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# CHOLESKY FACTORIZATION WITH MATLAB

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## Abstract:

A linear system of equations with a positive definite symmetric matrix can be efficiently solved using Cholesky decomposition. In this paper, the software MATLAB was used to compute the Cholesky factorization for the symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 7 \\ 2 & 13 & 23 & 38 \\ 4 & 23 & 77 & 122 \\ 7 & 38 & 122 & 294 \end{bmatrix}.$$

We also compared the result obtained by using the MATLAB built-in routine “chol()” with the solution obtained from direct method.

***Keywords*—**MATLAB, Symmetric Matrix, Upper Triangular Matrix, Decomposition and Positive Definite.

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## I. INTRODUCTION

A square matrix is said to be symmetric if it is equal to its transpose. i.e., a symmetric matrix is one where $a_{ij} = a_{ji}$ for all i and j. In other words, $[A] = [A]^T$.Such systems occur commonly in bothmathematical and engineering/science problem contexts.Special solution techniques are available for such systems. They offer computationaladvantages because only half the storage is needed and only half the computation time isrequired for their solution.One of the most popular approaches involves Choleskyfactorization (also calledCholesky decomposition). This algorithm is based on the fact that a symmetric matrix canbe decomposed, as in $A = U^T U$.That is, the resulting triangular factors are the transpose of each other.The terms of $A = U^T U$ can be multiplied out and set equal to each other. The factorization can be generated efficiently by recurrence relations.

## IIMATLAB FUNCTION:

MATLAB has a built-in function chol that generates the Cholesky factorization. It has thegeneral syntax,U = chol(X)where U is an upper triangular matrix so that U'*U = X. In this paper, we clearly showed howit can be employed to generate both thefactorization and a solution for the same matrix.

## III.PROCEDURE FOR CHOLESKY FACTORIZATION OF A SYMMETRIC POSITIVE DEFINITE MATRIX:

If a matrix A is symmetric and positive definite, we can find its LU decomposition such that the upper triangular matrix U is the transpose of the lower triangular matrix L, which is called Cholesky factorization.

Consider the Cholesky factorization procedure for a 4X4 matrix.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} = \begin{bmatrix} u_{11} & 0 & 0 & 0 \\ u_{12} & u_{22} & 0 & 0 \\ u_{13} & u_{23} & u_{33} & 0 \\ u_{14} & u_{24} & u_{34} & u_{44} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} = \begin{bmatrix} u_{11}^2 & u_{11}u_{12} & u_{11}u_{13} & u_{11}u_{14} \\ u_{12}u_{11} & u_{12}^2+u_{22}^2 & u_{12}u_{13}+u_{22}u_{23} & u_{12}u_{14}+u_{22}u_{24} \\ u_{13}u_{11} & u_{13}u_{12}+u_{23}u_{22} & u_{13}^2+u_{23}^2+u_{33}^2 & u_{13}u_{14}+u_{23}u_{24}+u_{33}u_{34} \\ u_{14}u_{11} & u_{14}u_{12}+u_{24}u_{22} & u_{14}u_{13}+u_{24}u_{23}+u_{34}u_{33} & u_{14}^2+u_{24}^2+u_{34}^2+u_{44}^2 \end{bmatrix}$$

………………(i)

Equating every row of the matrices on both sides of equation(i) yields

$$u_{11} = \sqrt{a_{11}}$$
$$u_{12} = \frac{a_{12}}{u_{11}}$$
$$u_{13} = \frac{a_{13}}{u_{11}}$$
$$u_{14} = \frac{a_{14}}{u_{11}}$$
$$u_{22} = \sqrt{a_{22} - u_{12}^2}$$
$$u_{23} = \frac{(a_{23} - u_{13}u_{12})}{u_{22}}$$
$$u_{24} = \frac{(a_{24} - u_{14}u_{12})}{u_{22}}$$
$$u_{33} = \sqrt{a_{33} - u_{23}^2 - u_{13}^2}$$
$$u_{34} = \frac{(a_{43} - u_{24}u_{23} - u_{14}u_{13})}{u_{33}}$$
$$u_{44} = \sqrt{a_{44} - u_{34}^2 - u_{24}^2 - u_{14}^2}$$

All the above can be combined into two formulas as

$$u_{kk} = \sqrt{a_{kk} - \sum_{i=1}^{k-1} u_{ik}^2} \quad for\ k = 1{:}N \quad ……………(ii)$$

$$u_{km} = \frac{(a_{km} - \sum_{i=1}^{k-1} u_{im}u_{ik})}{u_{kk}} \quad for\ m = k+1{:}N\ \ and\ k = 1{:}N \quad ………(iii)$$

## IV STATEMENT OF THE PROBLEM:

**PROBLEM:** Use MATLAB programme code to compute the Cholesky factorization for the symmetric matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 7 \\ 2 & 13 & 23 & 38 \\ 4 & 23 & 77 & 122 \\ 7 & 38 & 122 & 294 \end{bmatrix}$$ and also compare the result obtained by using the MATLAB built-in routine "chol()" with the solution obtained from direct method.

## V DIRECT SOLUTION FOR THE PROBLEM:

For the first row (i = 1), Eq. (ii) is employed to compute

$$u_{11} = \sqrt{a_{11}} = \sqrt{1} = 1$$

Then, Eq. (iii) can be used to determine

$$u_{12} = \frac{a_{12}}{u_{11}} = \frac{2}{1} = 2$$
$$u_{13} = \frac{a_{13}}{u_{11}} = \frac{4}{1} = 4$$
$$u_{14} = \frac{a_{14}}{u_{11}} = \frac{7}{1} = 7$$

For the second row (i = 2):

$$u_{22} = \sqrt{a_{22} - u_{12}^2} = \sqrt{13-4} = \sqrt{9} = 3$$
$$u_{23} = \frac{(a_{23} - u_{13}u_{12})}{u_{22}} = \frac{23-8}{3} = 5$$
$$u_{24} = \frac{(a_{24} - u_{14}u_{12})}{u_{22}} = \frac{38-14}{3} = 8$$

For the third row (i = 3):

$$u_{33} = \sqrt{a_{33} - u_{23}^2 - u_{13}^2} = \sqrt{77-25-16} = 6$$
$$u_{34} = \frac{(a_{34} - u_{24}u_{23} - u_{14}u_{13})}{u_{33}} = \frac{122-40-28}{6} = 9$$

For the fourth row (i = 4):

$$u_{44} = \sqrt{a_{44} - u_{34}^2 - u_{24}^2 - u_{14}^2} = \sqrt{294-81-64-49} = 10$$

Thus, the Cholesky factorization yields

  

$$U = \begin{bmatrix} 1 & 2 & 4 & 7 \\ 0 & 3 & 5 & 8 \\ 0 & 0 & 6 & 9 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

This factorization can be verified as follows

$$U^T U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 10 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 & 7 \\ 0 & 3 & 5 & 8 \\ 0 & 0 & 6 & 9 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 4 & 7 \\ 2 & 4+9 & 8+15 & 14+24 \\ 4 & 8+15 & 16+25+36 & 28+40+54 \\ 7 & 14+24 & 28+40+54 & 49+64+81+100 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 4 & 7 \\ 2 & 13 & 23 & 38 \\ 4 & 23 & 77 & 122 \\ 7 & 38 & 122 & 294 \end{bmatrix} = A$$

After obtaining the factorization, it can be used to determine a solution for a right-hand-side vector {b} which is the sum of the rows of [A]

$$b = \begin{bmatrix} 14 \\ 76 \\ 226 \\ 461 \end{bmatrix}$$

First, an intermediate vector{d} is created by solving $U^T\{d\} = \{b\}$

Then, the final solution can be obtained by solving $[U]\{X\} = \{d\}$

**VI SOLUTION USING MATLAB**

**CODE:** Make a MATLAB routine "cholesky()" , which implements the formulas in (ii) and (iii) to perform Cholesky factorization for the matrix A

$$= \begin{bmatrix} 1 & 2 & 4 & 7 \\ 2 & 13 & 23 & 38 \\ 4 & 23 & 77 & 122 \\ 7 & 38 & 122 & 294 \end{bmatrix}.$$

The matrix is entered in standard fashion as

```
>>A = [1 2 4 7; 2 13 23 38; 4 23 77 122; 7 38 122 294];
```

A right-hand-side vector that is the sum of the rows of [A] can be generated as

```
>> b=[sum(A(1,:)); sum(A(2,:)); sum(A(3,:)); sum(A(4,:))];
b =    14
       76
       226
       461
```

Next, the Cholesky factorization can be computed with

```
>> U = chol(A)

U =

     1     2     4     7
     0     3     5     8
     0     0     6     9
     0     0     0    10
```

We can test that this is correct by computing the original matrix as

```
>> U'*U

ans =

     1     2     4     7
     2    13    23    38
     4    23    77   122
     7    38   122   294
```

To generate the solution, we first compute

```
>> d=U'\b

d =
    14
    16
    15
    10
```

And then use this result to compute the solution

```
>> X=U\d

X =

     1
     1
     1
     1
```

  

**CONCLUSION:**

We computed the Cholesky factorization for the symmetric matrix A $= \begin{bmatrix} 1 & 2 & 4 & 7 \\ 2 & 13 & 23 & 38 \\ 4 & 23 & 77 & 122 \\ 7 & 38 & 122 & 294 \end{bmatrix}$ with the aid of the software MATLAB. We observed that Cholesky's method provides an efficient way to decompose asymmetric matrix and that the resulting triangular matrix and its transpose can beused to evaluate right-hand-side vectors efficiently. Also, the solutions obtained by Direct method and by MATLAB code are the same.

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