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A Brief Review On Advanced Real Analysis and its Applications

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Abstract

Real analysis is a branch of mathematics of a mathematical analysis with dealthesequenceofrealnumber,seriesofrealnumberandrealvaluedfunc- tion of a real function or real variable.In particular it deals the series con- vergent,divergent,continuity,differentiationandlimit.Althoughrealanalysis isdistinguishedfromcomplexanalysiswhichdealswiththeoriesconcerning the properties of complex numbers and functions of complex variables, the twowerenottotallyseparatedandweredevelopedalmostsimultaneouslyes- peciallytowardtherecentcentury.Inthispaperwepresentashorthistory of real analysis.

Keyword: Sequence,Series,Cauchy’sPrinciple,TestforConvergentSeries

1 Introduction

Asstatedearlier,realanalysisisamathematicalbranchthatwasdeveloped to explore the study of numbers and functions, including crucial concepts like limits and continuity.These ideas form the foundation of calculus and its applications.Real analysis has emerged as a critical tool in various ap- plications.Let’s take a quick look at some of the key concepts within real analysis.

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Realnumber

Thesetcontainingtheallrationalnumberandallirrationalnumberiscalled a real number.

Example

real number include rational number positive and negative integer ,natural fraction and irrational number example 3, 0, 1.5, 3/2, , and so on are real numbers.

Interval

Leta,bbeatwoelementinIR thesubsetxϵR:$a < x < b$ issaidtoanopen intervalandpointa,biscalledaendpointofintervalandlengthofinterval is defined is b-a

2 Realfunction

Let$X$beanon-emptyset.Afunction$f$:$X$to$R$iscalledarealvalued functionon$X$.ForeveryxϵXandtheimageisdenotedby$f(x)$Forexample, thefunction f;C to Rdefined by $f(z)=|z+1|$, z belong to C is real valued functionofcomplexnumbers.LetDbeanon-emptysubsetofrealnumber.A functionf:Dto$R$issaidtobearealvaluedfunctionofrealnumber.A functionisalsocalledarealfunctionandDissaidtobeadomainof$f$.The set$f(D)$=$f(x)$:xϵ Disarangeof$R$

Example

LetcϵRandf:RtoRbedefinedby$f(x)$=c,xϵR.Therangeoffunction$f$

isthe singleton setc.$f$is called a Constantfunction.

3 RealSequence

Amappingf: NtoRissaidtobeasequenceinRorsequenceofrealnumber.Theimageoffis$f(1)$,$f(2)$,$f(3)$...isarealnumber.Theimageofnth $f(n)$isanthelementofarealsequence

Note

Foraeverysequenceofdomainisusetosetofnaturalnumberbecausenat- ural number is the best ordering of other numbers.

Example

Let$f$:$N$to$R$bedefinedby$f(n)=n,n\epsilon$N.Then$f(I)=1,f(2)=2,...$ The sequence is denoted by n.It is also denoted by $\{1,2,3,...\}$.

4 BoundedSequence

ASequence$f(n)$ is said to be a bounded sequence there exit a real number Gsuchthatf(n)islessequaltoGforallnEN.Gissaidtobeaupperbound of a sequence.

Arealsequence($f(n)$)issaidtobeaboundedbelowthereexitareal number g such that f(n) greater than equal to g for all n belong to N.Then gissaidtobelowerboundofsequence.

ASequenceissaidtobeaboundedsequenceifandonlyifitisbounded above as well as bounded below.The range of a sequence is a bounded set. **Theorem**

ASequencecanhaveatmostonelimit.

5 Convergentsequence

A real sequence $f(n)$is said to be a convergent sequence ,then it has a limit belongto$R$ iflimitntendstoinfinite$f(n)$=lwherelisafinite.

example

Thesequence($1/n$)convergesto0.

6 Divergentsequence

Arealsequence($f(n)$)issaidtobeadivergent.

Adivergentsequenceisoneinwhichthesequencedoesnotapproach a finite, specific value as we move to the higher terms of the sequence.In mathematics the limit of a sequence is the value to which the terms of the sequence tend to.A sequence can be divergent or convergent.A divergent sequence has a limit that is either infinite, or is undefined.The series is referred to as convergent if the sequence has a finite limit.The series is referredtoasdivergentifthesequenceofpartialsumsisadivergentsequence.

Thereisnofinitelimittodiversesequences.

5,7,9,11,13,.,2n + 3

Limit=infinite

Theorem

ASequencedivergingtoinfiniteisunboundedabovebutboundedbelow.

7 Series

InfiniteSeries

Let$u_n$beasequence.thesequence$s_n$isdefinedbyu_1=u_2,s_2=u_2+u_2,...is representedbythesymbolu_1+u_2+u_3+...,whichissaidtobeaninfiniteseries generatedbyasequenceu_n.

Theseriesisdenotedbysummationn=1toinfiniteor to be the Σ u_n-$u_n$issaid nth term if series.

The element of the sequence $s_n$is called the partial sums of the series summation$u_n$andthesequence $s_n$iscalledthesequenceofpartialsumsof the series summation u_n.

If$u_n$isrealsequence,thensummation$u_n$issequenceofarealnumber.

Theinfiniteseries$u_n$issaidtobeconvergentordivergentifthesequence $s_n$isconvergentordivergent.

Example

Letusconsidertheseries1/1.2+1/2.3+1/3.4+...

Lettheseriesbesummationn=1toinfiniteu_n.Thenu_n=1/n(n+1).Let s_n=u_n+u_n+u_n+......+u_nThen s_n=1/1.2 + 1/2.3 +..................... 1/n(n+1)=1-1/n+1, and limit s_n=1.

Hencetheseriessummation$u_n$isconvergentandsumoftheseriesis1.

Theorem

Let$m$beanaturalnumber.Thentwoseriesu_1+u_2+u_3+...andu_m+1+ u_m+2+ ....convergeordivergenttogether.

Theorem

Ifsummation$u_n$andsummation$v_n$betwoconvergentserieshavingthesums s and t respectively,then

(i) theseries Σ (u_n+v_n)convergestothesums+t.

(ii) the series summation ku-n,k being a real number, converges to the sum ks.

Cauchy’sprincipleofconvergence

Anecessaryandsufficientconditionoftheconvergentoftheseriessummation $u_n$is the corresponding to a pre-assigned positive E,there exit a natural number m such that

$|u_n+1+u_n+2++u_n+p|$<Eforallnlessequaltomandforevery naturalnumberp.

Theorem

Anecessaryconditionfortheconvergenceofaseriessummation$u_n$is limit

u_n= 0.

Seriesofpositiveterm

A series summation $u_n$is said to be a positive terms if $u_n$is a positive real number. for all nϵ N.

Testsforconvergenceofaseriesofpositiveterms

Theconvergenceordivergenceofaparticularseriesisdecidedbyexamining thesequenceofpartialsumsoftheseries.Inmostcasestheexpression for s_n(the nth partial sum) becomes not so nice as can be easily handled to determine its nature in a straightforward manner.Some other elegent methods will be applied to the series that will decide the convergence of the series without prior knowledge of the nature of the sequence s_n.These methods, called ’tests for convergence’, will be discussed here.

8 Comparisontest

Letwehavetwoseriessummation$a_n$andSummation$b_n$witha_n,$b_n$greater equal to 0 for all n and an less equal $b_n$for all n.Then,

Ifsummation$b_n$isconvergentthensoissummationan. If summation $a_n$is divergent then so is summation b_n.

9 D’Alembert’sratiotest

Letsummation$u_n$beaseriesofapositivepositiverealnumberandlimit

u_n+1/u_n=l.

Thensummation$u_n$isconvergentifl<1,andsummation$u_n$isdivergent if l >1.

Note

Whenl=1,thetestisfailstogiveadecision.

Letu_n=1/n.Thensummation$u_n$isadivergentseriesandlimitu_n+1/u_n =1.

Letu_n=1/n^2.Thensummation$u_n$isaconvergentseriesandlimu_n+1/u_n =1.

10 Cauchy'sroottest

Letsummation$u_n$beaseriesofpositiverealnumbersandletlimit$(u_n)^{1/n}$ =l.

Thensummation$u_n$isconvergentifl<1,summation$u_n$isdivergentifl >1.

11 Generalformofratiotest

let $\sum u_n$*beaseriesofpositiverealnumbersandletlimitupperbar*u-n+1/u-n=*R*,*limitlowerbar*u_n+ 1/u_n=r.

Then $\sum u_n$*isconvergentifRlessthen*1, $\sum u_n$*divergentifrisgreaterthan*1.

12 Limitofafunction

LetfbearealfunctiondefinedonadomainDsubsetofIR.Inorderthat .fmayhave a limit l(ϵ IR) at a point c, for :1:sufficiently close to c, $f(x)$ should be arbitrarily close to l.For this to be meaningful, it is necessary that c be a limit point of the domain D. Keeping this requirement in view, we give the formal definition.

13 Definition

LetDsubsetofJRandf:DtolRbeafunction.Letcbealimitpoint of D. A real number l is said to be a limit off at c if corresponding to any neighbourhood V of l there exists a neighbourhood W of c such that f(x) ϵV for all x ϵ [W - c] intersection D.

Thisisexpressed by the symbollim x tendsto c$f(x)$ equalto l.

14 Applications

SequentialCriterion

LetDcIRandf: Dto$R$ beafunction. LetcbealimitpointofDandlϵRThenlimxtendsto$f(x)$equaltolifandonlyifforevery sequence$x_n$inD-(c)D-cconvergingtoc,thesequence$f(x_n)$convergestol.

Proof

Letlimxtendstoc$f(x)$equaltol.Thenforapre-assignedpositiveϵ,there existsapositiveδsuchthat

l-ϵ.$less$thanf(x)lessthenl+ϵ.forallxϵN'(e,δ)intersectionD....................................(i)

Let$x_n$beasequenceinD-econvergingtoe.

Sincelimx_n=c,thereexistsanaturalnumberksuchthate-δlessthan

$x_n$lessthanc+δforallngreaterthenequaltok.

Thereforefrom(i)l-ϵlessthan$f(x_n)$lessthan1+ϵforallngreaterthan equal to k.

Thisprovesthatlim$f(x_n)$equaltol.

Sequentialcriterionforcontinuity

Let$DcR$and$f$:$D$to$R$beafunction.LetcϵD∩D'.$f$iscontinuousatc ifandonlyifforeverysequence$x_n$in$D$convergingtoc,thesequence$f(x_n)$ convergesto$f(c)$.

Proof

Let$f$becontinuousat$c$.Let$x_n$beasequencein$D$ suchthat.limx_n=c.
Since$f$iscontinuousat$c$,forapre-assignedpositiveϵ,thereexbtsapositive

δsuchthat$|f(x)-f(c)|<\epsilon$forall$x\epsilon$N(c,δ)$\cap D$.
Since limx_n= c,t.here exists a natural number $m$such that $|x_n - c| < \delta$for all n≥m. Thereforeforalln≥m,$x_n\epsilon$ N(c,δ)andthisimplies$x_n\epsilon$ N(c,δ)$\cap D$*foralln*≥m, since $x_n\epsilon$ D for all nϵN.
Wehave$|f(x_n)-f(c)|<f$foralln≥m.

Thisshowsthatlim$f(x_n)=f(c)$.

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