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Fractional Differential Operators and Eulerian Integral of H- Function

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Abstract

A large number of Fractional Differential Operators and Eulerian integral of H- Function have been presented. Here, in this paper, our aim is to establishing two fractional calculus formulas involving Eulerian integral of H-function. by using generalized fractional integration operators given by Saigo and Maeda [M. Saigo, N. Maeda, Varna, Bulgaria, (1996), 386–400]. All the results derived here being of general character, they are seen to yield a number of results (known and new) regarding fractional integrals.In this paper we use fractional differential operators ${}_{\alpha}D_{x}^{\mu}$ to derive a number of key formulas of one and multivariable H-function. We use the generalized Leibnitz's rule for fractional derivatives in order to obtain one of the aforementioned formulas, which involve a product of two multivariable's H-function. It is further shown that ,each of these formulas yield interesting new formulas for certain multivariable hypergeometric function such as generalized Lauricella function (Srivastava-Dauost)and Lauriella hypergeometric function some of these application of the key formulas provide potentially useful generalization of known result in the theory of fractional calculus.

Keywords: Fractional differential operator, Generalized fractional integral operators, multivariable H-function,

Subject Classification 2020 MSC: 26A33, 33C45, 33C60.

1. INTRODUCTION AND DEFINITIONS

Fractional calculus which are derivatives and integrals of arbitrary (real and complex) orders have found many applications in a variety of fields ranging from natural science to social science. In recent years, it has turned out that many phenomena in engineering, physics, chemistry and other sciences can be described very successfully by means of models using mathematical tools deduced from fractional calculus

Fractional derivatives are also used in modeling many chemical processes, mathematical biology and many other problems in physics and engineering (see, e.g., [3]–[1], [4], [5], [6]). Under various fractional calculus operators, the computations of image formulas for special functions of one or more variables are important from the point of view of the usefulness of these results in the evaluation of generalized integrals and the solution of differential and integral equations (see, e.g., [7], [8],[9], [10], [11], [12], [13] and [14] and so on). Motivated essentially by diverse applications of fractional calculus we establish three image formulas for the product of multivariable H-function and general class of polynomials involving left and right sided fractional integral operators of Saigo-Meada [15]. By virtue of the unified nature of our results, a large number of new and known results involving Saigo, Riemann-Liouville and Erd´elyi-Kober fractional integral operators and several special functions are shown to follow as special cases of our main results. The generalized fractional integral operators of arbitrary order involving Appell function F3 in the kernel were defined and investigated by Saigo and Maeda [15, p. 393, Eqs. (4.12) and (4.13)]

The present work is an attempt in the direction of obtaining fractional calculus formula by utilizing series expression method, introduced by srivastava [16]. The name general class of polynomials, itself indicates the importance of the results, because we can derive a number of fractional calculus formulae for various classical orthogonal polynomials.

Differential operator ${}_{\alpha}D_x^{\mu}$ is defined by [5, p.49; 3; 9; 17, P-356]

$$
{}_{\alpha}D_x^{\mu} f(\mathrm{x}) = \begin{cases} \dfrac{1}{\sqrt{-\mu}} \displaystyle\int_{\alpha}^{x} (x-t)^{-\mu-1} f(t)dt \quad , & [\mathrm{Re}(\mu) < 0] \\ \dfrac{d^m}{dx^m}\, {}_{\alpha}D_x^{\mu-m} f(x), & [0 \le \mathrm{Re}(\mu) < \mathrm{m}] \end{cases} \qquad ...(1.1)
$$

Where m is a positive integer

For $\alpha = 0$,(1.1) Defines the classical Riemann-Liouville fractional derivative of order μ (or-μ) when α→∞ (1.1) may be identified with the definition of the well known Weyl fraction derivative of order μ (or-μ) [1,chap.13);3] the special case of fractional calculus operator ${}_{\alpha}D_x^{\mu}$ when α=0 is written as D_x^{μ} thus we have

$$D_x^{\mu} = {}_0D_x^{\mu} \qquad \text{... (1.2)}$$

Throughout the present paper .we assume that the convergence and existence condition corresponding appropriately to the ones detained above are satisfied by each of the various H-function involved in our results which are presented in the following sections

(i) The H-Function Defined by Saxena and kumbhat [18] is an extension of Fox's H-Function on specializing the parameters , H-Function can be reduced to almost all the known special function as well as unknown

The Fox's H-Function of one variable is defined and represented in this Paper as follows [see Srivastava et al [22] ,pp 11-13]

$$H[x] = H_{P,Q}^{M,N}\left[x \,/ \begin{matrix} (a_j,\alpha_j)_{1,P} \\ (b_j,\beta_j)_{1,Q} \end{matrix} \right] = \frac{1}{2\pi\omega} \int_{\theta=N-1} \theta(\xi) x^{\xi} d\xi \quad \text{...(1.3)}$$

$$\theta(\xi) = \frac{\prod_{i=1}^{n} \Gamma b_j - \beta_j \xi \prod_{j=1}^{N} \Gamma 1 - a_j - \alpha_j \xi}{\prod_{i=M=1}^{Q} \Gamma 1 - b_j + \beta_j \xi \prod_{j=N+1}^{P} \Gamma a_j - \alpha_j \xi} \qquad \text{....(1.4)}$$

For condition of the H-Function of one variable (1.13) and on the contour L we refer to srivastava et al [20]

Let α, β *and* η be complex numbers,and let x ∈ R_+= (0, ∞) Follwing Saigo [19} Fractional integral (Re(α)>0 and derivative Re(α) < 0 of first kind of a function f(x) on R_+ are defined respectively in the forms:

$$I_{0,\ x}^{\alpha,\beta,\eta}(\mathrm{f}) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1}\, 2F1\left(\alpha+\beta\,,\ -\eta\ ;\ \alpha\ ;\ 1-\frac{t}{x}\right) f(t)dt\ ; \qquad \text{...(1.5)} \qquad \mathrm{Re}(\alpha)>0$$

$$\frac{d^n}{dt^n} I_{0,\ x}^{\alpha+n\,\beta-n\,,\eta-n}(\mathrm{f})\ ,\ 0< \mathrm{Re}(\alpha)+n<1 \qquad (n= 1, 2, 3, \ldots\ldots\ldots\ldots),$$

Where 2F1(a, b; c; z) is Gauss's hypergeometric function

Let α, β,η and λ be complex numbers. Then there hold the following formulae. . the R.H.S. has a definite meaning

$$I_{0,\ x}^{\alpha,\beta,\eta}\ t^{\lambda} = \frac{\Gamma(1+\lambda)\Gamma(1+\lambda-\beta+\eta)}{\Gamma(1+\lambda-\beta)\Gamma(1+\lambda+\alpha+\eta)} x^{1-\beta} \qquad \text{.....(1.6)}$$

provided that Re(α)> max [0, Re(β-η)] — 1

2.MAIN RESULT

In this section we shall prove our main formulas on fractional derivative and Eulerian integral of multivariable H- Function

Results -I

$$D_x^{\mu}\left\{x^{v}\, I_{0,x}^{\alpha+\beta+\eta}\{t^{\lambda} H_{P,Q}^{M,N}\left[zt^{k}\Big|_{(bQ,BQ)}^{(ap\ AP)}\right]\right.$$

$$= x^{v+\lambda-\beta-\mu}\, H_{P+3\ \ Q+3}^{M\ \ \ N+3}\left[zx^{k}\Big|_{(bQ,BQ)\ (-\lambda+\beta,k)(-\lambda-\alpha-\eta,k)(-v-\lambda+\beta+\mu,k)}^{(-\lambda,k)(-\lambda-\eta+\beta,k)(-v-\lambda+\beta,k)(a\rho,A\rho)}\right]$$

Provided that (in addition to the appropriate convergence and existence condition) that min (μ , α , β , η , λ)> 0 and Re((μ + α + β + η + λ) > 0

Results -II

$$D_x^{\mu}\left\{x^{v}\, I_{0,x}^{\alpha+\beta+\eta}\ [t^{k}\ H\ \{z_1 t^{\rho_1} \ldots\ldots z_r t^{\rho_r}\ \}]\right.$$

$$= x^{\lambda+k-\beta-\mu}\, H_{p+3,q+3\ \ p_1,q_1\ldots\ldots p_r,q_r}^{0,n+3\ :\ m_1,n_1\ldots\ldots m_r,n_r}\left[\begin{matrix} z_1 x^{p_1} \\ \cdot \\ \cdot \\ \cdot \\ z_r x^{p_r}\end{matrix}\Bigg|_{(-k+\beta,\ \rho_1\ldots\ldots\rho_r)(-k-\alpha-\eta,\ \rho_1\ldots\ldots\rho_r)\ (-\lambda-k-\mu+\beta,\rho_1\ldots\ldots\rho_r\ )}^{(-k,\ \rho_1\ldots\ldots\rho_r)(-k+\beta-\eta,\ \rho_1\ldots\ldots\rho_r)\ (-\lambda-k+\beta,\ \rho_1\ldots\ldots\rho_r)}\right.$$

$$\times \left.\begin{matrix} \left(a_j,\ \alpha_j^1\ldots\ldots\alpha_j^{(r)}\right)_{1,p}\left(c_j^1\,\gamma_j^1\right)\right)_{1,p_1}\ldots\ldots\left(c_j^{(r)}\,\gamma_j^{(r)}\right)_{1,p_r} \\ \\ \left(b_j,\ \beta_j^1\ldots\ldots\beta_j^{(r)}\right)_{1,q}\left(d_j^1\,\delta_j^1\right)_{1,q_1}\ldots\ldots\left(d_j^{(r)}\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right]$$

Provided that (in addition to the appropriate convergence and existence condition) that min (μ , α , β , η , λ)> 0 and Re((μ + α + β + η + λ) > 0

Proof:-Result –I First using The Fox's H-Function of onevariable. then we shall utilize following definition introduced by srivastava [16]. then apply Fractional integral formula

$$I_{0,\ x}^{\alpha,\beta,\eta}\ \left(t^{\lambda}\right)\ = \frac{\Gamma(1+\lambda)\Gamma(1+\lambda-\beta+\eta)}{\Gamma(1+\lambda-\beta)\Gamma(1+\lambda+\alpha+\eta)}\, x^{1-\beta}$$ provided that Re(α)> max [0, Re(β-η)]— 1,

Now using by mellin barnes type contour integral for H- function for multivariable

We use the fractional derivative operator $D_x^{u}\ (x^{\mu}\ )$ and after simplification we get required result .

Proof:-Result –2 First using The Fox's H-Function of multivariable. then we shall utilize following definition introduced by srivastava [16]. then apply Fractional integral formula

$$I_{0,\ x}^{\alpha,\beta,\eta}\ (t^{\lambda})\ = \frac{\Gamma(1+\lambda)\Gamma(1+\lambda-\beta+\eta)}{\Gamma(1+\lambda-\beta)\Gamma(1+\lambda+\alpha+\eta)}x^{1-\beta}$$ provided that Re(α)> max [0, Re(β-η)]— 1,

Now using by mellin barnes type contour integral for H- function for multivariable
We use the fractional derivative operator $D_x^u\ (x^{\mu}\ )$ and after simplification we get required result .

3.Conclusion

In this paper we get fractional differential operator formulae involving special function and general class of polynomials. Here we presented two very generalized and unified theorems associated with the generalized fractional integral operators given by Saigo-Maeda. result can be specialized to yield a large number of simpler results. The main results may find potentially useful applications in a variety of areas.In this paper we get fractional differential operator formulae involving special function and general class of polynomials.

In this paper we get fractional differential operator formulae involving Eulerian integral of H- function. Here we presented two very generalized and unified theorems associated with the generalized fractional integral operators given by Saigo-Maeda. The main results may find potentially useful applications in a variety of areas.

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