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SENSITIVITY ANALYSIS OF BAYESIAN RIDGE ESTIMATOR IN THE PRESENCE OF MULTICOLLINEARITY WITH APPLICATION TO PENA DATA

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ABSTRACT
This research is aimed at conducting a sensitivity study, evaluating the performance of Bayesian estimators against Bayesian ridge methods under conditions of multicollinearity. The Bayesian schemes involves the introduction of a prior together with the likelihood which resulted in the posterior distribution that is not tractable, hence the use of numerical method i.e Gibbs sampler. Different ridge parameters k were introduced to remedy the effect of multicollinearity .Pena data was used to carry out Sensitivity analysis varying the priors with Bayesian estimate only ,Bayesian with Hoerl and Kennard ridge, Bayesian with Fayose and Ayinde ridge and Lukman and Ayinde ridge. The trace plot was draw for convergence.The sensitivity of the different priors were examined to know the impact of the priors, how sensitive the estimators are in the presence of multicollinearity. The introduction of Bayesian ridge estimators using different ridge parameter estimators shows promising improvement since the values of the estimates become more stable than the Bayesian estimator without multicolinearity, especially, for Lukman and Ayinde (2017) estimators which makes a good estimator to be used. The Trace plot shows relatively stable horizontal mean and variance, indicating convergence.

Keywords - Bayesian statistics, prior distributions, sensitivity analysis,Logistic Regression.

INTRODUCTION

Multicollinearity is a statistical phenomenon that occurs when two or more predictor variables in a regression model are highly correlated with each other. In other words, it is a condition in which there is a strong linear relationship between two or more independent variables in a regression analysis.

There are many research scenarios within the Bayesian context where informative (or user-specified) priors have an impact on final model estimates. Some examples include research with models such as the latent growth mixture model (Depaoli et al., 2017b; van de Schoot et al., 2018), the confirmatory factor analytic model (Golay et al., 2013), and logistic regression (Heitjan et al., 2008).

Van Erp, S., Mulder, J., &Oberski, D. L. (2018).. In their paper, examine the effectiveness of three distinct default priors: empirical Bayes priors, which are unique, noninformative improper priors, and ambiguous proper priors. They discover that these three default Bayesian structural equation modeling(BSEM) algorithms may perform substantially differently, especially with small samples, based on a simulation research. For this reason, a default BSEM study requires a thorough preceding sensitivity analysis

Depaoli, S.et al (2020). In their paper emphasizes the importance of conducting prior sensitivity analysis, even when using so-called "diffuse" priors. A small simulation study demonstrates how priors can significantly affect estimates and help to understand the role of priors and the importance of sensitivity analysis in Bayesian methods.

Likelihood-Based Methods and Bayesian Method:
Comparative studies between Bayesian and likelihood-based methods have been conducted in various contexts. Gelman et al. (2014) and Albert (2009) provide comprehensive overviews of Bayesian data analysis, while Agresti (2002) and Cameron and Trivedi (2013) delve into likelihood-based methods for logistic and negative binomial regression.
However, the specific focus on comparing Bayesian estimators with likelihood-based methods in the presence of multicollinearity is limited. This research objective aims to bridge this gap by conducting a sensitivity study, evaluating the performance of Bayesian estimators against likelihood-based methods under conditions of multicollinearity

Sensitivity of Bayesian Posterior Simulation Method:
Sensitivity analysis is a useful tool for researchers to compare final model findings based on the original (or reference) prior to results obtained with alternative priors. Sensitivity analyses are generally advised, and a checklist (Depaoli and van de Schoot, 2017) has been produced to help with the transparent interpretation and conduct of such analyses. Numerous Bayesian experts (e.g., Muthén and Asparouhov, 2012; Kruschke, 2015) support this recommendation. The following publications provide applicable works that use a sensitivity analysis of priors: Müller (2012), Depaoli et al. (2017a), or van de Schoot et al (2014).
The sensitivity of Bayesian posterior simulation methods to multicollinearity is an underexplored area. While research by Gelman et al. (2013) and Brooks and Gelman (1998) provides foundational insights into Bayesian posterior simulation methods, their robustness in the context of generalized linear models under multicollinearity remains to be systematically investigated.
An important component of any Bayesian analysis is the prior distribution of the unknown model parameters. Often, researchers rely on default priors, which are constructed in an automatic fashion without requiring substantive prior information. However, the prior can have a serious influence on the estimation of the model parameters, which affects the mean squared error, bias, coverage rates, and quantiles of the estimates.

Materials and Methodology.
Bayesian Ridge Regression: In Bayesian ridge regression, a prior distribution is specified for the regression coefficients. The prior distribution shrinks the coefficients towards zero, effectively reducing the impact of multicollinearity. The strength of the shrinkage is controlled by hyperparameters, which can be estimated from the data or set based on prior knowledge.
Bayesian Logistic Regression is a variation of logistic regression that incorporates Bayesian principles for estimating the model parameters and making probabilistic inferences about them. Unlike traditional (frequentist) logistic regression, which uses maximum likelihood estimation (MLE) to estimate the parameters, Bayesian Logistic Regression provides a probability distribution over the parameters themselves. This makes it possible to express uncertainty about the parameter estimates and to perform Bayesian model selection and hypothesis testing.
Prior Distribution: In Bayesian Logistic Regression, a prior distribution for the model parameters is gotten. This prior reflects the beliefs about the parameters before observing any data. It encapsulates any prior knowledge or assumptions gotten about the values. We assume a normal prior on β.

$$\beta_{j \sim N(\mu_j, \sigma^2{}_j)}$$

Likelihood: Like the traditional logistic regression, Bayesian Logistic Regression uses a likelihood function that models the probability of observing the data given the model parameters. For binary classification, the likelihood is typically the binomial likelihood.

Likelihood = $\pi(x_i)^{y_i}\left(1-\pi(x_i)\right)^{1-y_i}$(1)

where $\pi(x_i)$represents the probability of the event for i with covariate vector $x_i\, andy_i$ indicates the presence of $y_i = 1$, or absence y=0 of the event i.From the classical logistic regression $\pi(x_i)$ is given by:

$$\pi(x_i) = \frac{e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}{1+e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}} \qquad (2)$$

In effect the likelihood contribution from ith subject i

$$\text{Likelihood} = \left[\frac{e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}{1+e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}\right]^{y_i}\left[1-\frac{e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}{1+e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}\right]^{(1-y_i)} \qquad (3)$$

Given that individual subjects are assumed independent from each other, the likelihood function over a data set of n subject is the

$$\text{Likelihood} = \prod_{i=1}^{n}\left[\left[\frac{e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}{1+e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}\right]^{y_i}\left[1-\frac{e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}{1+e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}\right]^{(1-y_i)}\right] (4)$$

Posterior Distribution: The goal of Bayesian Logistic Regression is to compute the posterior distribution over the model parameters. This posterior distribution represents the updated beliefs about the parameters after observing the data. It is proportional to the product of the prior distribution and the likelihood function.

Posterior ∝ Likelihood × Prio

$$\text{Posterior} = \prod_{i=1}^{n}\left[\left[\frac{e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}{1+e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}\right]^{y_i}\left[1-\frac{e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}{1+e^{\beta_0+\beta_1x_1+\cdots+\beta_kx_k}}\right]^{(1-y_i)}\right]$$

$$\text{x}\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi\sigma_j}}exp\left\{-\frac{1}{2}\left(\frac{\beta_j-\mu_j}{\sigma_j}\right)^2\right\} (5)$$

Bayesian Logistic Regression offers a more principled and flexible approach to logistic regression modeling, especially when dealing with small sample sizes or when prior information is available. It provides a richer understanding of parameter uncertainty and allows for more comprehensive probabilistic inference. However, it typically requires more computational resources and expertise in Bayesian methods compared to traditional logistic regression.

Bayesian methods allow for the estimation of these hyperparameters as well, which can lead to more robust model tuning.

Logistic Regression Using Pólya-Gamma Latent Variables was used in this paper to address the presence of multicollinearity. Polson et al. (2012) proposed an alternative Gibbs sampler for logistic and negative binomial models.The approach introduces a vector of latent variables, Z_i, that are scale mixtures of normals with independent Pólya-Gamma precision terms rather than Gamma precision terms as in the t-link model

A random variable ω is said to have a Polya-Gamma distribution with parameters b>0 and c∈ ℜ, if

$$\omega\sim PG(b,c) \overset{\text{m}}{=} \frac{1}{2\pi^2}\sum_{k=1}^{\infty}\frac{d_k}{(k-1/2)^2+c^2/(4\pi^2)} (6)$$

where $d_k's$ are independently distributed according to a Ga(b,1) distribution giving an important property of the PG(b,c) density – namely that for a∈ ℜ and $\eta\epsilon\Re$

$$\frac{(e^{\eta})^a}{(1+e^{\eta})^b} = 2^{-b}e^{k\eta}\int_0^{\infty}e^{-\omega\eta^2/2}\rho(\omega/b,0)\,d\omega (7)$$

Where k = a-b/2 and $\rho(\omega/b,0)$ denotes a PG (b,0) density.

Here the ridge – type estimator of □ was examined for logistic model. Different levels of ridge-type parameter (k), namely Hoerl and Kennard (1970), Lukman and Ayinde (2015) and Fayose and Ayinde (2019) was introduced, and the posterior mean was examined.

Likelihood:

This is the joint probability density function (p.d.f) for the model

$$\text{Logistic} \quad f(x;\mu,s) \quad \frac{e^{-(x-\mu)/s}}{s\left(1+e^{-(x-\mu)/s}\right)^2} \quad x\in(-\infty,+\infty) \quad s>0$$

A Gibbs sampler for logistic models was proposed by Polson *et al.* (2012). This involves the use of a vector of latent random variables Y_j that are scale mixtures of normal with independent polya-gamma precision terms rather than Gamma precision terms as in t-link models. A polya-gamma random variable X with parameters (a,b) with $a>0$ and $b\in\Re$ is given

$$f(x|a,b) = \frac{1}{2\pi^2}\sum_{k=1}^{\infty}\frac{\omega_k}{\left(k-\frac{1}{2}\right)^2+\frac{b^2}{4\pi^2}} \qquad (8)$$

Where ω_k's are independently distributed according to a $Gamma(b,1)$ distribution.

They further established a germane property of polya-gamma density that made it useful as a sampler for logistic model:

$$\frac{(e^{\eta})^a}{(1+e^{\eta})^b} = 2^{-b} e^{\kappa\eta} \int_0^{\infty} e^{-\frac{\omega\eta^2}{2}} p(\omega|b,0)\,d\omega, \qquad (9)$$

Here, $\kappa = a - \frac{b}{2}$ and $p(\omega|b,0)$ denotes a polya-gamma density with parameters$(b,0)$.
The integrand on the right hand side is the kernel of a normal density with precision ω (i.e the conditional density of η) times the prior for ω.
Hence, we can
The LHS of equation (1) has the same function form as the probability parameter logistic regression model.
Hence, from the likelihood of binary response vector, the Bernoulli likelihood has the same form as the LHS of equation (1). So, with these properties of polya-gamma and its connection with logistic regression model, Polson et al (2012) shows that the full conditional distribution of β given Y and ω is $p(\beta|Y=y,\omega) \propto \pi(\beta)\exp\left[-\frac{1}{2}(z-X\beta)^T W(z-X\beta)\right]$ (10)
It is clear that the random variable Z follows Normal distribution with mean ν and a variance$W^{-1} = \tau I$.
Thus, assuming a $N_p(\beta_0, T_0^{-1})$ prior for β, the full conditional for β given $Z = z$ and W is

$$N_p(m, V),$$

where $V = (T_0 + X^T W X)^{-1}$, $m = V(T_0\beta_0 + X^T W z)$, different K in the ridge used namely.
Ridge Estimator (HoerlAnd Kennard, (1970a)
Hoerl and Kennard

$$\hat{k}_i(HK) = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}$$, i = 1, 2, 3, p.

where $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-p}$ and it is the Mean Square Error from the OLS regression, α_i is the i[th] element of the vector, and is also the regression coefficient from the OLS regression. $\alpha_i = Q'\hat{\beta}$ where Q is an orthogonal matrix. p is the number of regressors and n is the sample size
Ridge Estimator (LukmanAndAyinde(2017)

Lukman and Ayinde $\hat{k}_i(LA) = \frac{\hat{\sigma}^2}{\lambda_i \hat{\alpha}_i^2}$

where λ = (λ_i) = 1,2,3, … , p

Ridge Estimator (FayoseAndAyinde, (2019)

$$KGRFA = \hat{k}_i^{Min}(FA) = \frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\left\{\frac{\left[\left(\frac{\hat{\alpha}_i^4\lambda_{Min}^2}{4\hat{\sigma}^2}\right)+\left(\frac{6\hat{\alpha}_i^4\lambda_{Min}}{\hat{\sigma}^2}\right)\right]^{\frac{1}{2}} - }{\left(\frac{\hat{\alpha}_i^2\lambda_{Min}}{2\hat{\sigma}^2}\right)}\right\}$$

where $\lambda_{Min} = Min(\lambda_i) = 1,2,3,\ldots,p$
Different high levels of collinearity among regressors were chosen to be:
High Positive Collinearity (HPC) when ρ = 0.80, 0.85, 0.90, 0.95,0.99 and 0.999 with
Sample sizes10,20,30,50,100,200,300 and 500.Three explanatory variables were used for the different levels of multicollinearity with increasing sample sizes after which seven explanatory variables were also used for the different levels of multicollinearity with the increasing sample sizes.
In this study, model estimation was carried out using Bayesian approach via the Gibbs sampler of the Markov Chain Monte Carlo simulation and pena data was used in the analysis to carry out Sensitivity analysis varying the priors with Bayesian estimate only ,Bayesian with Hoerl and Kennard ridge, Bayesian with Fayose and Ayinde ridge and Lukman and Ayinde ridge.The trace plot was drawnfor convergence.

Data Analysis

Sensitivity Analysis

Table 1: Bayesian Only

Estimate	precision	Mean(SD) β0	Mean(SD) β1	Mean(SD) β2	Mean(SD) β3	Mean(SD) β4
0.2	0.05	-4.27(3.02)	1.75(0.66)	**-0.07(0.02)**	0.10(0.06)	-0.44(0.22)
	0.10	-2.84(2.48)	1.60(0.63)	**-0.07(0.02)**	0.09(0.06)	-0.46(0.22)
	1.00	-0.22(0.95)	**1.02(0.46)**	**-0.06(0.02)**	**0.07(0.05)**	-0.42(0.18)
0.4	0.05	-4.19(3.02)	1.79(0.68)	-0.07(0.03)	0.10(0.06)	-0.46(0.23)
	0.10	-2.72(2.47)	1.57(0.62)	**-0.07(0.02)**	0.09(0.06)	-0.46(0.22)
	1.00	**-0.04(0.94)**	1.04(0.47)	**-0.06(0.02)**	**0.06(0.05)**	**-0.43(0.17)**
0.6	0.05	-4.15(3.03)	1.77(0.67)	**-0.07(0.02)**	0.10(0.06)	-0.45(0.22)
	0.10	-2.64(2.45)	1.56(0.62)	**-0.07(0.02)**	0.08(0.06)	-0.46(0.21)
	1.00	0.11(0.96)	1.06(0.47)	**-0.06(0.02)**	**0.06(0.05)**	-0.44(0.18)
1.0	0.05	-3.97(3.05)	1.74(0.66)	-0.07(0.03)	0.10(0.06)	-0.45(0.22)
	0.10	-2.40(2.45)	1.56(0.62)	**-0.07(0.02)**	0.08(0.06)	-0.47(0.21)
	1.00	0.45(0.97)	1.11(0.48)	**-0.06(0.02)**	**0.06(0.05)**	-0.47(0.18)
1.5	0.05	-3.78(3.02)	1.73(0.67)	-0.07(0.03)	0.09(0.06)	-0.46(0.23)
	0.10	-2.20(2.45)	1.55(0.63)	**-0.07(0.02)**	0.08(0.06)	-0.48(0.22)
	1.00	0.84(0.96)	1.15(0.49)	**-0.06(0.02)**	**0.05(0.05)**	-0.49(0.18)
2,0	0.05	-3.56(2.99)	1.69(0.66)	**-0.07(0.02)**	0.09(0.06)	-0.45(0.21)
	0.10	-1.88(2.42)	1.51(0.62)	**-0.07(0.02)**	0.08(0.06)	-0.48(0.21)
	1.00	1.27(0.97)	1.19(0.50)	**-0.06(0.02)**	**0.04(0.05)**	-0.51(0.19)
3.0	0.05	-3.25(3.01)	1.69(0.67)	-0.07(0.03)	0.09(0.06)	-0.47(0.22)
	0.10	-1.38(2.44)	1.49(0.62)	**-0.07(0.02)**	0.07(0.06)	-0.50(0.21)
	1.00	2.10(0.96)	1.35(0.53)	**-0.07(0.02)**	**0.03(0.05)**	-0.58(0.19)
4.5	0.05	-2.56(2.97)	1.62(0.66)	**-0.07(0.02)**	0.08(0.06)	-0.48(0.22)
	0.10	-0.59(2.47)	1.45(0.63)	**-0.07(0.02)**	0.07(0.06)	-0.52(0.22)
	1.00	3.35(0.98)	1.59(0.57)	-0.07(0.02)	**0.01(0.05)**	-0.67(0.20)
5.0	0.05	-2.47(2.96)	1.63(0.66)	-0.07(0.02)	0.08(0.06)	-0.49(0.23)
	0.10	-0.32(2.47)	1.45(0.64)	-0.07(0.02)	0.07(0.06)	-0.54(0.22)
	1.00	3.75(0.96)	1.67(0.60)	-0.08(0.02)	**0.00(0.05)**	-0.70(0.21)
6.0	0.05	-1.96(2.97)	1.59(0.65)	-0.07(0.02)	0.08(0.06)	-0.50(0.22)
	0.10	0.17(2.45)	1.42(0.63)	-0.07(0.02)	0.06(0.06)	-0.55(0.22)
	1.00	4.64(0.97)	1.91(0.64)	-0.08(0.03)	-0.01(0.06)	-0.78(0.22)

Table 2: HoerlAndKennard Ridge With Bayesian

Estimate	precision	Mean(SD) β0	Mean(SD) β1	Mean(SD) β2	Mean(SD) β3	Mean(SD) β4
0.2	0.05	-3.28(2.53)	1.30(0.53)	**-0.06(0.02)**	0.06(0.05)	-0.28(0.15)
	0.10	-2.37(2.17)	1.20(0.51)	**-0.05(0.02)**	**0.06(0.04)**	-0.30(0.15)
	1.00	**-0.25(0.94)**	**0.83(0.40)**	**-0.05(0.02)**	**0.04(0.04)**	(0.06)
0.4	0.05	-3.17(2.53)	1.29(0.53)	**-0.06(0.02)**	**0.06(0.04)**	-0.29(0.15)
	0.10	-2.30(2.19)	1.19(0.51)	**-0.05(0.02)**	**0.05(0.04)**	-0.30(0.15)
	1.00	-0.07(0.95)	0.83(0.41)	**-0.05(0.02)**	**0.04(0.04)**	**-0.31(0.13)**
0.6	0.05	-3.15(2.57)	1.30(0.54)	**-0.06(0.02)**	0.06(0.05)	-0.29(0.15)
	0.10	-2.22(2.18)	1.17(0.50)	**-0.05(0.02)**	**0.05(0.04)**	-0.30(0.15)
	1.00	**0.05(0.94)**	**0.85(0.40)**	**-0.05(0.02)**	**0.04(0.04)**	**-0.32(0.13)**
1.0	0.05	-3.06(2.60)	1.30(0.54)	**-0.06(0.02)**	0.06(0.05)	-0.29(0.15)
	0.10	-2.04(2.19)	1.17(0.51)	**-0.05(0.02)**	**0.05(0.04)**	-0.31(0.14)
	1.00	**0.39(0.94)**	0.87(0.41)	**-0.05(0.02)**	**0.03(0.04)**	**-0.33(0.13)**
1.5	0.05	-2.88(2.53)	1.28(0.54)	**-0.06(0.02)**	0.06(0.05)	-0.29(0.15)
	0.10	-1.88(2.19)	1.15(0.51)	**-0.05(0.02)**	0.05(0.04)	-0.31(0.15)
	1.00	0.78(0.95)	0.89(0.41)	**-0.05(0.02)**	0.03(0.04)	**-0.35(0.13)**
2.0	0.05	-2.77(2.53)	1.27(0.53)	**-0.06(0.02)**	0.06(0.04)	-0.30(0.15)
	0.10	-1.63(2.22)	1.14(0.51)	**-0.05(0.02)**	0.05(0.04)	-0.32(0.14)
	1.00	1.16(0.95)	0.92(0.42)	**-0.05(0.02)**	0.02(0.04)	-0.37(0.14)
3.0	0.05	-2.54(2.54)	1.25(0.53)	**-0.06(0.02)**	0.06(0.05)	-0.30(0.15)
	0.10	-1.22(2.18)	1.10(0.51)	**-0.05(0.02)**	0.04(0.04)	-0.32(0.15)
	1.00	**1.96(0.94)**	0.98(0.43)	**-0.06(0.02)**	0.01(0.04)	-0.41(0.14)
4.5	0.05	-2.07(2.54)	1.19(0.52)	**-0.06(0.02)**	0.05(0.05)	-0.31(0.15)
	0.10	-0.63(2.19)	1.06(0.50)	**-0.05(0.02)**	0.04(0.04)	-0.34(0.14)
	1.00	3.16(0.95)	1.10(0.45)	**-0.06(0.02)**	-0.01(0.04)	-0.47(0.14)
5.0	0.05	-1.98(2.51)	1.21(0.53)	**-0.06(0.02)**	0.05(0.04)	-0.32(0.15)
	0.10	-0.39(2.19)	1.05(0.51)	**-0.05(0.02)**	0.04(0.04)	-0.34(0.15)
	1.00	3.54(0.95)	1.14(0.46)	**-0.06(0.02)**	-0.02(0.04)	-0.48(0.14)
6.0	0.05	-1.67(2.50)	1.16(0.52)	**-0.05(0.02)**	0.05(0.04)	-0.32(0.15)
	0.10	-0.01(2.19)	1.01(0.50)	**-0.05(0.02)**	0.04(0.04)	-0.35(0.15)
	1.00	4.34(0.95)	1.26(0.48)	**-0.07(0.02)**	-0.03(0.04)	-0.53(0.15)

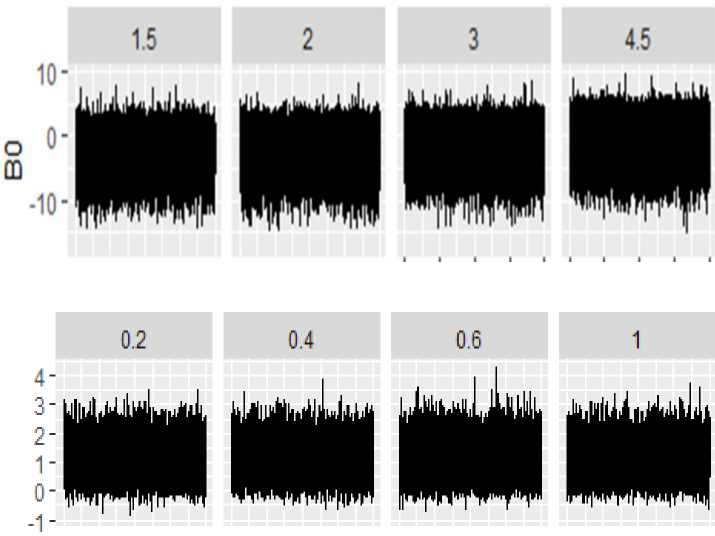
Table 3:Fayose And Ayinde Ridge With Bayesian

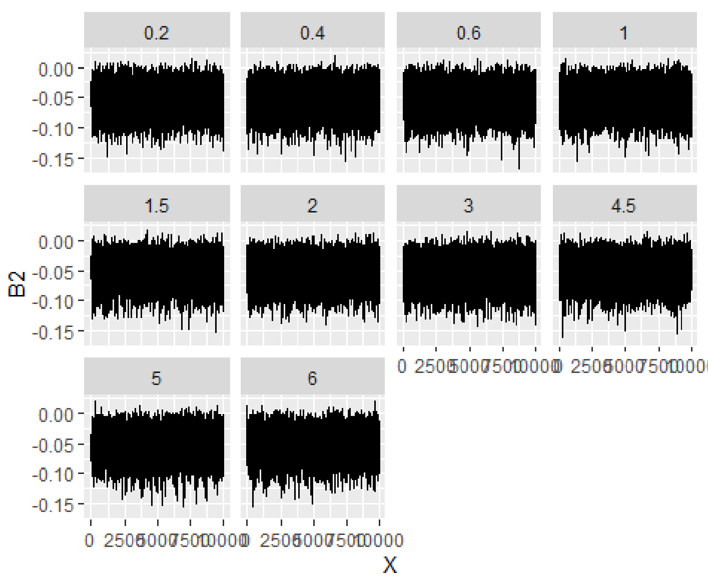
Estimate	precision	Mean(SD) β0	Mean(SD) β1	Mean(SD) β2	Mean(SD) β3	Mean(SD) β4
0.2	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	0.10	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	1.00	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.20(0.12)
0.4	0.05	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	0.10	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.21(0.12)
	1.00	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.20(0.12)
0.6	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	0.10	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	1.00	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.20(0.12)
1.0	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	0.10	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	1.00	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.20(0.11)
1.5	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	0.10	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.21(0.12)
	1.00	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.19(0.11)
2.0	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.21(0.12)
	0.10	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.11)
	1.00	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.19(0.11)
3.0	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.10(0.04)	-0.21(0.12)
	0.10	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.20(0.12)
	1.00	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.18(0.11)
4.5	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.21(0.12)
	0.10	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.21(0.12)
	1.00	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.08(0.04)	-0.17(0.11)
5.0	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.21(0.12)
	0.10	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.20(0.12)
	1.00	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.08(0.04)	-0.16(0.11)
6.0	0.05	0.001 (0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.21(0.12)
	0.10	0 .001(0.01)	0.01(0.04)	-0.05(0.02)	0.09(0.04)	-0.20(0.12)
	1.00	0.001 (0.01)	0.02(0.04)	-0.05(0.02)	0.08(0.04)	-0.16(0.11)

Table 4:Lukman AndAyinde Ridge With Bayesian

Estimate	precision	Mean(SD) β0	Mean(SD) β1	Mean(SD) β2	Mean(SD) β3	Mean(SD) β4
0.2	0.05	-5.96(**2.80**)	1.28(**0.57**)	**-0.07(0.02)**	0.06(**0.05**)	-0.06(**0.06**)
	0.10	-4.24(**2.29**)	1.02(**0.49**)	**-0.06(0.02)**	0.05(**0.05**)	-0.06(**0.06**)
	1.00	-0.58(**0.95**)	**0.46(0.36)**	**-0.05(0.02)**	0.02(**0.04**)	-0.08(**0.06**)
0.4	0.05	-5.94(**2.72**)	1.27(**0.55**)	**-0.07(0.02)**	0.06(**0.05**)	-0.06(**0.06**)
	0.10	-4.13(**2.30**)	1.01(**0.50**)	**-0.06(0.02)**	0.04(**0.05**)	-0.06(**0.06**)
	1.00	**-0.45(0.94)**	**0.47(0.36)**	**-0.06(0.02)**	0.01(**0.04**)	-0.08(**0.06**)
0.6	0.05	-5.85(**2.74**)	1.27(**0.56**)	**-0.06(0.02)**	0.06(**0.05**)	-0.06(**0.06**)
	0.10	-4.09(**2.26**)	1.00(**0.50**)	**-0.06(0.02)**	0.04(**0.05**)	-0.06(**0.06**)
	1.00	**-0.27(0.94)**	**0.47(0.36)**	**-0.05(0.02)**	0.01(**0.04**)	-0.08(**0.06**)
1.0	0.05	-5.69(**2.71**)	1.24(**0.55**)	**-0.06(0.02)**	0.06(**0.05**)	-0.06(**0.06**)
	0.10	-3.97(**2.26**)	0.99(**0.50**)	**-0.06(0.02)**	0.04(**0.05**)	-0.06(**0.06**)
	1.00	**0.01(0.94)**	**0.47(0.36)**	**-0.06(0.02)**	0.01(**0.04**)	-0.08(**0.06**)
1.5	0.05	-5.52(**2.71**)	1.22(**0.55**)	**-0.06(0.02)**	0.06(**0.05**)	-0.06(**0.06**)
	0.10	-3.70(**2.28**)	0.95(**0.50**)	**-0.06(0.02)**	0.04(**0.05**)	-0.07(**0.06**)
	1.00	0.40(**0.93**)	**0.48(0.36)**	**-0.06(0.02)**	0.00(**0.04**)	-0.09(**0.06**)
2.0	0.05	-5.45(**2.72**)	1.21(**0.56**)	**-0.06(0.02)**	0.05(**0.05**)	-0.06(**0.06**)
	0.10	-3.48(**2.25**)	0.94(**0.50**)	**-0.06(0.02)**	0.04(**0.04**)	-0.07(**0.06**)
	1.00	0.79(**0.94**)	0.48(**0.37**)	**-0.06(0.02)**	-0.01(**0.04**)	-0.09(**0.06**)
3.0	0.05	-5.16(**2.68**)	1.18(**0.54**)	**-0.06(0.02)**	0.05(**0.05**)	-0.06(**0.06**)
	0.10	-3.06(**2.22**)	0.89(**0.49**)	**-0.06(0.02)**	0.03(**0.04**)	-0.07(**0.06**)
	1.00	1.54(**0.94**)	0.50(**0.38**)	**-0.06(0.02)**	-0.03(**0.04**)	-0.10(**0.06**)
4.5	0.05	-4.60(**2.66**)	1.11(**0.54**)	**-0.06(0.02)**	0.05(**0.05**)	-0.06(**0.06**)
	0.10	-2.43(**2.21**)	0.82(**0.49**)	**-0.06(0.02)**	**0.03(0.04)**	-0.07(**0.06**)
	1.00	2.71(**0.95**)	0.53(**0.39**)	**-0.06(0.02)**	**-0.05(0.04)**	-0.11(**0.06**)
5.0	0.05	-4.45(**2.66**)	1.09(**0.55**)	**-0.06(0.02)**	0.04(**0.05**)	-0.06(**0.06**)
	0.10	-2.20(**2.19**)	0.79(**0.48**)	**-0.06(0.02)**	**0.02(0.04)**	-0.07(**0.06**)
	1.00	3.12(**0.95**)	0.54(**0.39**)	**-0.06(0.02)**	**-0.06(0.04)**	-0.11(**0.06**)
6.0	0.05	-4.14(**2.60**)	1.05(**0.54**)	**-0.06(0.02)**	0.04(0.05)	-0.06(**0.06**)
	0.10	-1.82(**2.22**)	0.75(**0.49**)	**-0.06(0.02)**	**0.02(0.04)**	-0.08(**0.06**)
	1.00	3.89(**0.95**)	0.57(**0.42**)	**-0.07(0.02)**	**-0.08(0.04)**	-0.12(**0.06**)

TRACE PLOT





Discussion and Results

The sensitivity of the different priors was examined to know the impact of the priors, how sensitive the estimators are in the presence of multicollinearity.

From the sensitivity analysis carried out, evaluating the performance of Bayesian estimators against Bayesian ridge methods under conditions of multicollinearity. Varying the priors with Bayesian estimate only,Bayesian with Hoerl and Kennard ridge, Bayesian with Fayose and Ayinde ridge and Lukman and Ayinde ridge.

Table 1 shows the mean and standard deviation using only Bayesian estimate and Table 2 showsthe one of Hoerl and Kennard ridge with Bayesian which was relatively stable compare to that of Bayesian only when the priors were varied. Table 4 shows the result of lukman and Ayinde ridge with Bayesian and the result of varying the priors gives a better stability than the previous two andlastly Table 3 is the result of Fayose and Ayinde ridge(2019) with Bayesian which bring about the best stability and this makes it a good estimator to be used.The trace plot was drawn for convergence and it showing a relatively stable horizontal mean and variance, indicating convergence.

Conclusion

Comparing the posterior estimates of models with various prior specifications allowed us to assess the robustness of the findings. The posterior estimates between models will have a low percentage of deviation if priors have little effect on the outcomes. In case the priors exert a noteworthy influence, a greater percentage divergence between the models will be observed.

Tables 1 to 4 shows the sensitivity of the estimators in the presence of multicollinearity, the dataset used has been confirmed by several authors among which is Aladeitan et al., 2023. Hence, the sensitivity of the classical and bayesian estimators is high towards change in scenarios. But the introduction of Bayesian ridge estimators using different ridge parameter estimators shows promising improvement since the values of the estimates become more stable than the Bayesian estimator without multicolinearity, especially, for Lukman and Ayinde (2017) estimators which makes a good estimator to be used.The Trace plotshows a relatively stable horizontal mean and variance, indicating convergence.

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