RESEARCH PAPER OPEN ACCESS

Research of Numerical Method for Solving Ordinary Differential Equations

Rekha Vishwakarma[1], Garima Singh[2], Nidhi Gargav*[3]
(Department of Mathematics , Barkatullah University Institute of Technology, and Bhopal(M.P)
Email: rekhavishwa2002@gmail.com)
(Department of Mathematics, Barkatullah University Institute of Technology, and Bhopal(M.P)
Email: drgarimasingh709@gmail.com)
(Department of Mathematics, Barkatullah University Institute of Technology, and Bhopal(M.P)
Email: Nidhigargav1911@gmail.com)

--************************----------------------------------

Abstract:

The Runge-Kutta Method is a technique of numerical iterating ordinary differential equation (ODE) .These days the exigency for solving real world obstacle has been enhanced .The obstacle in question here is a traffic flow problem involving another parameters solving vehicular traffic flow based on partial differential equation (PDE) are more complex and time consuming then can't be done in real time .So (ODE) mathematical model are constructed from other RK method is more credible to another iteration method .These accurate solution can be widely used to solve the enhancing exigency for traffic flow optimization .In This paper the construction of an (ODE) Mathematical model is a done for a real world obstacle specifically traffic flow is discussed and a numerical solution is given as an example we have also compared the solution obtained from the Runge-Kutta Method with Improved Euler's method .Thus showing the solution obtained from an order RK method (for first order ODE) is more accurate.

Keywords — **Runge -Kutta method ,ordinary differential equation, traffic flow , partial differential equation , mathematical model ,Improved Euler's method**

--************************----------------------------------

INTRODUCTIONS:

Traffic flow is the perusal of the movement of different vehicles between two points for a certain periods time and the interactions between them. With the aim of understanding and providing the optimum solution. Here we consider a real time traffic flow problem. Which is converted into a mathematical model to get the optimal solution and it is used to verify the traffic density between two traffic signals in a city. Traffic flow are mainly based on Partial Differential Equations. But is very complex to solve and derive the solution. and make this model more suitable. We use numerical methods to get the solution numerical differential is used for evaluation because it addresses two issue accurate and time. Here we consider the RK method as it reduces the time spend and relatively more accurate and precise answer. To use this method we convert the partial differential equation to Ordinary Differential Equation because the Runge-kutta method is only applicable to (ODE) the fourth order RK method for (1st order ODE is more accurate) than first ,second ,and third order R-K method. The real world problem is converted to a mathematical model and the solution to be obtained is discussed below. The outcome of the model given is a numerical solution to vehicular density and velocity of traffic flow between consecutive signal in real time situation.

CONSTRUCTION OF A TRAFFIC FLOW PROBLEM :

VARIABLES (or) PARAMETER

In this paper, we consider ,only vehicular density (ρ) and velocity (v) to be relevant variables though there could be many other factors affecting the system such as accidents, on/off ramps ,length of roadway ,the spacing between cars ,etc. Thus ignoring the other factors, let F(ρ ,v) represent the Traffic Flow, in which we define ρ(x ,t) as the vehicular density ,in particular for car –density and v(x ,t)as the velocity at a point x and time t. The mathematical model for the above data will be Partial Differential Equation (PDE).Hence the obtained PDE will be converted into a pair of Ordinary Differential Equation (ODE) and solved further.

ODE MATHEMATICAL MODEL :

Let the function ,F(ρ ,v) **=0,**

Here, ρ is a function of x and t .The PDE is given as follow

$$\frac{\partial \rho}{\partial x}+\frac{\partial \rho}{\partial t}=0 \qquad \text{..........(1)}$$

Let us assume that ρ(x,t) =U (x)W (t), i.e., ρ can be written as the product of two functions, one depend only x, the other depends only on t.thus we get,

$$W\frac{dU}{dx}+U\frac{dW}{dt}=0 \qquad \text{...........(2)}$$

The above equation is reconstructed to collect the variable X in one side, and variable t in the side.

$$\frac{1}{U}\frac{dU}{dx}=-\frac{1}{W}\frac{dW}{dt} \qquad \text{...........(3)}$$

Let as introduce a common constant “a”, such that

$$\frac{1}{U}\frac{dU}{dx}=-\frac{1}{W}\frac{dW}{dt}=a \qquad \text{...........(4)}$$

The given PDE is now separated to form two ODE ‘s

$$\frac{1}{U}\frac{dU}{dx}=a \qquad \text{.............(5)}$$

$$\frac{1}{W}\frac{dW}{dt}=-a \qquad \text{..........(6)}$$

The general solution for equations (5)and (6) are U(x)= $k_1\exp(ax)$ *and* $W(t)=k_2\exp(-at)$, where k_1 and k_2 are arbitrary constants.

Also, equation (5) and (6) Can be again reconstructed to form a PDE with general solution.

Similarly, consider v as a function of x and t. The above process is followed to find the general solutions for, $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial t}=B\frac{dA}{dx}+A\frac{dB}{dt}=0$ as $A(x)=l_1\exp(rx)$ & $B(t)=l_2\ exp(\text{-rt})$, where r is a common constant and $1_1,1_2$ arbitrary constants.

NUMERICAL SOLUTION:

Ⅳ ORDER Runge- kutta Method :

For a given Ordinary Differential Equation ,

the Ⅳ ORDER R. K method is given by,

K_1=hf (x ,y)

K_2= hf $(x+\frac{h}{2}, y+\frac{k_1}{2})$

K_3= hf $(x+\frac{h}{2}, y+\frac{k_2}{2})$

K_4 = hf (x+h , y+k_3)

And $\Delta y=\frac{1}{6}(k_1+2k_2+2k_3+k_4)$ and

$y(x+h)=y(x)+\Delta$ where , h=Δx

A. *ILLUSTRATION:*

Consider an ODE of first order for one parameters , say v(t , x).

Let the differential equation be, **x’= t+x** with initial condition, x(0)=1. Assume h=0.2. Now let us find the value of x(2.0).

From the given, x_0 = x(0)=1 → t_0 =0 ; x_0 = 1 and h=0.2

By □ ORDER Runge-kutta method,

K_1 = hv (t_0, x_0) = 0.2

K_2 = hv $(t_0+\frac{h}{2}, x_0+\frac{k_1}{2})$ = 0.24

K_3 = hv $(t_0+\frac{h}{2}, x_0+\frac{k_2}{2})$ = 0.244

K_4 = hv (t_0+h , x_0+k_3) = 0.2888

$\Delta x=\frac{1}{6}(k_1+2k_2+2k_3+k_4)$ = 0.2428

X_1 = x(0.2) = $x_0+\Delta x$ = 1.2428

Now ,we can find x(0.4) with initial value

x_1 1.2428 and t_1 = 0.2

K_1 = hv (t_1 ,x_1) = 0.2885

K_2 = hv $(t_1+\frac{h}{2}, x_1+\frac{k_1}{2})$ = 0.3374

K_3= hv $(t_1+\frac{h}{2}, x_1+\frac{k_2}{2})$ = 0.3423

K_4 = hv (t_1+h , x_1+k_3) = 0.3970

$\Delta x=\frac{1}{6}(k_1+2k_2+2k_3+k_4)$ = 0.3408

X_2 = x(0.4) = $x_1+\Delta x$ = 1.5836

And continuing the process until the time we need to calculate,

For x(2.0) with initial value $x_9 = 9.2919$ and $t_9 = 1.8$

$K_1 = hv(t_9, x_9) = 2.2183$

$K_2 = hv(t_9 + \frac{h}{2}, x_9 + \frac{k_1}{2}) = 2.4602$

$K_3 = hv(t_9 + \frac{h}{2}, x_9 + \frac{k_2}{2}) = 2.4844$

$K_4 = hv(t_9 + h, x_9 + k_3) = 2.7552$

$\Delta x = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 2.4771$

$X_{10} = x(2.0) = x_9 + \Delta x = 11.7690$

The value of x_{10} represents, at t = 2.0 second, the distance covered is **x =**11.7690 meter. Therefore the velocity of traffic flow will be, v =11.7690$^{m}/_{s}$.

A similar process can be followed to find the vehicular density, assuming N to be the Number of vehicles with a distance between the signal as 2000 meter (I.e, 2 km) for a particular period of time.

Consider the same problem, to find the solution using Improved Euler's method.

$X^*_{n+1} = x_n + h^* f(t_n, x_n)$. n=0,1,2,..........9,

$X_{n+1} = x_n + \frac{h}{2}\{f(t_n, x_n) + f(t_n + h, x^*_{n+1})\}$

$X_1 = X(0.2)$

$= x_0 + \frac{h}{2}\{f(t_0, x_0) + f(t_0 + h, x^*_{n+1})\}$

$= 1.24$

$X_2 = x(0.4)$

$= x_1 + \frac{h}{2}\{f(t_1, x_1) + f(t_1 + h, x^*_{n+1})\}$

$= 1.5768$

....

$X_{10} = x(2.0)$

$= x_9 + \frac{h}{2}\{f(t_9, x_9) + f(t_9 + h, x^*_{n+1})\}$

$= 11.9744$

The solution obtained using Improved Euler's method varies when compared to Ⅳ ORDER R-K method. A graphical solution, comparing R-K method and Improved Euler's method is shown below

Table -2 →Comparision Between RK and Improved Euler's method

EXACT SOLUTION:

An Exact value problem is an ODEs together with some initial value X' = t+x x_0 = X(0) = 1

General solution to is $\mathbf{x_1 = x(0.2) = t = 2e^x - (x+1)}$

$x_0 = 1$

$x_1 = 1.24$

....

$X_{10} = 11.7781$

GRAPHICAL SOLUTION:

The value obtained from Improved Euler's method and RK method

Table 1- Comparision of accuracy between Improved Euler's method and RK method

Method X_n	RK Method	Improved Euler's Method	Exact Value
X_0	1	1	1
X_1	1.2428	1.24	1.24
X_2	1.5836	1.5768	1.5836
X_3	2.0441	2.0316	2.0442
X_4	2.6509	2.6305	2.6510
X_5	3.4362	3.4052	3.4364
X_6	4.4364	4.3943	4.4402
X_7	5.7056	5.6450	5.7103
X_8	7.3001	7.2149	7.3060
X_9	9.2919	9.4741	9.2992
X_{10}	11.7690	11.9744	11.7781

The graph provides comparison of accuracy between RK method and Improved Euler's method

Table-1.1 Comparison to Solving in ODEs Accuracy to Runge-Kutta Rules

RK METHOD	11.7690
IMPROVED EULER'S METHOD	11.9744
EXACT VALUE	11.7781

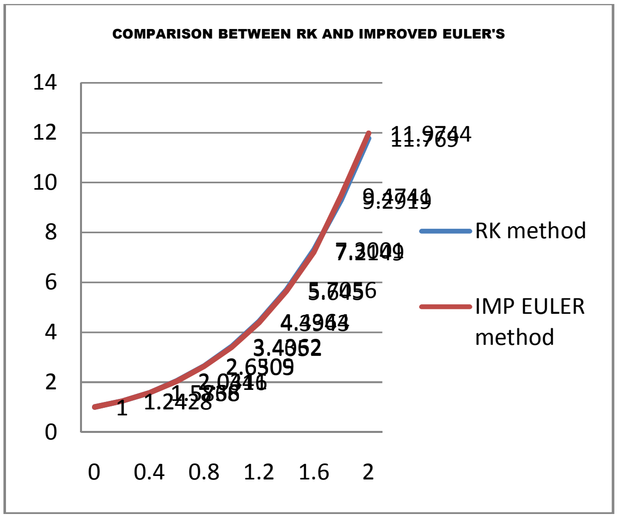
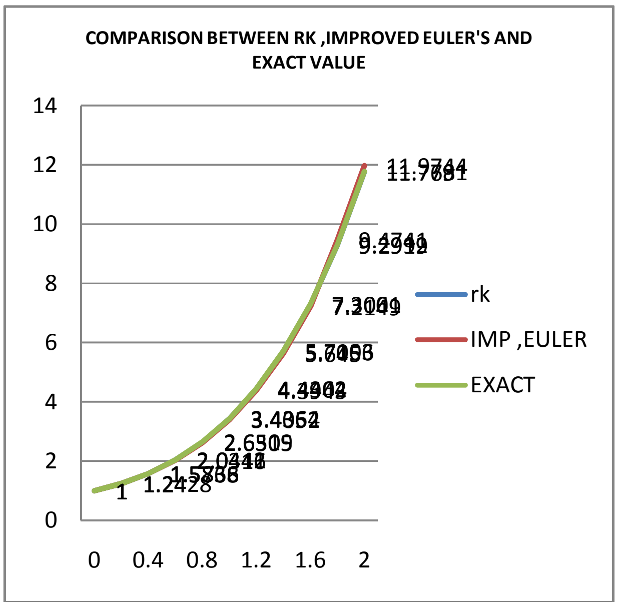


Table-2.1Comparision between RK, Improved Euler's Method and Exact value



From the above table ,we observe that the value obtained from the Improved Euler's method and RK method are the same and Even the exact value is same.

ADVANTAGES:

- The solution obtained is more accurate and precise
- It is mainly used to solve a complex problem ,physically or geometrically
- The numerical approach enable the solution of a complex problem with a great number of simple operation
- It is used for temporal discretizition of ordinary differential equation
- Time consumption is comparatively low when compared to other analytical method

APPLICATION:

- The Applications of the Runge kutta method has a greater effect in areas area such as
- Kinetic magnetic induction system ,Navier-Stoker system

CONCLUSION :

Traffic Flow is the perusal of the movement two vehicles and the complex in flow is identified and it is converted into a mathematical model to obtain the mathematical solution. Most of the traffic flow problem that use partial differential equations to deduce the solution but it is complicated to solve the equation and so we use numerical method to solve the problem when PDE is converted into ODE and then numerical is used as it decrease the time consumption. We use Ⅳ order Runge-kutta method (for first order ODE) which is more accurate and precise. We have also provided a comparative study between the reliability of the RK method and IMPROVED EULER'S method. Both method are given the same answers provided in this paper. and this method looks simple, but more complicated situation arise when boundary condition are introduced. The above traffic flow analysis given the traffic density between two traffic signal in the city and it is applied to verify the density and given the accurate time to pass the signal and can be used to find the shortest route to reach the destinations. This Method can also be converted into program code and the solution is obtained within a short period. Thus RK method is the best alternative method to solve the traffic flow analysis for a city of obtain an accurate solution.

ACKNOWLEDGMENT

The author assertively express their gratitude to the reviewers and the Editor-in-chief for their prompt response and invaluable suggestion.

REFERENCES:

[1] "A STUDY ON APPLICATION OF RUNGE- KUTTA METHOD WITH RESPECT TO TRAFFIC FLOW PROBLEM", International Journal of Emerging Technologies and Innovative Research (www.jetir.org), ISSN:2349-5162, Vol.6, Issue 6, page no.121-126, June 2019, Available :http://www.jetir.org/papers/JETIR1907774.pdf

[2] "A study on Numerical Exact Solution of Improved Euler's and Rung – Kutta Method **"**International Journal of Novel Research in Physics Chemistry &Mathematics (www.noveltyjournals.com),ISSN 2394-9651,Vol.3,Issue 1,pp: (1-5),Month: January-April 2016,http://www.noveltyjournal.com

[3] B. Rai , D.P.Choudhury, H.I. Freedman, A Course in Original Differential Equation,2002 Narosa Publishing House ,Reprint 2004- Revised.

[4]http://www.flow3d.com/resources/cfd-101/numerical-issues/implicit-versus-explicit-numerical-method/

[5] http:/www.public.asu.edu/~hhuange38/pde_slides_sepvar-heat.pdf

[6] http://fluid.ippt.pan.pl/metro/CDROM-PL/kursy/METRO-pdf-en/metro-ippt-lecture11.pdf

[7] http://www.inmath.com/differential-equation/12-runge-kutta-rk4-des.php

[8] Pierce, Rod. "Sorry We Can't Find It (404)" Math Is Fun. Ed. Rod Pierce. 25 Apr 2024. 25 Apr 2024 http://www.mathsisfun.com/calculas/separation-variable.html

[9] http://en.wikipedia.org/wiki/Runge -kutta

[10] **P**.Kandasamy , **k**.Thilagavathy , **K**.Gunavathi –"Numerical Methods"– pp.379-385–s.Chand & Company Ltd.

[11] *Weisstein ,eri W* "numerical Integration. "from *MathWorld* -.Awolfarm Web Resource. https://mathworld.wolfram.com/NumericalIntegration.html

[12] *William E.Boyce,Richard C.Diprima* , Elementary Differential Equation and Boundary value problem,6th ed,1997,by John Wiley &Sons, Inc.

