

Solving Intuitionistic Fuzzy Maximal Flow Problem Using Intuitionistic Trapezoidal Fuzzy Number

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Abstract:

Amit Kumar and Manjot Kaur proposed a new algorithm to find the fuzzy maximal flow between source and sink by representing the flow as normal trapezoidal fuzzy numbers. There are several papers in the literature in which generalized fuzzy numbers are used for solving real life problems. But in this paper we introduced new algorithm to find the intuitionistic fuzzy maximal flow between source and sink by representing the flow as normal intuitionistic trapezoidal fuzzy numbers. To illustrate the new algorithm a numerical example is solved. If there is no vagueness about the flow between source and sink then the proposed algorithm gives the same result as in crisp maximal flow problems.

Keywords —: Intuitionistic fuzzy maximal flow problem; ranking function; Generalized Intuitionistic trapezoidal fuzzy numbers.

I. INTRODUCTION

The maximal flow problem is one of the classical problems of network optimization. It provides very useful models in a number of practical contexts including electrical powers, traffics, communication networks, oil pipeline, logistics and power system. The maximal flow problem was proposed by Fulkerson and Dantzig (1955) originally and solved by specializing the simplex method for the linear programming, and Ford and Fulkerson (1956) solved it by augmenting path algorithm. Kim and Roush (1982) is one of the first on this subject. They developed the fuzzy flow theory, presenting the conditions to obtain an optimal flow by means of definitions on fuzzy matrices. Chanas, S., Kolodziejczyk, W. (1982) studied flow is a real number and the capacities have upper and lower bounds with a satisfaction function. Again they studied (1986) the integer flows in network with fuzzy capacity constraints. Chanas, S., Delgado, M., Verdegay, J.L. (1983) investigated the Fuzzy optimal flow on imprecise structures. Kumar et al., proposed a new algorithm to find fuzzy maximal

flow between source and sink by using ranking function. Liu, S.T. and Kao, C. (2004) developed Network flow problems with fuzzy arc lengths. Ji, X., Yang L., and Shao, Zhen (2006) studied chance constrained maximum flow problem with arc capacities. Amit Kumar and Manjot Kaur (2010) developed new algorithm for solving fuzzy maximal flow problems using generalized trapezoidal fuzzy numbers. Jayagowri (2017) proposed new algorithm for solving intuitionistic fuzzy maximal flow problem using Intuitionistic fuzzy triangular number. In this paper a new algorithm is introduced for solving the intuitionistic fuzzy maximal flow problem using Intuitionistic Trapezoidal fuzzy number. This paper is organized as follows; In Section 2, some basic definitions, ranking function and, the arithmetic operations of Trapezoidal fuzzy numbers are reviewed. In Section 3, new algorithm is proposed for solving the intuitionistic fuzzy maximal flow. In section 4, an illustrative example is proved our theorem. In Section 5, Results and discussion are explicated.

IPRELIMINARIES

In this section some basic definitions, arithmetic operations and ranking function are reviewed.

2.1 Definition

A Fuzzy number $\tilde{A}=(a,b,c,d)$ is said to be a trapezoidal fuzzy if its membership function is given by

$$\mu_A(x) = \left\{ \begin{array}{ll} 0, & -\infty < x \leq a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & b \leq x \leq c \\ \frac{x-d}{c-d}, & c < x \leq d \\ 0, & d \leq x < \infty \end{array} \right\} \text{ Where } a,b,c,d \in \mathbb{R}$$

2.2 Definition

A Trapezoidal fuzzy number $\tilde{A}=(a,b,c,d)$ is said to be nonnegative trapezoidal fuzzy number if and only if $a \geq 0$.

2.3 Definition

A Trapezoidal fuzzy number $\tilde{A}=(a,b,c,d)$ is said to be zero trapezoidal fuzzy number if and only if $a=b=c=d=0$.

2.4 Trapezoidal Intuitionistic Fuzzy Number

An Intuitionistic fuzzy number $A = \{ \langle a_1, a_2, a_3, a_4; w \rangle \langle b_1, b_2, b_3, b_4; w \rangle \}$ is said to be a generalized trapezoidal Intuitionistic fuzzy number if its membership function and non-membership function are given by

$$\mu_A(x) = \left\{ \begin{array}{ll} \frac{(x-a_1)}{(a_2-a_1)} w & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{(x-a_4)}{(a_3-a_4)} w & a_3 \leq x \leq a_4 \end{array} \right\}$$

$$\gamma_A(x) = \left\{ \begin{array}{ll} \frac{(x-b_1)}{(b_2-b_1)} w & b_1 \leq x \leq b_2 \\ 1 & b_2 \leq x \leq b_3 \\ \frac{(x-b_4)}{(b_3-b_4)} w & b_3 \leq x \leq b_4 \end{array} \right\}$$

2.5 Ranking of Intuitionistic trapezoidal fuzzy number:

A trapezoidal intuitionistic fuzzy number $A = \{ \langle a_1, a_2, a_3, a_4; w \rangle \langle b_1, b_2, b_3, b_4; w \rangle \}$ is completely defined by it's the membership and non-membership functions as follows.

$$L_\mu(x) = \frac{x-a_1}{a_2-a_1} w ; a_1 \leq x \leq a_2 \quad \& \quad R_\mu(x) = \frac{x-a_4}{a_3-a_4} w ; a_2 \leq x \leq a_4$$

$$L_\gamma(x) = \frac{x-b_1}{b_2-b_1} w ; b_1 \leq x \leq b_2 \quad \& \quad R_\gamma(x) = \frac{x-b_4}{b_3-b_4} w ; b_2 \leq x \leq b_4$$

Then L^{-1} and R^{-1} are inverse functions of functions L and R respectively,

Then the Graded Mean Integration Representation of membership function and non-membership function are

$$P_\mu(A) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} \quad \text{and} \quad P_\gamma(A) = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

2.6 Arithmetic operations of Trapezoidal Intuitionistic fuzzy number:

If $\tilde{A}' = \{ \langle a_1, a_2, a_3, a_4 \rangle ; \langle b_1, b_2, b_3, b_4 \rangle \}$ and $\tilde{B}' = \{ \langle a'_1, a'_2, a'_3, a'_4 \rangle ; \langle b'_1, b'_2, b'_3, b'_4 \rangle \}$

are two intuitionistic fuzzy numbers we define,

Addition :

$$\tilde{A}^I + \tilde{B}^I = \left\{ \left(a_1 + a'_1, a_2 + a'_2, a_3 + a'_3, a_4 + a'_4 \right); \left(a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3, a'_4 + b'_4 \right) \right\}$$

Subtraction:

$$\tilde{A}^I - \tilde{B}^I = \left\{ \left(a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1 \right); \left(a'_1 - b'_4, a'_2 - b'_3, a'_3 - b'_2, a'_4 - b'_1 \right) \right\}$$

2.7 Arithmetic operations of Trapezoidal fuzzy numbers

In the existing algorithm the following arithmetic operations are used:

Let $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers then

(i) $\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$

(ii) $\tilde{A} \ominus \tilde{B} = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$

To overcome the shortcomings, the subtraction operation used in existing algorithm is replaced by the following operation

Let $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers then

$$\tilde{A} \ominus \tilde{B} = (a, b, c, d)$$

Where,

$$\left\{ \begin{array}{l} d = \text{maximum}(0, (d_1 - d_2)) \\ c = \text{maximum}(0, \text{minimum}(d, (c_1 - c_2))) \\ b = \text{maximum}(0, \text{minimum}(c, (b_1 - b_2))) \\ a = \text{maximum}(0, \text{minimum}(b, (a_1 - a_2))) \end{array} \right\}$$

3. NEW ALGORITHM

In this section a new algorithm is proposed for solving the intuitionistic fuzzy maximal-flow problems. It is very easy to understand and apply for solving intuitionistic fuzzy maximal-flow problems in real life situations. The intuitionistic fuzzy maximal flow algorithm is based on finding breakthrough paths with net positive flow, between

the sour and sink nodes. Consider an arc (i, j) with initial intuitionistic fuzzy capacities $(\langle \tilde{I}f\tilde{c}_{\mu_{ij}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ij}}; \max(w_2, w'_2) \rangle >$

to $\langle \tilde{I}f\tilde{c}_{\mu_{ji}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ji}}; \max(w_2, w'_2) \rangle >$

represent these intuitionistic fuzzy residuals. As portions of these intuitionistic fuzzy capacities are committed to the flow in the arc, the intuitionistic fuzzy residuals (or remaining intuitionistic fuzzy capacities) of the arc are updated. We use the notation

$(\langle \tilde{I}f\tilde{c}_{\mu_{ij}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ij}}; \max(w_2, w'_2) \rangle >$

to $\langle \tilde{I}f\tilde{c}_{\mu_{ji}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ji}}; \max(w_2, w'_2) \rangle >$

represent these intuitionistic fuzzy residuals. For a node j that receives flow from node i we attach a label $[\langle \tilde{I}f\tilde{a}_{\mu_j}, \tilde{I}f\tilde{a}_{\gamma_j} \rangle, i]$ where $\langle \tilde{f}a_{\mu_j}, \tilde{I}f\tilde{a}_{\gamma_j} \rangle$ is the intuitionistic fuzzy flow from node i to j . The steps of the algorithm are summarized as follows:

Step 1. For all arcs (i, j) , set the residual intuitionistic fuzzy capacity equal to the initial intuitionistic fuzzy capacity i.e.,

$(\langle \tilde{I}f\tilde{c}_{\mu_{ij}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ij}}; \max(w_2, w'_2) \rangle >$

$\langle \tilde{I}f\tilde{c}_{\mu_{ji}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ji}}; \max(w_2, w'_2) \rangle >$

$(\langle \tilde{I}f\tilde{c}_{\mu_{ij}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ij}}; \max(w_2, w'_2) \rangle >$

$\langle \tilde{I}f\tilde{c}_{\mu_{ji}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ji}}; \max(w_2, w'_2) \rangle >$

Let $\tilde{I}f\tilde{a}_1 = (\langle \alpha, \alpha, \alpha, \infty; 1 \rangle \langle \alpha, \alpha, \alpha, \infty; 0 \rangle)$ and label source 1 with $[\langle \alpha, \alpha, \alpha, \infty; 1 \rangle \langle \alpha, \alpha, \alpha, \infty; 0 \rangle]$ set $i=1$ go to step 2.

Step 2. Determine S_i , the set of unlabelled nodes j that can be reached directly from node i by arcs with positive residuals (i.e.

$\langle \tilde{I}f\tilde{c}_{\mu_{ij}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ij}}; \max(w_2, w'_2) \rangle >$ if it

is a non-negative fuzzy number for all $j \in S_i$). If $S_i \neq \emptyset$, go to step 3. Otherwise, go to step 5.

Step 3. Determine $k \in S_i$ such that maximum R

$(\langle \tilde{I}f\tilde{c}_{\mu_{ij}}; \min(w_1, w'_1), \tilde{I}f\tilde{c}_{\gamma_{ij}}; \max(w_2, w'_2) \rangle >$

$= R(\langle \tilde{I}f_{\mu_{ik}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ik}}; \max(w_2, w_2') \rangle)$
 Set $(\langle \tilde{I}f_{\mu_k}, \tilde{I}f_{\gamma_k} \rangle) = (\langle \tilde{I}f_{\mu_{ik}}, \tilde{I}f_{\gamma_{ik}} \rangle)$ and label node k with $[\langle \tilde{I}f_{\mu_k}, \tilde{I}f_{\gamma_k} \rangle, i]$ If $k = n$, the sink node has been labeled, and a breakthrough path is found, go to step 5. Otherwise, set $i = k$, and go to step 2.

Step 4. (Backtracking). If $i = 1$, no breakthrough is possible; go to step 6. Otherwise, let r be the node that has been labeled immediately before the current node i and remove i from the set of nodes adjacent to r . Set $i = r$, and go to step 2.

Step 5. (Determination of Residuals). Let $N_p = (1, k_1, k_2, \dots, k_n)$ define the nodes of the p^{th} breakthrough path from source node 1 to sink node n . Then the maximal flow along the path is computed as

$$\tilde{I}f_p = \text{minimum}(\langle \tilde{I}f_{\mu_{k_1}}, \tilde{I}f_{\gamma_{k_1}} \rangle, \langle \tilde{I}f_{\mu_{k_2}}, \tilde{I}f_{\gamma_{k_2}} \rangle, \dots, \langle \tilde{I}f_{\mu_{k_n}}, \tilde{I}f_{\gamma_{k_n}} \rangle) / \min(\langle \tilde{I}f_{\mu_{k_2}}, \tilde{I}f_{\gamma_{k_2}} \rangle, \dots, \langle \tilde{I}f_{\mu_{k_n}}, \tilde{I}f_{\gamma_{k_n}} \rangle)$$

The residual capacity of each arc along the breakthrough path is decreased by $\tilde{I}f_p$ in the direction of the flow and increased in the reverse direction i.e., for nodes i and j on the path, the residual flow is changed from the current

$$\langle \tilde{I}f_{\mu_{ij}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ij}}; \max(w_2, w_2') \rangle$$

$$\langle \tilde{I}f_{\mu_{ji}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ji}}; \max(w_2, w_2') \rangle$$

to

$$1. (\langle \tilde{I}f_{\mu_{ij}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ij}}; \max(w_2, w_2') \rangle) \ominus \tilde{I}f_p$$

if the flow is from i to j .

$$2. (\langle \tilde{I}f_{\mu_{ji}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ji}}; \max(w_2, w_2') \rangle) \oplus \tilde{I}f_p$$

if the flow is from j to i .

To restore the previous condition any nodes that were removed in step 4. Set $i = 1$, and return to step 2 to attempt a new breakthrough path.

Step 6. (Solution). Given that m breakthrough paths have been determined, the intuitionistic

fuzzy maximal flow in the network is $\tilde{F} = (\tilde{I}f_1 \oplus \tilde{I}f_2 \oplus \dots \oplus \tilde{I}f_m)$ where m is the number of iterations that get no breakthrough.

4. ILLUSTRATIVE EXAMPLE

In this section the improved algorithm is illustrated by solving a numerical example.

Example 1. Consider the network shown in Figure 1. The bidirectional fuzzy capacities are shown on the respective arcs. For example,

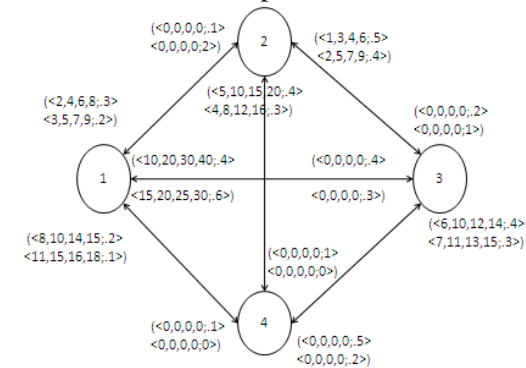


Fig 1

The algorithm is applied in the following manner.

Iteration 1. Set the residuals intuitionistic fuzzy capacity

$$\langle \tilde{I}f_{\mu_{ij}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ij}}; \max(w_2, w_2') \rangle = \langle \tilde{I}f_{\mu_{ji}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ji}}; \max(w_2, w_2') \rangle$$

initial capacities

$$\langle \tilde{I}f_{\mu_{ij}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ij}}; \max(w_2, w_2') \rangle$$

$$\langle \tilde{I}f_{\mu_{ji}}; \min(w_1, w_1'), \tilde{I}f_{\gamma_{ji}}; \max(w_2, w_2') \rangle$$

Step 1. Set

$$\tilde{I}f_1 = (\langle \infty, \infty, \infty, \infty; 1 \rangle \langle \infty, \infty, \infty, \infty; 0 \rangle)$$

and label node 1 with $[(\langle \infty, \infty, \infty, \infty; 1 \rangle \langle \infty, \infty, \infty, \infty; 0 \rangle) -]$

Set $i = 1$.

Step 2. $S_1 = \{2, 3, 4\} (\neq \emptyset)$

Step 3. $k = 3$, because maximum

$$\{R(\tilde{I}f_{12}), R(\tilde{I}f_{13}), R(\tilde{I}f_{14})\} = R(\tilde{I}f_{13})$$

That is

$$\text{maximum}(\langle 5,0.3 \rangle \langle 6,.02 \rangle; \langle 25,0.4 \rangle) \langle 22.5,0.6 \rangle$$

$$; \langle 11.8,0.2 \rangle \langle 15.2,0.1 \rangle = \langle 25,0.4 \rangle \langle 22.5,0.6 \rangle$$

Set

$$\tilde{I}f a_3 = \tilde{I}f c_{13} = \langle 10,20,30,40;0.4 \rangle \langle 15,20,25,30;0.6 \rangle$$

,and label node 3 with

$$[\langle 10,20,30,40;0.4 \rangle \langle 15,20,25,30;0.6 \rangle; -] \text{Set } i = 3,$$

and repeat step 2.

Step 4: $S_3 = \{4\}$ here we have only one point therefore obviously the only one point is our maximum point that is max

$$\{R(\tilde{I}f c_{34})\} = R(\tilde{I}f c_{34})$$

$$\text{Set } \tilde{I}f a_4 = \tilde{I}f c_{34} = \langle 6,10,12,14;0.4 \rangle$$

$$\langle 7,11,13,15;0.3 \rangle \text{ and label node 4 with}$$

$$[\langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle; -]$$

Step 5. The break through path is achieved.

1→3→4. The break through path is determined from the labels starting at node 4 and moving backward to node 1 and the breakthrough path is 1→3→4. Thus, $N_1 = \{1,3,4\}$ and COMPLETED

$$\tilde{I}f_1 = \min\{\tilde{I}f a_1, \tilde{I}f a_3, \tilde{I}f a_4\} = \min(\langle \infty, \infty, \infty, \infty; 1 \rangle$$

$$\langle \infty, \infty, \infty, \infty; 0 \rangle) \langle 10,20,30,40;0.4 \rangle$$

$$\langle 15,20,25,30;0.6 \rangle$$

$$\langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle$$

$$= (\langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle)$$

The Intuitionistic fuzzy residual capacities along path N_1 are

$$(\tilde{I}f c_{13}, \tilde{I}f c_{31}) = (\langle 10,20,30,40;0.4 \rangle \langle 15,20,25,30;0.6 \rangle$$

$$\ominus \langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle$$

$$\langle 0,0,0,0;0.4 \rangle \langle 0,0,0,0;0.3 \rangle$$

$$\oplus \langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle$$

$$= (\langle 10 - 14, 20 - 12, 30 - 10, 40 - 6; \min(0.4, 0.4) \rangle$$

$$\langle 15 - 15, 20 - 13, 25 - 11, 30 - 7; \max(0.6, 0.3) \rangle$$

$$\langle 0 + 6, 0 + 10, 0 + 12, 0 + 14; \min(0.4, 0.4) \rangle$$

$$\langle 0 + 7, 0 + 11, 0 + 13, 0 + 15; \max(0.3, 0.3) \rangle$$

$$= (\langle -4, -8, 20, 34; 0.4 \rangle \langle 0, 7, 14, 23; 0.6 \rangle,$$

$$\langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle)$$

$$(\tilde{I}f c_{34}, \tilde{I}f c_{43}) = (\langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle$$

$$\ominus \langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle$$

$$\langle 0,0,0,0;0.5 \rangle \langle 0,0,0,0;0.2 \rangle$$

$$\oplus \langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle$$

$$= (\langle -8, -2, 2, 8; 0.4 \rangle \langle -8, -2, 2, 8; 0.3 \rangle$$

$$\langle 6,10,12,14;0.4 \rangle \langle 7,11,13,15;0.3 \rangle$$

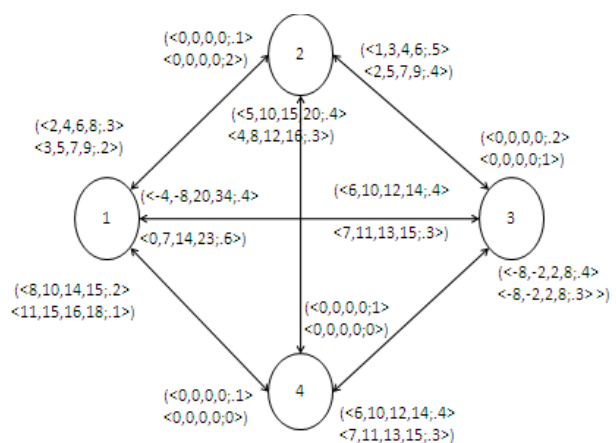


Fig 2 obtained Network after iteration 1

Iteration 2.

Repeating the procedure described in the previous iteration, at the starting node 1, the obtained break through is 1→4. And

$$\tilde{f}_2 = \langle 8,10,14,15;0.2 \rangle \langle 11,15,16,18;0.1 \rangle$$

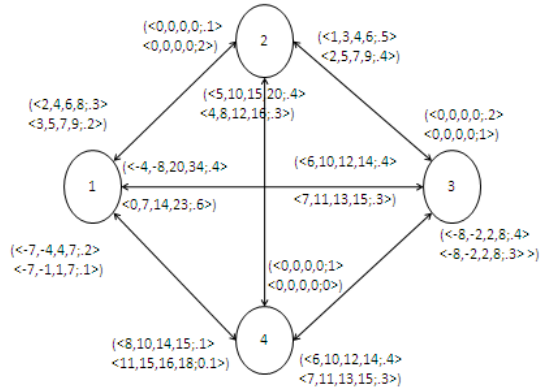


Fig 3 obtained Network after iteration 2

Iteration 3.

Repeating the procedure described in the previous iteration, at the starting node 1, the obtained break through is 1→2→4. And

$$\tilde{f}_3 = \langle 2,4,6,8,0.3 \rangle \langle 3,5,7,9;0.2 \rangle$$

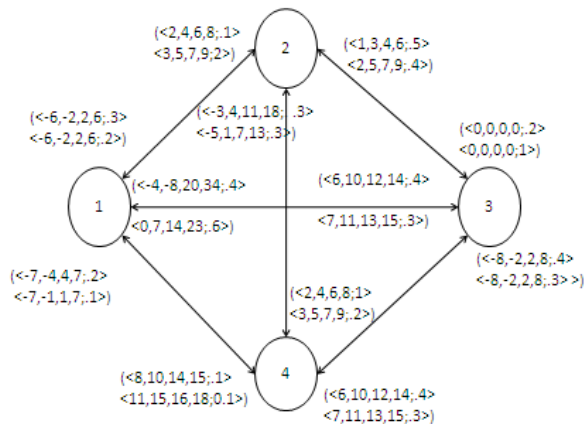


Fig 4 obtained Network after iteration 3

Iteration 4.

Repeating the procedure described in the previous iteration, at the starting node 1, the obtained break through is 1→3→4. And

$$\tilde{f}_4 = \langle -4,-8,20,34;0.4 \rangle \langle 0,7,14,23;0.6 \rangle$$

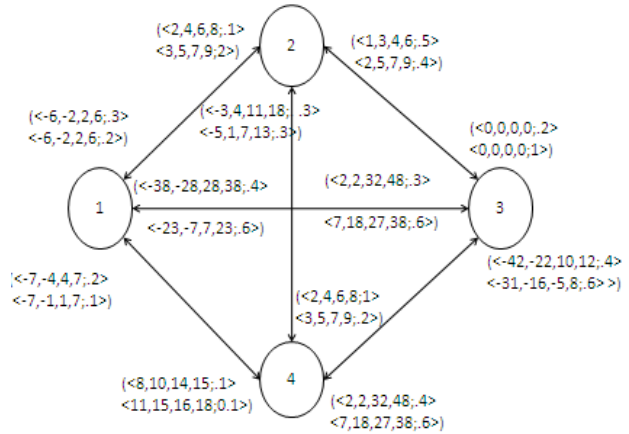


Fig 5 obtained Network after iteration 4

More iterations are not possible after 4th iteration because there is no way out to reach at sink from source. The Intuitionistic fuzzy maximal flow is:

$$\begin{aligned} \tilde{f} &= \tilde{f}_1 + \tilde{f}_2 + \tilde{f}_3 + \tilde{f}_4 = \langle 6,10,12,14;4 \rangle \\ &\langle 7,11,13,15;0.3 \rangle \\ &\oplus \langle 8,10,14,15;0.2 \rangle \langle 11,15,16,18;0.1 \rangle \oplus \\ &\langle 2,4,6,8;0.3 \rangle \langle 3,5,7,9;0.2 \rangle \rangle \\ &\langle -4,-8,20,34;0.4 \rangle \langle 0,7,14,23;0.6 \rangle \rangle \\ &= \langle 12,16,52,71;0.2 \rangle \langle 21,38,50,65;0.6 \rangle \end{aligned}$$

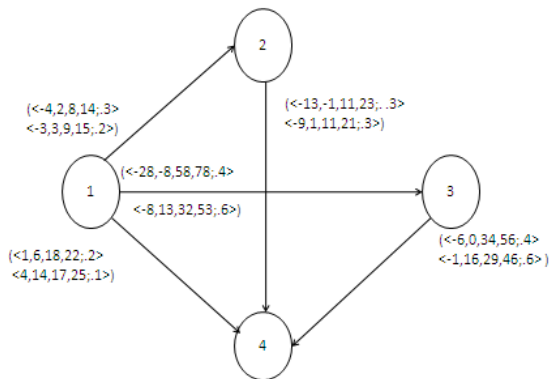
Intuitionistic fuzzy optimal flow in different arc

Table 1

Arc	$(\tilde{f}^c_{\mu_{ij}}, \tilde{f}^c_{\gamma})$	R	Intuitionistic fuzzy flow amount	Direction
	$(\tilde{f}^c_{\mu_{ji}}, \tilde{f}^c_{\gamma})$	$(\tilde{f}^c_{\mu_{ij}})$	$(\tilde{f}^c_{\mu_{ji}})$	
	$(\tilde{f}^c_{\mu_{ij}}, \tilde{f}^c_{\gamma})$	R	$(\tilde{f}^c_{\mu_{ij}})$	
	$(\tilde{f}^c_{\mu_{ji}}, \tilde{f}^c_{\gamma})$	$(\tilde{f}^c_{\mu_{ij}})$	$(\tilde{f}^c_{\mu_{ji}})$	

(1,2)	<- 4,2,8,14;0.3> <- 3,3,9,15;.2> <(-8,-6,-4,- 2;.1) (-9,-7,-5,-3;.2>	<10.6> <-5,-6>	<- 4,2,8,14;0. 3> <- ,3,9,15;.2>	1-2
(1,3)	<-28,- 8,58,78;.4> <- 8,13,32,53;.6> <-14,-6,0,1;.1> <-3,-7,-6,- 9;0.5>	<25,22.5 22.5>	<-28,- 8,58,78;.4> <- 8,13,32,53; .6>	1-3
(1,4)	<1,6,18,22;0.2 > <4,14,17,25;.1 > <(-15,-14,-10,- 8;.1) (-18,-16,-15,- 11;.1>	<11.8,15. 2> <-11.8,- 15.2>	<1,6,18,22; 0.2> <4,14,17,2 5;.1>	1-4
(2,3)	<-5,-1,1,5;0.5> <-7,- 2,2,7;0.4> <0,0,0,0;0.2> <0,0,0,0.1>	<.0,0> <.0,0>	--	---
(2,4)	<-13,- 1,11,23;.3> <- 9,1,11,21;.3> <-8,-6,-4,- 2;.1> <-9,-7,-5,- 3;0.2>	<5,6> ; <-5,-6>	<(1,2,3,10;.2 > <4,3,10,13; .4>	2-4
(3,4)	<- 6,0,34,56;0.4> <- 1,16,29,46;0.6 > <-48,-32,-2,- 2;.3> <-38,-27,-18,- 70;.6>	<19,7,22. 5>; <- 19.7,- 22.5>	<- 6,0,34,56;0 4> <- ,16,29,46;0 .6>	3-4

Fig 6 No breakthrough path



5. Result and discussion

The obtained result can be explained as follows:

- 1) The amount of flow between source and sink is greater than 30 and less than 115 units.
- 2) Maximum number of persons are in favor that amount of flow will be 60 units.
- 3) The percentage of favourness for remaining flow can be obtained as follow:

Let x represents the amount of flow, then

- 1) the percentage of the favourness for membership function and non-membership function

are $\langle 12,16,52,71;0.2 \rangle \langle 21,38,50,65;0.6 \rangle$

$$\mu_A(x) = \left\{ \begin{array}{l} \frac{(x-12)}{4} \cdot 0.2 ; 12 \leq x \leq 16 \\ 1 ; 16 \leq x \leq 52 \\ \frac{(x-71)}{19} \cdot 0.2 ; 52 \leq x \leq 71 \end{array} \right\}$$

$$\text{and } \gamma_A(x) = \left\{ \begin{array}{l} \frac{(x-38)}{17} \cdot 0.6 ; 21 \leq x \leq 38 \\ 1 ; 38 \leq x \leq 50 \\ \frac{(x-65)}{15} \cdot 0.6 ; 50 \leq x \leq 65 \end{array} \right\}$$

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