Imai, Dromion and lump solutions of the Ishimori-I equation, Prog. Theor. Phys. 98(5) (1997) 1013–1023.

[16] J. Satsuma, M.J. Ablowitz, Two-dimensional lumps in non-linear dispersive sys-tems, J. Math. Phys. 20(7) (1979) 1496–1503.

[17] H. O. Roshid, W. X. Ma, Dynamics of mixed lump-solitary waves of an extended (2+1)-dimensional shallow water wave model,Phys. Lett. A 382 (45) (2018) 3262-3268.

[18] W. Q. Peng, S.F. Tian, T. T. Zhang, Analysis on lump, lumpoff and rogue waves with predictability to the $(2+1)$-dimensional B-type Kadomtsev–Petviashviliequation,Phys.Lett.A 382(38) (2018) 2701-2708.

[19] Y. Liu, X. Wen, Fission and fusion interaction phenomena of mixed lump kink solutions for a generalized (3+1)-dimensional B-type Kadomtsev-Petviashvili equation, Mod. Phys. Lett. B 32(15) (2018) 1850161.

[20] X. Zhang, Y. Chen, Y. Zhang, Breather, lump and X soliton solutions to nonlocal KPequation, Comput. Math.Appl., 74 (2017) 2341–2347.

[21] S. M. Rayhanul Islam. Applications of the exp(-Φ(ξ))-Expansion Method to Find Exact Traveling Wave Solutions of the Benney-Luke Equation in Mathematical Physics. American J. Appl. Math. 3 (2015), 100-105.