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Unsteady Heat and Mass Transfer MHD flow of an UCM fluid over a stretching surface with Destructive/Generative Chemical Reaction

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ABSTRACT

It is well known that the stagnation points have a definite role in groundwater flow because the separation streamlines passing through them describe different flow regions. In this work, the solution of unsteady boundary layer MHD flow of a Maxwell fluid over a stretching surface in the presence of destructive/generative chemical reaction is obtained. The problem was transformed to ordinary differential system of equations. The solution for the non-magnetic case is chosen as an initial guess and the iterations using Euler scheme are continued till convergence within prescribed accuracy is achieved, with the corrections incorporated in subsequent iterative steps until convergence, which is used to obtain the values of our initial guesses. Finally, the resulting guesses together with the system was solved using generalized Thomas' algorithm. The system of equations has to be solved in the infinite domain $0 < \eta < \infty$. A finite domain in the η-direction can be used instead with η chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. To assess the accuracy of the present work, the results of skin friction coefficient for the steady motion in the absence of suction/blowing with $n = 1$ are compared with the available results as shown in Table 1 and found to be in agreement with previous work. The result obtained are displayed in Tables and Figures, and discussed. Qualitative explanation for UCM fluid behaviour have been made. Some of the results obtained shows that when the magnetic parameter increases, the velocity decreases while the temperature and concentration increases. The effect of magnetic field and unsteadiness parameter on the UCM fluid increases the temperature field while concentration field is enhanced with increasing generative chemical reaction and magnetic parameters.

Keywords **—Unsteady flow, Upper Convected Maxwell (UCM) fluid, MHD flow, Mass transfer, chemically reactive species, Shooting method.**

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I. INTRODUCTION

The boundary layer flows of non-Newtonian fluid have received much importance due to its numerous industrial and engineering applications. In view of non-Newtonian fluids diverse rheological properties cannot be examined through one constitutive relationship between shear stress and rate of strain. For any boundary layer, Maxwell model is used to predict stress relaxation and also excludes the effects of shear dependent viscosity. The first person to investigate the boundary layer flow of a viscous fluid was Sakiadis [1], the flow is caused due to the motion of the rigid plane sheet in its own plane. Due to entrainment to ambient fluid, this situation represents a different class of boundary-layer problem which has a solution substantially different from that of boundary-layer flow over a semi-infinite flat plate. Erickson et al. [2] extended this problem to the moving surface in the presence of suction or blowing. Crane [3] considered the moving sheet, and the velocity is proportional to the distance from the slit. In general, these types of flows occur in the drawing of plastic films and artificial fibers. Gupta and Gupta [4] investigated heat and mass transfer over a stretching sheet with suction or blowing. Similarity solution of MHD boundary layer flow problem of an electrically conducting incompressible fluid over a stretching surface in the presence of transverse magnetic field was studied by Pavlov [5].

Chakrabarti and Gupta [6] extended the problem to the temperature distribution in the MHD boundary layer flow due to stretching surface with suction. Non-Newtonian fluids with convective heat and mass transfer finds many industrial applications such as nuclear fuel slurries, paper coating, liquid metals, movement in biological fluids, plastic extrusion, material processing and crystal growing. Andersson et al. [7] discussed the momentum and mass diffusion of the flow with chemical reactive species over a stretching sheet. Takhar et al. [8] presented the mass transfer with magnetohydrodynamic (MHD) flow in a viscous electrically conducting fluid by a stretching sheet with nonzero velocity. The problem of second grade fluid with a porous medium was extended by Akyildiz et al. [9]. The effects of suction/blowing with heat absorption/generation over a porous stretching surface in the presence of boundary layer flow were analyzed by Layek et al. [10].

It is well known that the stagnation points have a definite role in groundwater flow because the separation streamlines passing through them describe different flow regions. The stagnation point flow over a stretching surface is classic problem in fluid mechanics. Sadeghy et al. [11] presented the stagnation-point flow of upper-convected Maxwell fluid. Mahapatra et al. [12] reinvestigated the stagnation-point flow problem towards a stretching plate taking different stretching and stagnation flow velocities. They analyzed that two different types of boundary layer are formed near the stretching plate depending on the ratio of the stretching and stagnation flow velocities. Dual solutions in mixed convection flow near a stagnation point on a vertical porous plate has been studied by Ishak et al. [13]. Hayat et al. [14] presented the MHD flow of micropolar fluid near a stagnation-point towards a non-linear stretching surface. Abbas et al. [15] studied the mixed convection in the stagnation point flow of Maxwell fluid towards a vertical stretching surface. Recently, Hayat and Nawaz [16] presented the unsteady stagnation point flow of viscous fluid caused by an impulsively rotating disk.

Different analytical techniques such as LSM, DTM, OHAM, and HPM, were studied by Ghasemi et al. [17, 18] and Vatani et al. [19]. The HAM was first devised in 1992 by Liao Shijun of Shanghai Jiaotong University in his PhD dissertation Liao [20] and further modified it in 1997 to introduce a non-zero auxiliary parameter, referred to as the convergence-control parameter, c_0 Liao [21], and also formulate an extension to construct a homotopy on a differential system in general form Liao [22]. The convergence-control parameter is a non-physical variable that provides a simple way to verify and enforce convergence of a solution series. The capability of the HAM to naturally show convergence of the series solution is unusual in analytical and semi-analytic approaches to nonlinear partial differential equations.

In the last twenty years, the HAM has been applied to solve a growing number of nonlinear ordinary/partial differential equations in science, finance, and engineering, as reported by Liao [23] and Vajravelu [24]. Further, a unified wave model applied with the HAM, Liao [25] admits not only the traditional smooth progressive periodic/solitary waves, but also the progressive solitary waves with peaked crest in finite water depth. This model shows peaked solitary waves are consistent solutions along with the known smooth ones. Additionally, the HAM has been applied to many other nonlinear problems such as nonlinear heat transfer, Abbasbandy [26] the limit cycle of nonlinear dynamic systems Chen, Liu [27], the American put option Zhu [28] the exact Navier–Stokes equation was given by Turkyilmazoglu [29], similarly, the option pricing under stochastic volatility Park and Kim [30], the application of Homotopy analysis method applied to electrohydrodynamic flow, the Poisson–Boltzmann equation for semiconductor devices and others Mastroberardino [31], Nassar [32].

The HAM has recently been reported to be useful for obtaining analytical solutions for nonlinear frequency response equations. Such solutions are able to capture various nonlinear behaviors such as hardening-type, softening-type or mixed behaviors of the oscillator Tajaddodianfar [33], [34] These analytical equations are also useful in prediction of chaos in nonlinear systems Tajaddodianfar [35].

Despite the contributions of various authours and existing results on flow of an UCM fluid over a stretching surface, the discussion on including combined heat and mass transfer effect has not been adequately reported. Hence, we revisit the work [36] to include the impact of combined effect heat and mass transfer as well as the influence of generative/destructive chemical reaction. Therefore, existing results in the literature are compared with the present study as a special case.

II. MATHEMATICAL FORMULATION

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A. Transient unsteady-state flow and mass transfer

We consider the unsteady and incompressible MHD flow and mass transfer of an electrically conducting upper convected Maxwell fluid over a stretching surface. The flow is induced due to the stretching surface by applying equal and opposite forces by the x-axis and considering the flow to be bounded to the region $y > 0$. The mass flow and unsteady fluid start at $t = 0$. The sheet appears out of a slit at origin and moves with velocity $U(x,t) = bx/(1-\alpha t)$ where b and α are positive constants both having dimensions $time^{-1}$, b is the rate of stretching and $b/(1-\alpha t)$ is the rate of stretching with time. In case of polymer, the material properties of the sheet vary with time. A uniform magnetic field of strength B_0 is along the y −axis. The induced magnetic field is trifling, which is a valid assumption on a scale under the small magnetic Reynolds number and the external field is zero. The problem of heat and mass transfer in the flow along a flat plate that contains a species, say A is slightly soluble in B. C_w be the concentration at the plate surface and C_∞ be the solubility of A in B and in the concentration of species far away from the plate is A. Let the rate of reaction of the species A with B be an nth-order homogenous chemical reaction with constant n. The flow geometry and coordinate system are shown in Fig. 1. (Source: [36]).

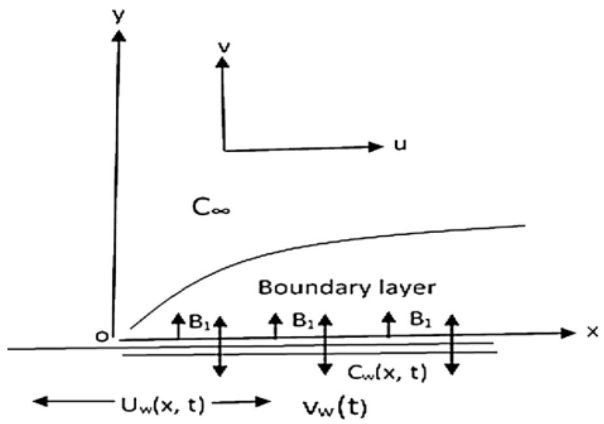

Fig. 1: Flowgeometrycoordinate

It is desirable to study the system by the boundary layer analysis [37]. The governing equations of the model [36] and [38] are modified to include the impact of heat and mass buoyancy, energy system with space heat generation or absorption. Under the aforementioned assumptions using Boussinesq approximations the system of equation governing the Unsteady Heat and Mass Transfer MHD flow of an UCM fluid over a stretching surface with generative/destructive chemical reaction are stated as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v_{n_f}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2(x)}{\rho_{n_f}}u -\lambda\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial y\partial x}\right) +\frac{g}{\rho_{n_f}}\left(\beta_\tau(T-T_\infty)+\beta_c(C-C_\infty)\right) \quad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho_{n_f}c_p}\frac{\partial^2 T}{\partial y^2} +\left(\frac{\kappa u_w(x)}{\rho_{n_f}c_p x v}\right)\frac{A^*(T_w-T_\infty)}{bx}(u-U) +\left(\frac{\kappa u_w(x)}{\rho_{n_f}c_p x v}\right)B^*(T-T_\infty) \quad (3)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{D_m}{\rho_{n_f}}\frac{\partial^2 C}{\partial y^2} -\frac{k'v}{\tau\rho_{n_f}}\frac{\partial}{\partial y}\left((C-C_\infty)\frac{\partial T}{\partial y}\right)+k_r^2(C-C_\infty)^n \quad (4)$$

where $A^*, B^* > 0$.

The appropriate initial and boundary conditions relevant to the problem are

$$t\le 0: u=u(x), v(x)=-v_0, T=T_w, C=C_w \ \forall y$$

$$t>0:\begin{cases} u=\dfrac{bx}{1-\alpha t}, v_w(t)=-v_0, -\dfrac{\kappa}{h}\dfrac{\partial T}{\partial y}=(T_w-T), \\ -\dfrac{D_B}{h_2}\dfrac{\partial C}{\partial y}=(C_w-C), y=0 \\ u\to 0, T\to T_\infty, C\to C_\infty \quad as\ y\to\infty \end{cases} \quad (5)$$

Here, u and v are the components of velocity in x and y directions respectively, λ is the relaxation time, ν is the fluid kinematic viscosity, σ is the fluid conductivity, B_0 is the uniform magnetic field, ρ is the density of fluid, C is the species concentration, D is the coefficient of diffusion in the diffusing species of the fluid and k_r is the rate of reaction constant of order n.

Where the surface temperature and concentration of the sheet is assumed to vary by both the sheet and time, in accordance with $T_w(x,t)=T_\infty+bx(1-\alpha t)^{-2}$ and $C_w(x,t)=C_\infty+bx(1-\alpha t)^{-2}$ respectively. The wall temperature and concentration $T_w(x,t)$, $C_w(x,t)$ increases (reduces), if b is positive (negative) and is in proportion to x. Moreover, the amount of temperature and concentration increase (reduce) along the sheet increases with time. Here $v_w(t)=-v_0/\sqrt{(1-\alpha t)}$ at is the velocity of suction $v_0>0$ or blowing $v_0<0$. The expression for$U_w(x,t), v_w(t), C_w(x,t), \lambda(t)$, $\kappa_r(t)$ is valid for time $t<\alpha^{-1}$.

A. *B. Method of Solution*

To seek for solution, we sought for a stream function $\psi(x,y,t)$ which must identically satisfied equation, such that

$$u=\frac{\partial\psi}{\partial y}, \quad \text{and} \quad v=-\frac{\partial\psi}{\partial x}$$

where

$$\psi(x,y,t)=\left(\frac{bv}{1-\alpha t}\right)^{1/2}xf(\eta), \eta(y,t)=\left(\frac{b}{1-\alpha t}\right)^{1/2}y, \quad (6)$$

where $\eta$is similarity variable and $\psi$is stream function defined thus by (6), the velocity component are

$$u=\frac{bx}{1-\alpha t}f'(\eta), \qquad v=-\left(\frac{bv}{1-\alpha t}\right)^{1/2}f(\eta) \quad (7)$$

The temperature and concentration is represented as

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

where

$$T_w(x,t)=T_\infty+bx(1-\alpha t)^{-2}, C_w(x,t)=C_\infty+bx(1-\alpha t)^{-2}$$

Using Eqs. (5)–(7) on Eqs. (2) - (4), the transform boundary value problems becomes:

$$f'''(\eta)+f(\eta)f''(\eta)-A\left(f'(\eta)+\frac{1}{2}\eta f''(\eta)\right)$$
$$-Haf'(\eta)+\lambda_0\big(2f(\eta)f'(\eta)f''(\eta)-f'(\eta)^2f''(\eta)\big) \quad (8)$$
$$-\big(f'(\eta)\big)^2+Grt\,\theta(\eta)+Grc\,\theta(\eta)=0$$

$$\theta''(\eta)-Pr\left(A\frac{1}{2}\eta-f(\eta)\right)\theta'(\eta)$$
$$+A^*(f'(\eta)-1)+B^*\theta(\eta)=0 \quad (9)$$

$$\phi''(\eta)-Sc\left(A\frac{1}{2}\eta-f(\eta)\right)\phi'(\eta)-\beta \quad (10)$$
$$\big(\phi'(\eta)\theta'(\eta)+\phi(\eta)\theta''(\eta)\big)+\delta\phi(\eta)^n=0$$

The corresponding initial-boundary conditions becomes

$$f'(0)=1, f(0)=S, Bi\theta'(0)=\theta(0)-1,$$
$$\omega\,\phi'(0)=\phi(0)-1 \quad (11)$$
$$f'(\infty)\to 0, \theta(\infty)\to 0, \phi(\infty)\to 0$$

The dimensionless parameters in Eqs. (8)–(11) are the unsteadiness parameter A, the Maxwell parameter β, the Magnetic parameter Ha, the Schmidt number Sc, the reaction rate parameter δ and the suction/blowing parameter S, which can be represented as follows

$$A=\frac{\alpha}{b}, S=-\frac{v_0}{\sqrt{\nu b}}, Ha=\frac{\sigma B_0^2}{\rho b}(1-\alpha t), Sc=\frac{\nu}{Dm},$$
$$\delta=\frac{\kappa_r(C_w-C_\infty)^{n-1}}{\rho b}, \beta=\frac{\kappa_r\nu(T_w-T_\infty)}{\tau Dm},,$$
$$Grc=\frac{\beta_c g}{\rho b x^2} Grt=\frac{\beta_\tau g}{\rho b x^2}, \omega==\frac{k}{h}\sqrt{\frac{b}{\nu(1-\alpha t)}}$$
$$Pr=\frac{\nu}{\kappa}\rho c_p A^*=\frac{A1}{1-\alpha t}, Bi=\frac{D_m}{h_2}\sqrt{\frac{b}{\nu(1-\alpha t)}}$$

It is worth mentioning that the chemical reaction parameter δ is a real number, $\delta<0$ indicates the destructive chemical reaction, $\delta>0$ denotes the generative chemical reaction, and $\delta=0$ for the non-reactive species. It follows that for suction S is positive and S is negative for blowing, and this parameter is used to controlling the normal flow strength and direction at the boundary.

$$B(t)=B_0\sqrt{\frac{b}{1-\alpha t}}, \lambda=\lambda_0\frac{1-\alpha t}{b}$$

C. *Skin friction, Mass and heat transfer coefficients.*

The physical quantities of engineering interest in this problem such as the skin-friction parameter, the plate surface temperature, Nusselt number and the Sherwood number can be easily computed. For practical purposes, the functions $f(\eta), \theta(\eta)$ and $\phi(\eta)$ allow us to determine the skin friction coefficient and mass transfer rates.

For local similarity case, integration over the entire plate is necessary to obtain the total skin-friction, total heat and mass transfer rate. These parameters characterize the wall heat and mass transfer rates, and are as follows:

$$c_{f_x}=\frac{\tau_w}{\rho_{nf}U_w^2}, \quad Nu_x=\frac{xq_w}{k_{nf}(T_w-T_\infty)}, \quad (12)$$
$$Sh_x=\frac{xq_m}{D_m(C_w-C_\infty)}$$

where τ_w represents the skin friction along the surface, q_w the heat flux from the surface and are respectively given as

$$\tau_w=\mu\left[\frac{\partial u}{\partial y}\right]_{y=0}, q_w=-\kappa\left[\frac{\partial T}{\partial y}\right]_{y=0}, q_m=-D_m\left[\frac{\partial C}{\partial y}\right]_{y=0} \quad (13)$$

where q_w and q_m, represents the heat and mass fluxes at the surface respectively.

Applying the similarity transformation on (12) subject to (13) dimensionless skin-friction, wall mass and heat transfer rate are obtained as

Table 1: Comparison of $f''(0)$ for different values of Ha in the absence of the parameters $\beta=S=\gamma=Sc=0, n=1$.

Results	$Ha=0.0$	$Ha=0.5$	$Ha=1.0$	$Ha=1.5$	$Ha=2.0$
Anderson et al. [7]	-1.000000	-1.224900	-1.414000	-1.581000	-1.732000
Prasad et al. [40]	-1.000174	-1.224753	-1.414449	-1.581139	-1.732203
Mukhopadhyay et al. [41]	-1.000173	-1.224753	-1.414450	-1.581140	-1.732203
Palani et al. [36]	-1.000000	-1.224745	-1.414214	-1.581139	-1.732051
Present Result	-1.000476	-1.224774	-1.414216	-1.581139	-1.732051

Table 2: Comparison of $f''(0)$ for different values of M, when $\beta=Ha=S=\gamma=Sc=0, n=1$

Results	Sharidan et al. [42]	Chamkha et al. [43]	Bhattacharyya et al. [44]	Palani et al. [36]	Present study
$A=0.4$	-	-	-	-	-1.134968
$A=0.8$	1.261042	1.261512	1.261487	1.261043	-1.261203
$A=1.2$	1.377722	1.378052	1.377910	1.377724	-1.377838
$A=1.6$	-	-	-	-	-1.486047
$A=2.0$	-	-	-	-	-1.587348

Table 3: Effect of space heat source on skin-friction, Sherwood and Nusselt numbersfor different values of A, when $\beta=Ha=S=\gamma=Sc=0, n=1$

$A^*=-0.5, B^*=-0.3$	-1.15053	0.01596	-0.10011
$A^*=-0.2, B^*=-0.1$	-1.16347	0.04424	-0.17005
$A^*=0.1, B^*=0.1$	-1.21737	0.14166	-0.67996
$A^*=0.2, B^*=0.1$	-1.27086	0.30101	-1.26526

$$C_f = \sqrt{Re_x} c_{f_x} = \left.\frac{\partial^2 f(\eta)}{\partial \eta^2}\right|_{\eta=0}, Nu = \frac{Nu_x}{\sqrt{Re_x}} = -\left.\frac{\partial \theta(\eta)}{\partial \eta}\right|_{\eta=0}, Sh = \frac{Sh_x}{\sqrt{Re_x}} = \left.\frac{\partial \phi(\eta)}{\partial \eta}\right|_{\eta=0} \quad (14)$$

$Re_x = \dfrac{U_w x}{\nu}$ is the local Reynolds number.

III. NUMERICAL SOLUTION OF THE PROBLEM

Governing Eqs. (8) - (10) subject to conditions (11) are solved numerically using Runge–Kutta fourth order method along with shooting technique. The higher order nonlinear partial differential equations are converted into first order simultaneous linear differential equations and then transformed to initial value problem (Jain et al. [39]). In this method the third-order nonlinear Eq. (8) and second order Eqs. (9) and (10) are reduced to a system of first order differential equations. The solution for the non-magnetic case is chosen as an initial guess and the iterations using Euler scheme are continued till convergence within prescribed accuracy is achieved, with the corrections incorporated in subsequent iterative steps until convergence, which is used to obtain the values of our initial guesses. Finally, the resulting guesses together with the system was solved using generalized Thomas' algorithm. The system of equations has to be solved in the infinite domain $0 < \eta < \infty$. A finite domain in the η-direction can be used instead with η chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance.

Grid-independence studies show that the computational domain $0 < \eta < \eta_\infty$ can be divided into intervals each of uniform step size which equals 0.02. This reduces the number of points between $0 < \eta < \eta_\infty$ without sacrificing accuracy. The value $\eta_\infty = 1$ was found to be adequate for all the ranges of parameters studied here.

To validate our method and results, we compare with the existing results in the literature when:

(i). $\beta = S = \gamma = Sc = 0, n = 1$, the result of Anderson et al. [7], Prasad et al. [40], Mukhopadhyay et al. [41] and Palani et al. [36] becomes a case in the current report (Table 1),

(ii). $\beta = Ha = S = \gamma = Sc = 0, n = 1$, the results of Palani et al. [36], Sharidan et al. [42], Chamkha et al. [43] and Bhattacharyya et al. [44] becomes a special case of the current report (Table 2).

IV. RESULT AND DISCUSSION

To assess the accuracy of the present method, the results of skin friction coefficient $f''(0)$ for the steady motion in the absence of suction/blowing with $n = 1$ are compared with the available results as shown in Table 1 and found to be in agreement with previous work. Table 2 show the comparison of skin friction for different values of unsteadiness A, when $\beta = Ha = S = \delta = Sc = 0, n = 1$ and found to be in agreement with previous work. Table 3 shows the effect of space heat source on skin-friction, Sherwood and Nusselt numbers for different values of A, when $\beta = Ha = S = \delta = Sc = 0, n = 1$. In addition, Table 4 shows the results of various governing parameters effects on wall rate transfer.

Table 4: Results of various governing parameters effects on wall rate transfer

Parameters	$f''(0)$	$\phi'(0)$	$\theta'(0)$	Parameters	$f''(0)$	$\phi'(0)$	$\theta'(0)$
$A = 0.0$	-1.14419	-0.13516	-0.48375	$Ha = 0.0$	-0.96583	-0.04819	-0.46687
$A = 0.2$	-1.18810	0.07321	-0.43720	$Ha = 0.5$	-1.18810	0.07321	-0.43720
$A = 0.4$	-1.22291	0.44500	-0.39262	$Ha = 1.0$	-1.37944	0.18789	-0.41449
$A = 0.8$	-1.25202	2.10868	-0.32578	$Ha = 1.5$	-1.54927	0.29437	-0.39662
$Bi = -1.2$	-1.04980	-0.14097	-1.58778	$S = -0.4$	-0.92897	0.31829	-0.30002
$Bi = -0.6$	-1.15466	0.01703	-0.70844	$S = -0.2$	-1.02114	0.23322	-0.35093
$Bi = 0.7$	-1.20147	0.09660	-0.32989	$S = 0.0$	-1.12822	0.13000	-0.40716
$Bi = 1.4$	-1.23295	0.15391	-0.07989	$S = 0.1$	-1.18810	0.07321	-0.43720
$\alpha = 0.0$	-1.18920	0.07356	-0.43692	$\delta = -0.2$	-1.22530	-0.57153	-0.42107
$\alpha = 0.8$	-1.18487	0.07201	-0.43800	$\delta = -0.1$	-1.22156	-0.49219	-0.42249
$\alpha = 1.6$	-1.18080	0.07013	-0.43901	$\delta = 0.1$	-1.20717	-0.23105	-0.42860
$\alpha = 2.2$	-1.17793	0.06857	-0.43970	$\delta = 0.2$	-1.18810	0.07321	-0.43720
$n = 1$	-1.17332	0.24535	-0.44588	$Grt = -0.5$	-1.54871	0.75728	-0.36061
$n = 2$	-1.18810	0.07321	-0.43720	$Grt = -0.2$	-1.37965	0.32016	-0.39875
$n = 3$	-1.20656	-0.18859	-0.42823	$Grt = 1.0$	-0.76824	-0.16078	-0.50139
$n = 4$	-1.21161	-0.27365	-0.42614	$Grt = 4.0$	0.45228	-0.38402	-0.61591
$\gamma = 0.00$	-1.15947	0.05578	-0.44126	$Grc = 0.0$	-1.27135	0.22422	-0.41334
$\gamma = 0.05$	-1.17376	0.06458	-0.43920	$Grc = 0.2$	-1.11771	-0.00280	-0.45328
$\gamma = 0.10$	-1.18810	0.07321	-0.43720	$Grc = 0.6$	-0.88058	-0.14419	-0.49418
$\gamma = 0.15$	-0.84482	-0.43904	-0.69782	$Grc = 1.2$	-0.57843	-0.23742	-0.53193

Table 4 shows the results of various governing parameters effects on wall rate transfer. From the table, we see that the skin-friction coefficient increases as the thermal Grt and mass Grashof number Grc, chemical reaction parameter, and mass slip parameter α increase and decreases as unsteadiness parameter A, convective heat transfer B_i, reaction order n and Suction S increases. Also, the rate of heat transfer (Nusselt number) is seen to increase due to increases in unsteadiness A, convective heat transfer B_i, chemical reaction order δ and magnetic parameter Ha while the rate decreases with increasing mass slip parameter α, thermal and mass Grashof number, suction and reaction order. Lastly, the rate of mass transfer at the surface (Sherwood number) is predicted to increase as a result of increasing unsteadiness A, convective heat transfer B_i, magnetic parameter $Ha$and chemical reaction parameter δ while it decreases when mass slip parameterα, reaction order n Suction S, thermal GrashofGrt and mass Grashof number Grc is increased.

The numerical calculations are carried out for different values of the physical parameters involved in equations: unsteadiness parameter A, Magnetic parameter Ha, chemical reaction parameter δ, order of chemical reaction n and suction/blowing parameter S. In order to analyze salient features of the problem, the numerical results are shown in figures and physical explanations are discussed for all cases in terms of graphs as shown in Fig. 2-16.

Fig. 2 show the nature of several values of unsteadiness parameter A on the velocity profile. It is seen that the velocity along the surfaces increases with the increase of A, and this caused reduction in the thickness of the momentum boundary layer near the wall. The effects of Magnetic parameter Ha is depicted on Fig. 3. The fluid velocity is found to decrease with increasing values of Ha. This is due to the fact that applied magnetic field produces a drag in the form of Lorentz force thereby decreasing the magnitude of velocity.The fluid velocity is found to decrease with increasing suction ($S > 0$)as seen in Fig.4. Both the thermal and mass Grashof number are augmented with increasing velocity as displayed in Fig. 5.

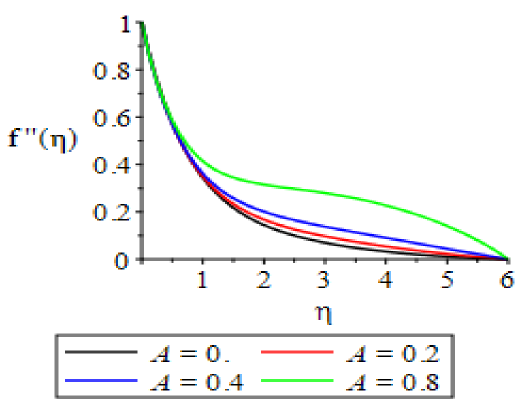

Fig.2: Effect of unsteadiness parameter on velocity

Fig. 6-11 display the behavior of A, Ha, S, G_{rt}, G_{rc} and B_i against the temperature profile of the system. Increase in unsteadiness parameter A and the Magnetic parameter Haresults in the increase in the temperature profile as seen in Fig. 6 and 7.

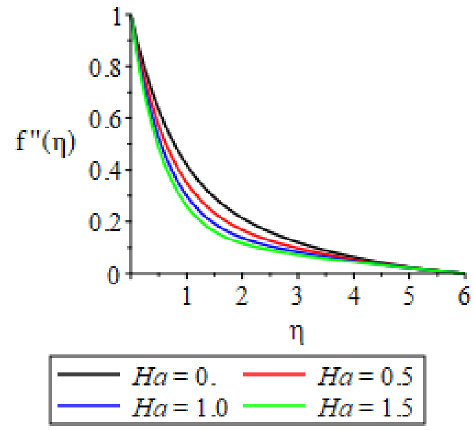

Fig.3: Effect of Hartmann number on velocity.

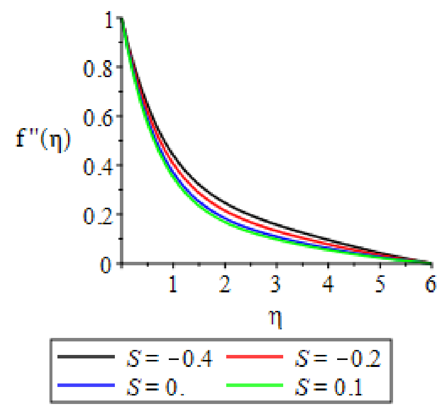

Fig.4: Effect of suction/injection parameter on velocity.

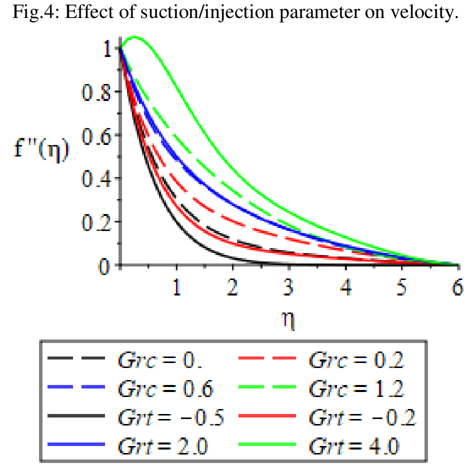

Fig.5: Effect of Grashof (thermal Grt, mass Grc) number on velocity

Though the magnetic parameter is not directly present in the energy equation, the temperature field is indirectly affected by the magnetic parameter through changes in the velocity field.While temperature decreases with increasing values of S, G_{rt}, G_{rc} and B_i as displayed in Fig. 8-11. Fig. 12 shows the effects of magnetic parameter on the concentration field ϕ. The species concentration increases within the boundary layer when Ha is augmented. The influences of chemical reaction parameter δ, convective heat transfer B_i and mass slip parameter α on ϕ are qualitatively similar to that of Ha as can be seen through Fig. 14, 17 and 18. As the suction S enlarges, the concentration distribution decreases as seen in Fig.13. Both the thermal and mass Grashof number are reduced with increasing concentration distribution as displayed in Fig. 15 and 16. The concentration profile against the chemical reaction order is exhibited in Fig. 19. Concentration distribution is a decreasing function of chemical reaction parameter.

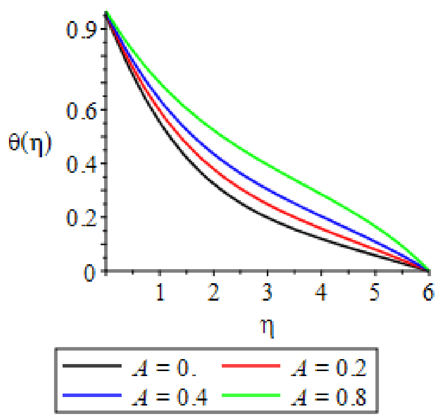

Fig.6: Effect of unsteadiness parameter on temperature distribution

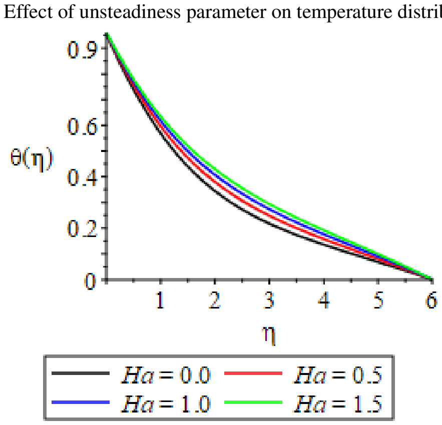

Fig.7: Effect of Hartmann number on temperature distribution.

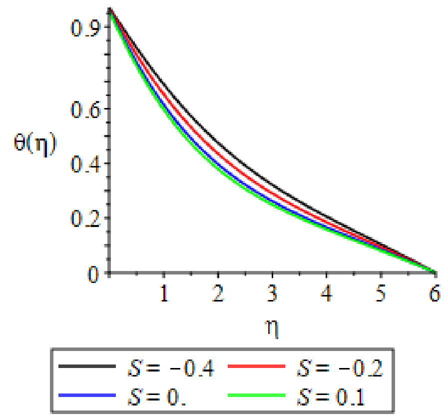

Fig.8: Effect of suction/injection parameter on temperature distribution.

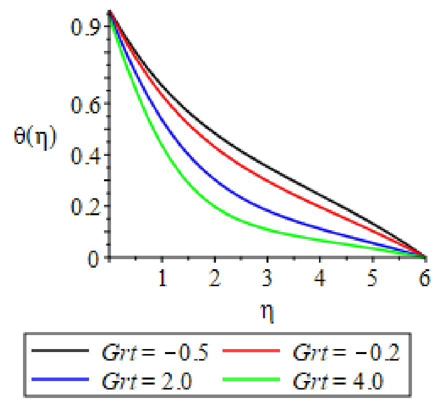

Fig.9: Effect of thermal Grashof number on temperature distribution.

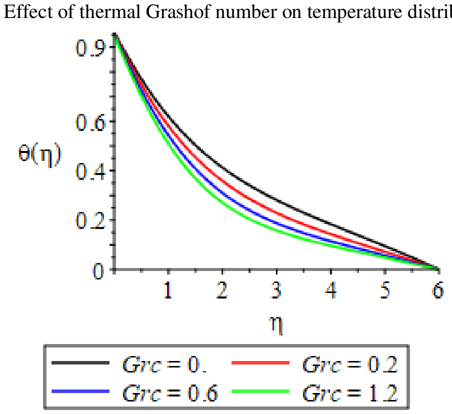

Fig.10: Effect of mass Grashof number on temperature distribution.

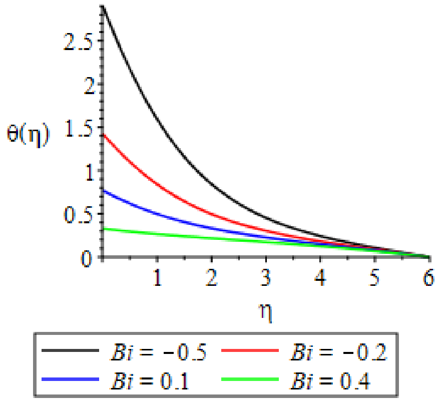

Fig.11: Effect of convective heat parameter on temperature distribution.

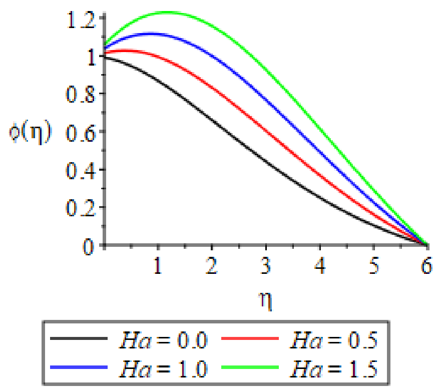

Fig.12: Effect of Hartmann number on chemical species distribution.

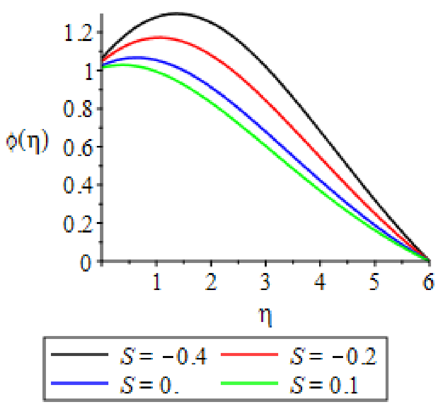

Fig.13: Effect of suction/injection parameter on chemical species distribution.

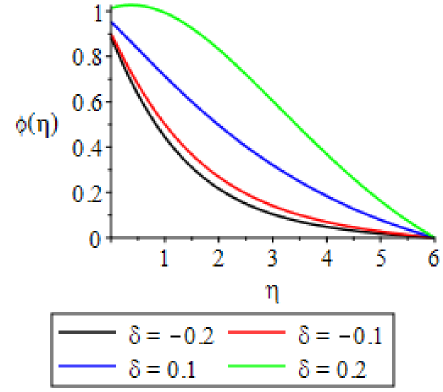

Fig.14: Effect of chemical reaction parameter on chemical species distribution

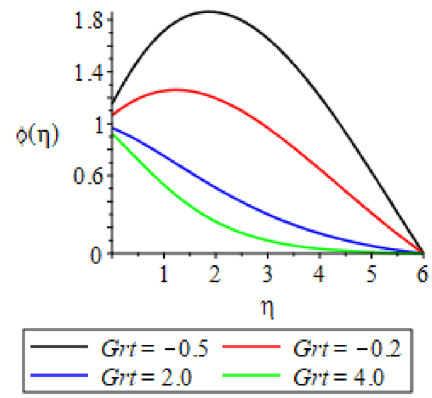

Fig.15: Effect of thermal Grashof number on chemical species distribution.

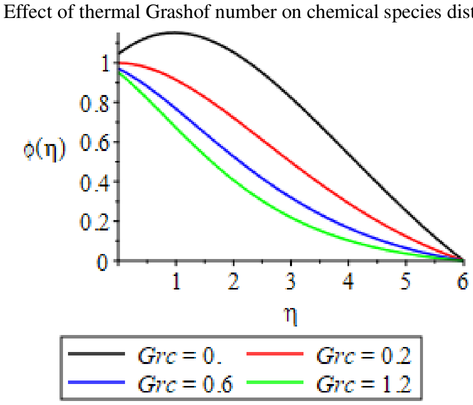

Fig.16: Effect of mass Grashof number on chemical species distribution.

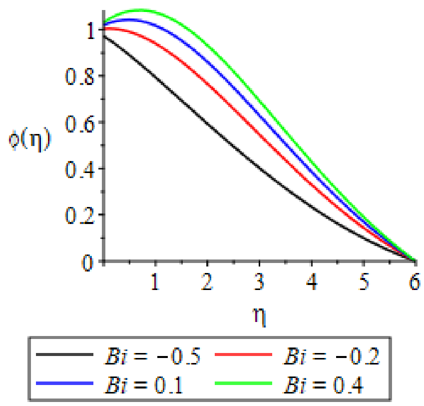

Fig.17: Effect of convective heat transfer on chemical species distribution.

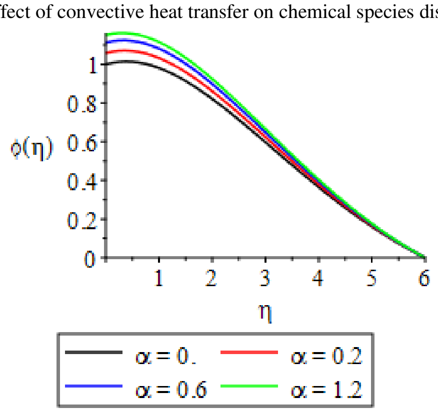

Fig.18: Effect of mass slip parameter on chemical species distribution.

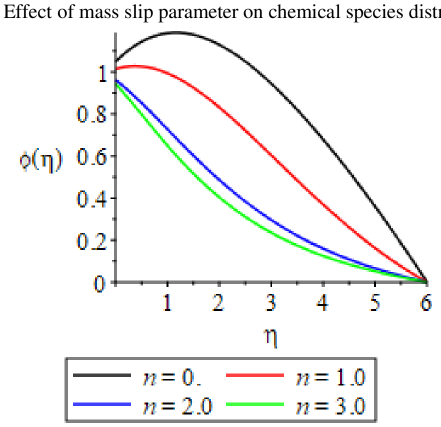

Fig.19: Effect of reaction order on chemical species distribution.

V. CONCLUSIONS

In this work, unsteady boundary layer MHD flow of a Maxwell fluid over a stretching surface in the presence of higher order constructive/destructive chemical reaction is obtained. Qualitative explanation for UCM fluid behavior is to provide some insights into the nature of underlying physical processes. The following observations have been made:

- We observe that, when the magnetic parameter increases, the velocity decreases while the temperature and concentration increases.
- The magnitude of skin friction coefficient is inversely proportional to the unsteadiness parameter while the Nusselt and Sherwood number are directly proportional to the unsteadiness parameter.
- The effect of magnetic field and unsteadiness parameter on the UCM fluid increases the temperature field
- The concentration field is enhanced with increasing chemical reaction and magnetic parameters.
- The magnitude of skin friction coefficient, Nusselt and Sherwood number are directly proportional to suction.
- is inversely proportional to the unsteadiness parameter while the
- Fluid velocity, temperatureand concentration decrease with increasing suction
- Concentration distribution is an increasing function of chemical reaction parameter. However, it has an opposite relation with chemical reaction order.

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