

# IMPLEMENTATION OF PAN-SAN SEQUENCE IN CRYPTOGRAPHY

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## Abstract:

In this manuscript, an application of the Pan-San sequence in cryptography by coding and decoding matrices with the support of the Unicode standard for information interchange, the approaches of finding errors, and correcting the errors are offered. As well, blocking methods are deliberated.

*Keywords* — Pan-San sequence, Coding and Decoding, Unicode.

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## 1. INTRODUCTION

Applications of Number theory enable Mathematical algorithms to make data unintelligible and secure, allowing authorized users to delete or update it. Cryptography is an application of Mathematics that enhances communication privacy through codes. It helps to maintain confidentiality and integrity in communications, ensuring data preservation from tampering. In Modern cryptographic systems, advanced Mathematics, including number theory, is used to explore the properties and relationships of numbers. The authors provide a strong fundamental grounding in [1]. To learn more about the application of cryptography see [2 – 10]. In [11],

the authors invented four novel sequences and their properties.

In this study, coding and decoding matrices are used to establish the application of the Pan-San sequence in cryptography. The procedure for detecting and correcting errors is described. Methods of blocking are also being explored.

## 2. Implementation of Pan-San numbers in coding and decoding

Consider the Pan-San sequence

$$0, t, 2t(t^2 + 1), 2t(t^2 + 1)^2 - t, 8t^2(t^2 + 1)^3 - 4t(t^2 + 1)^2, \dots$$

whose recurrence relation is defined by

$$P_{s,t} = 2t(t^2 + 1)P_{s-1,t} - P_{s-2,t}, \quad s, t \in \mathbb{N} - \{1\}$$

where  $P_{0,t} = 0$  and  $P_{1,t} = t$ .

Pan-San sequence for  $t = 2$  is stated as

$$(P_{s,2}) = (0, 2, 20, 198, \dots)$$

Let the matrix formulation of the sequence be

$A = \begin{pmatrix} 10 & -1 \\ 1 & 0 \end{pmatrix}$ . It is monitored by mathematical induction on  $s$  by

$$A^s = \begin{pmatrix} \frac{P_{s+1,2}}{2} & \frac{-P_{s,2}}{2} \\ \frac{P_{s,2}}{2} & \frac{-P_{s-1,2}}{2} \end{pmatrix} \text{ for all } s \in N \quad (1)$$

and it is clear that

$$\text{Det}(A^s) = \frac{1}{4} [(P_{s,2})^2 - (P_{s+1,2})(P_{s-1,2})]$$

Let  $C_m, D_m$  be the coding and decoding matrices.

After transforming Unicode for the text to be sent respectively.

Select the decoding matrix as

$$D_m = \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix} \quad (2)$$

where  $d_1, d_2, d_3,$  and  $d_4 \in Z^+$

Transform the chances of  $C_m$  and  $D_m$  as follows.

$$C_m = D_m \times A^s \quad (3)$$

This implies that

$$D_m = C_m \times (A^s)^{-1} \quad (4)$$

Hence, the coding matrix is

$$\begin{aligned} C_m &= \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix} \begin{pmatrix} \frac{P_{s+1,2}}{2} & \frac{-P_{s,2}}{2} \\ \frac{P_{s,2}}{2} & \frac{-P_{s-1,2}}{2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2}[(d_1 * P_{s+1,2}) + (d_2 * P_{s,2})] & -\frac{1}{2}[(d_1 * P_{s,2}) + (d_2 * P_{s-1,2})] \\ \frac{1}{2}[(d_3 * P_{s+1,2}) + (d_4 * P_{s,2})] & -\frac{1}{2}[(d_3 * P_{s,2}) + (d_4 * P_{s-1,2})] \end{pmatrix} \\ &= \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix} \quad (5) \end{aligned}$$

Then, transferring the code matrix to the receiver, one will obtain the message matrix  $D_m$  by decoding which is deliberated below.

$$\begin{aligned} D_m &= \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix} \begin{pmatrix} \frac{-P_{s-1,2}}{2} & \frac{P_{s,2}}{2} \\ \frac{-P_{s,2}}{2} & \frac{P_{s+1,2}}{2} \end{pmatrix} \\ &= \begin{pmatrix} -\frac{1}{2}[(c_1 * P_{s-1,2}) + (c_2 * P_{s,2})] & \frac{1}{2}[(c_1 * P_{s,2}) + (c_2 * P_{s+1,2})] \\ -\frac{1}{2}[(c_3 * P_{s-1,2}) + (c_4 * P_{s,2})] & \frac{1}{2}[(c_3 * P_{s,2}) + (c_4 * P_{s+1,2})] \end{pmatrix} \quad (6) \end{aligned}$$

The relationship between the coding matrix  $C_m$  and the decoding matrix  $D_m$  is provided by

$$\begin{aligned} \text{Det}(C_m) &= \text{Det}(D_m \times A^s) \\ &= \text{Det}(D_m) \times \text{Det}(A^s) \\ &= \text{Det}(D_m) \quad (7) \end{aligned}$$

Also, let us select the elements in the matrix  $D_m$  as given below

$$d_1 = -\frac{1}{2} [(c_1 P_{s-1,2}) + (c_2 P_{s,2})] > 0 \quad (8)$$

$$d_2 = \frac{1}{2} [(c_1 P_{s,2}) + (c_2 P_{s+1,2})] > 0 \quad (9)$$

$$d_3 = -\frac{1}{2} [(c_3 P_{s-1,2}) + (c_4 P_{s,2})] > 0 \quad (10)$$

$$d_4 = \frac{1}{2} [(c_3 P_{s,2}) + (c_4 P_{s+1,2})] > 0 \quad (11)$$

From (8), (9), (10) and (11), it is manifested that

$$\begin{aligned} \frac{P_{s,2}}{P_{s-1,2}} &< \frac{c_1}{c_2} < \frac{P_{s+1,2}}{P_{s,2}} \quad \text{and} \\ \frac{P_{s,2}}{P_{s-1,2}} &< \frac{c_3}{c_4} < \frac{P_{s+1,2}}{P_{s,2}} \\ \Rightarrow \frac{c_1}{c_2} &= \frac{c_3}{c_4} = \lambda \text{ (say)} \quad (12) \end{aligned}$$

The process of coding and decoding matrices is illustrated through the following example.

**Illustration 1**

Assume that the message to be sent is “TRUE”

Then, the corresponding text is

$$D_m = \begin{pmatrix} T & R \\ U & E \end{pmatrix}$$

Now, the coding and decoding procedure is explained below

Using Unicode standard for the corresponding Alphabets, the message matrix becomes

$$D_m = \begin{pmatrix} 54 & 62 \\ 55 & 45 \end{pmatrix}$$

The coding matrix for  $s = 1$  is given by

$$\begin{aligned} C_m &= D_m A^1 \\ &= \begin{pmatrix} 54 & 62 \\ 55 & 45 \end{pmatrix} \begin{pmatrix} 10 & -1 \\ 1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 602 & -54 \\ 595 & -55 \end{pmatrix} \end{aligned}$$

The decoding matrix for  $s = 1$  is

$$\begin{aligned} D_m &= \begin{pmatrix} 602 & -54 \\ 595 & -55 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 10 \end{pmatrix} \\ &= \begin{pmatrix} 54 & 62 \\ 55 & 45 \end{pmatrix} \end{aligned}$$

Ultimately, the Unicode aids in recognizing the correct word "TRUE".

**3. Error detection and correction**

The main goal of coding and decoding theory is the detection and rectification of errors in the code message  $C_m$ . The foremost objective is to employ the determinant property of the matrix as a check scenario for the delivered message  $C_m$ .

By calculating the determinants of  $C_m$  and  $D_m$  for  $s = 1$ , the receiver can determine whether the code message  $C_m$  is true or false by comparing the determinants received from the channel.

There are several scenarios to locate the damaged element, such as a single error, two errors, and so on.

Now analyze the first example, which has a single error in the code matrix  $C_m$ .

The first idea is that the communication channel acquired a "single" error in the coding matrix  $C_m$ .

The code matrix  $C_m$  clearly demonstrates the following four kinds of "single" errors.

(i)  $\begin{pmatrix} x_1 & c_2 \\ c_3 & c_4 \end{pmatrix}$

(ii)  $\begin{pmatrix} c_1 & x_2 \\ c_3 & c_4 \end{pmatrix}$

(iii)  $\begin{pmatrix} c_1 & c_2 \\ x_3 & c_4 \end{pmatrix}$

(iv)  $\begin{pmatrix} c_1 & c_2 \\ c_3 & x_4 \end{pmatrix}$

where each  $x_i$  ( $i = 1$  to  $4$ ) is the error element. To explore the four distinct scenarios described above, one may employ the following relationships.

$$x_1 c_4 - c_2 c_3 = Det(D_m) \tag{13}$$

$$c_1 c_4 - x_2 c_3 = Det(D_m) \tag{14}$$

$$c_1 c_4 - c_2 x_3 = Det(D_m) \tag{15}$$

$$c_1 x_4 - c_2 c_3 = Det(D_m) \tag{16}$$

Hence, from the above relations, it is evident that

$$x_1 = \frac{Det(D_m) + c_2 c_3}{c_4} \tag{17}$$

$$x_2 = \frac{-Det(D_m) + c_1 c_4}{c_3} \tag{18}$$

$$x_3 = \frac{-Det(D_m) + c_1 c_4}{c_2} \quad (19)$$

$$x_4 = \frac{Det(D_m) + c_2 c_3}{c_1} \quad (20)$$

The message matrix  $D_m$  contains positive integer elements, resulting in integer values from equations (17) to (20). If no integer values exist, single error scenarios are inaccurate or an error may arise in the checking element  $Det(D_m)$ . If  $Det(D_m)$  is wrong, use the relations indicated in (12) to test the accuracy of the code matrix  $C_m$ .

Similarly, one might examine the cases with double errors in the code matrix  $C_m$ . Consider the following scenario with double errors in  $C_m$ .

$$(i) \begin{pmatrix} x_1 & x_2 \\ c_3 & c_4 \end{pmatrix} \quad (21)$$

$$(ii) \begin{pmatrix} x_1 & c_2 \\ x_3 & c_4 \end{pmatrix} \quad (22)$$

$$(iii) \begin{pmatrix} x_1 & c_2 \\ c_3 & x_4 \end{pmatrix} \quad (23)$$

$$(iv) \begin{pmatrix} c_1 & x_2 \\ x_3 & c_4 \end{pmatrix} \quad (24)$$

$$(v) \begin{pmatrix} c_1 & x_2 \\ c_3 & x_4 \end{pmatrix} \quad (25)$$

$$(vi) \begin{pmatrix} c_1 & c_2 \\ x_3 & x_4 \end{pmatrix} \quad (26)$$

where  $x_1, x_2, x_3$  and  $x_4$  are error elements of  $C_m$ .

The relation in (7) provides the subsequent equations for the matrices presented from (21) to (26)

$$x_1 c_4 - x_2 c_3 = Det(D_m) \quad (27)$$

$$x_1 c_4 - c_2 x_3 = Det(D_m) \quad (28)$$

$$x_1 x_4 - c_2 c_3 = Det(D_m) \quad (29)$$

$$c_1 c_4 - x_2 x_3 = Det(D_m) \quad (30)$$

$$c_1 x_4 - x_2 c_3 = Det(D_m) \quad (31)$$

$$c_1 x_4 - c_2 x_3 = Det(D_m) \quad (32)$$

Also, from (12),  $c_1 \approx \lambda c_2$  and  $c_3 \approx \lambda c_4$ .

The equations from (27) to (32) are Diophantine equations with an infinite number of solutions. Hence the choices of  $x_i$  ( $i = 1$  to  $4$ ) must satisfy the equations from (27) to (32). Using such a similar technique, it is possible to fix all conceivable "triple" errors in the code matrix  $C_m$ .

#### 4. Blocking method of coding and decoding

This section explores new coding and decoding approaches based on the Pan-San sequence. Place our message in an even ordered matrix, by introducing special characters between two words, and the size of the final phase of the matrix is even. Dividing the message matrix  $D_m$  of size  $2t$  into the block matrices named as  $B_m$  ( $1 \leq m \leq t^2$ ) of size  $2 \times 2$  from left to right.

Assume that the matrices  $B_m$  and  $X_m$  possess the subsequent way

$$B_m = \begin{pmatrix} b_1^m & b_2^m \\ b_3^m & b_4^m \end{pmatrix} \quad \text{and}$$

$$X_m = \begin{pmatrix} x_1^m & x_2^m \\ x_3^m & x_4^m \end{pmatrix}$$

Replace the elements of  $A^s$  represented in (1) by

$$A^s = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$$

Let  $b$  denotes the number of block matrices  $B_m$ .

Selecting  $n$  accordance with  $b$  as follows.

$$n = \begin{cases} 3, & b \leq 0 \\ \lfloor \frac{b}{2} \rfloor, & b > 0 \end{cases}$$

$$D_m = \begin{pmatrix} H & A & V & E \\ \# & A & \# & G \\ O & O & D & \# \\ D & A & Y & \# \end{pmatrix}$$

#### 4.1. Pan-San Blocking Algorithm

##### Algorithm for Coding

1. Divide the matrix  $D_m$  into block matrix  $B_m$  ( $1 \leq m \leq t^2$ ).
2. Select  $n$ .
3. Determine  $b_i^m, (1 \leq i \leq 4)$ .
4. Compute  $|B_m| = u_m$
5. Construct the code matrix  $C_m = [u_m b_i^m]_{i=1,3,4}$ .

##### Algorithm for Decoding

1. Compute  $A^s$ .
2. Determine  $A_m, (1 \leq m \leq 4)$
3. Calculate  $a_1 b_3^m + a_3 b_4^m = x_3^m$   
( $1 \leq m \leq t^2$ )
4. Compute  $a_2 b_3^m + a_4 b_4^m = x_4^m$   
( $1 \leq m \leq t^2$ )
5. Find  $u_m = x_4^m(a_1 b_1^m - a_3 k_m) - x_3^m(a_2 b_1^m - a_4 k_m)$
6. Replace  $k_m = b_2^m$
7. Construct  $B_m$
8. Construct  $D_m$

An application of an above algorithm is illustrated as follows

##### Illustration 2

Consider the message text “**HAVE A GOOD DAY**” and the message matrix

##### Algorithm for Coding

1. Divide the message matrix  $D_m$  of order  $4 \times 4$  into  $2 \times 2$  and label it  $B_m$  ( $1 \leq m \leq 4$ ), from left to right.  
 $B_1 = \begin{pmatrix} H & A \\ \# & A \end{pmatrix}, B_2 = \begin{pmatrix} V & E \\ \# & G \end{pmatrix},$   
 $B_3 = \begin{pmatrix} O & O \\ D & A \end{pmatrix}, B_4 = \begin{pmatrix} D & \# \\ Y & \# \end{pmatrix}$
2. Since  $b = 4 > 3, n = 2$ . In the matrix  $D_m$ , Unicode is used for the elements which is shown in the table below.

$H = 48$	$A = 41$	$V = 56$
$E = 45$	$\# = 23$	$G = 47$
$O = 4F = 184$	$D = 44$	$Y = 59$

3. Elements of the blocks  $B_m (1 \leq m \leq 4)$  as presented in the following table.

$b_1^1 = 48$	$b_2^1 = 41$	$b_3^1 = 23$	$b_4^1 = 41$
$b_1^2 = 56$	$b_2^2 = 45$	$b_3^2 = 23$	$b_4^2 = 47$
$b_1^3 = 184$	$b_2^3 = 184$	$b_3^3 = 44$	$b_4^3 = 41$
$b_1^4 = 44$	$b_2^4 = 23$	$b_3^4 = 59$	$b_4^4 = 23$

4. The following table shows  $u_m$ , the determinants of the blocks  $B_m$ .

$u_1 =  B_1  = 1025$
$u_2 =  B_2  = 1597$
$u_3 =  B_3  = -552$
$u_4 =  B_4  = -345$

5. The following code matrix  $C_m$  is obtained by using the step 3 and step 4.

$$C_m = \begin{pmatrix} 1025 & 48 & 23 & 41 \\ 1597 & 56 & 23 & 47 \\ -552 & 184 & 44 & 41 \\ -345 & 44 & 59 & 23 \end{pmatrix}$$

#### Algorithm for Decoding

- From the value of  $A$  given in (1), it is denoted that  $A^2 = \begin{pmatrix} 99 & -10 \\ 10 & -1 \end{pmatrix}$
- Choose the elements of  $A^2$  as  $a_1 = 99, a_2 = -10, a_3 = 10, a_4 = -1$ .
- Estimate the elements  $x_3^m$  by using the relation  $a_1 b_3^m + a_3 b_4^m = x_3^m$  where  $m = 1, 2, 3, 4$ .  
Hence,  $x_3^1 = 2687, x_3^2 = 2747,$   
 $x_3^3 = 4766, x_3^4 = 6071$ .
- Find the elements  $x_4^m$  through the relation  $a_2 b_3^m + a_4 b_4^m = x_4^m$   
Hence,  $x_4^1 = -271, x_4^2 = -277,$   
 $x_4^3 = -48, x_4^4 = -613$ .

5. Calculate  $k_m$  from the relation  $u_m = x_4^m (a_1 b_1^m - a_3 k_m) - x_3^m (a_2 b_1^m - a_4 k_m),$   
 $m = 1, 2, 3, 4$ .

Thus,

$$k_1 = 41, k_2 = 45, k_3 = 184, k_4 = 23.$$

6. Rename each  $k_m$  as below:

$$k_1 = b_2^1 = 41, k_2 = b_2^2 = 45,$$

$$k_3 = b_2^3 = 184, k_4 = b_2^4 = 23.$$

7. Finally construct  $B_m$  as shown below

$$B_m = \begin{pmatrix} 48 & 41 & 56 & 45 \\ 23 & 41 & 23 & 47 \\ 184 & 184 & 44 & 23 \\ 44 & 41 & 59 & 23 \end{pmatrix}$$

The corresponding message matrix is

$$D_m = \begin{pmatrix} H & A & V & E \\ \# & A & \# & G \\ O & O & D & \# \\ D & A & Y & \# \end{pmatrix}$$

#### Illustration 3

Consider the message text “A BEAUTIFUL LIFE BEGINS AT HOME” and the corresponding message matrix is

$$D_m = \begin{pmatrix} A & \# & \# & B & E & A \\ U & T & I & F & U & L \\ \# & \# & L & I & F & E \\ \# & \# & B & E & G & I \\ N & S & \# & \# & A & T \\ \# & \# & H & O & M & E \end{pmatrix}$$

#### Algorithm for Coding

- Divide the message matrix  $D_m$  of order  $6 \times 6$  into  $2 \times 2$  and label it  $B_m (1 \leq m \leq 9)$ , from left to right.

$$\begin{aligned}
 B_1 &= \begin{pmatrix} A & \# \\ U & T \end{pmatrix} & B_2 &= \begin{pmatrix} \# & B \\ I & F \end{pmatrix} \\
 B_3 &= \begin{pmatrix} E & A \\ U & L \end{pmatrix} & B_4 &= \begin{pmatrix} \# & \# \\ \# & \# \end{pmatrix} \\
 B_5 &= \begin{pmatrix} L & I \\ B & E \end{pmatrix} & B_6 &= \begin{pmatrix} F & E \\ G & I \end{pmatrix} \\
 B_7 &= \begin{pmatrix} N & S \\ \# & \# \end{pmatrix} & B_8 &= \begin{pmatrix} \# & \# \\ H & O \end{pmatrix} \\
 B_9 &= \begin{pmatrix} A & T \\ M & E \end{pmatrix}
 \end{aligned}$$

2. Since  $b = 9 > 3$ , and  $n = 4$ . In the matrix  $D_m$ , Unicode is used for the elements which are shown below.

$A = 41$	$B = 42$	$E = 45$	$N = 4E = 180$
$H = 48$	$I = 49$	$U = 55$	$M = 4D = 176$
$T = 54$	$S = 53$	$F = 46$	$O = 4F = 184$
$\# = 23$	$G = 47$	$L = 4C = 172$	

3. Elements of the blocks  $B_m (1 \leq m \leq 9)$  as presented in the following table.

$b_1^1 = 41$	$b_2^1 = 23$	$b_3^1 = 55$	$b_4^1 = 54$
$b_1^2 = 23$	$b_2^2 = 42$	$b_3^2 = 49$	$b_4^2 = 46$
$b_1^3 = 45$	$b_2^3 = 41$	$b_3^3 = 55$	$b_4^3 = 172$
$b_1^4 = 23$	$b_2^4 = 23$	$b_3^4 = 23$	$b_4^4 = 23$
$b_1^5 = 172$	$b_2^5 = 49$	$b_3^5 = 42$	$b_4^5 = 45$
$b_1^6 = 46$	$b_2^6 = 45$	$b_3^6 = 47$	$b_4^6 = 49$
$b_1^7 = 180$	$b_2^7 = 53$	$b_3^7 = 23$	$b_4^7 = 23$
$b_1^8 = 23$	$b_2^8 = 23$	$b_3^8 = 48$	$b_4^8 = 184$
$b_1^9 = 41$	$b_2^9 = 54$	$b_3^9 = 176$	$b_4^9 = 45$

4. The following table shows  $u_m$ , the determinants of the blocks  $B_m$ .

$u_1 =  B_1  = 949$
$u_2 =  B_2  = -1000$
$u_3 =  B_3  = 5485$
$u_4 =  B_4  = 0$
$u_5 =  B_5  = 5682$
$u_6 =  B_6  = 139$
$u_7 =  B_7  = 2921$
$u_8 =  B_8  = 3128$
$u_9 =  B_9  = -7659$

5. The following code matrix  $C_m$  is obtained by using step 3 and step

$$C_m = \begin{pmatrix} 949 & 23 & 55 & 54 \\ -1000 & 42 & 49 & 46 \\ 5485 & 41 & 55 & 172 \\ 0 & 23 & 23 & 23 \\ 5682 & 49 & 42 & 45 \\ 139 & 45 & 47 & 49 \\ 2921 & 53 & 23 & 23 \\ 3128 & 23 & 48 & 184 \\ -7659 & 54 & 176 & 45 \end{pmatrix}$$

#### Algorithm for Decoding

1. By equation (1),  $A^4 = \begin{pmatrix} 9701 & -980 \\ 980 & -99 \end{pmatrix}$
2. The elements of  $A^4$  are  $a_1 = 9701$ ,  
 $a_2 = -980$ ,  $a_3 = 980$ ,  $a_4 = -99$ .
3. Estimate the elements  $x_3^m$  by using the relation  
 $a_1 b_3^m + a_3 b_4^m = x_3^m$  where  $m = 1$  to  $9$ .

Hence

$$x_3^1 = 586475, x_3^2 = 520429, x_3^3 = 702115,$$

$$x_3^4 = 702115, x_3^5 = 245663, x_3^6 = 503967,$$

$$x_3^7 = 245663, x_3^8 = 645968, x_3^9 = 1751476$$

4. Find the elements  $x_4^m$  employing the relation  
 $a_2b_3^m + a_4b_4^m = x_4^m$

Hence

$$x_4^1 = -59246, x_4^2 = -52574, x_4^3 = -70928$$

$$x_4^4 = -24817, x_4^5 = -45615, x_4^6 = -50911,$$

$$x_4^7 = -24817, x_4^8 = -65256, x_4^9 = -176935$$

5. Calculate  $k_m$  from the relation

$$u_m = x_4^m(a_1b_1^m + a_3k_m)$$

$$-x_3^m(a_2b_1^m + a_4k_m), m = 1, 2, \dots, 9.$$

Thus

$$k_1 = 23, \quad k_2 = 42, \quad k_3 = 41,$$

$$k_4 = 23, \quad k_5 = 49, \quad k_6 = 45,$$

$$k_7 = 53, \quad k_8 = 23, \quad k_9 = 54.$$

6. Rename  $k_m = b_2^m$  as below:

$$k_1 = b_2^1 = 23, \quad k_2 = b_2^2 = 42,$$

$$k_3 = b_2^3 = 41, \quad k_4 = b_2^4 = 23,$$

$$k_5 = b_2^5 = 49, \quad k_6 = b_2^6 = 45,$$

$$k_7 = b_2^7 = 53, \quad k_8 = b_2^8 = 23,$$

$$k_9 = b_2^9 = 54.$$

7. Finally construct  $B_m$  as shown below

$$B_m = \begin{pmatrix} 41 & 23 & 23 & 42 & 45 & 41 \\ 55 & 54 & 49 & 46 & 55 & 172 \\ 23 & 23 & 172 & 49 & 46 & 45 \\ 23 & 23 & 42 & 45 & 47 & 49 \\ 180 & 53 & 23 & 23 & 41 & 54 \\ 23 & 23 & 48 & 184 & 176 & 45 \end{pmatrix}$$

The corresponding message matrix is

$$D_m = \begin{pmatrix} A & \# & \# & B & E & A \\ U & T & I & F & U & L \\ \# & \# & L & I & F & E \\ \# & \# & B & E & G & I \\ N & S & \# & \# & A & T \\ \# & \# & H & O & M & E \end{pmatrix}$$

## 5. Conclusion

The aim of this manuscript is to provide the application of the PAN-SAN sequence in cryptography by coding and decoding. The Unicode standard is used to assess the principles by assuming certain secret codes for letters in the message text. It is possible to use various sequences in cryptography by changing the decoding process.

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