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MARKOVIAN QUEUING SYSTEM WITH INCENTIVIZED ENTRY IN M/M/1/N

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ABSTRACT:

In this paper, we learn and develop a finite capacity of single server constrained, Markovian queuing system with incentivized entry .This M/M/1/N model is developed a mathematical model and solve for steady-state solution and also performance measure and numerically illustration are explained in this paper. Discussed by financial analysis of model by developing a cost model of the system. The term incentivized entry caused by the condition experienced by the system after the release of the proposal business discounts are incentive entries. In queuing system, the further additions to existent customer behaviour in queuing model.

Keywords: Incentivized entry, Problematic model, queuing theory.

1.INTRODUCTION:

AgnerKrarup Erlang (1909) was a danish mathematician and the invented of queuing theory. In the early 20thcentuary, Erlang was head of a technical laboratory at the copenhagen telephone. Queuing system aspect of modern life that we encounter at every step in our daily life activities. Whether it happens at checkout counter in the supermarket or in accessing arises whenever a shared facility needs to be accessed for service by a large number of jobs or customers. In any queuing system it is important to know the customer behaviour. The behaviour of arrivals are 1)Balking 2)Reneging and 3)Jockeying. Haight(1957) consider an queuing with balking in M/M/1 queue model and Haight(1959) also justify the M/M/1 queuing with reneging. Ancker and gafarian(1963) studied by the effect of some queuing problem with balking and reneging in an M/M/1/N queuing model. G.Kannadasan and D.Devi(2021) describea single server queuing system with server vacations and impatience of customer have been broadly studied and queuing system analysis of FM/ FM/1 queue with single working vacation and impatience of customers. In the current volatile and competitive scenario business environment, business management efficiently to meet the expectations of customer and audiences are in a very important cumbersome work. Therefore companies know your performance levels in advance and quality of service manipulate. On the other hand, reverse balky the possibility and non-participation in the system of handling encouraged entry the percentage change in number of customer due to deals and discounts.

Jain and kumar (2014) detailed in an M/M/1/N queuing system with reverse balking. In mention discouraged arrival defined parameter λ/n+1 by processing of poisson. They mentioned customers are said looking at large system kept from participation size. Studied the life and death queuing process of a single server. A state dependent parameters$\lambda_n = \frac{\lambda}{n+1}, n \geq 0$ and $\mu_n = \mu, n \geq 1$.

Literature survey:

In this paper we first develop a steady-state queuing model with encouraged arrivals having Markovian queuing system in single server. The model is tested for validity. The following organized details are in this paper. section 2 deals with development of mathematical model while section 3 present steady-state solution. Performance measure derived in section 4. In presented the numerically illustration in section 5. Section 6 deals with financial analysis of the model. Conclusion and future possibility are given by section 7.

2.MATHEMATICALMODEL FORMULATION:

A single server Markovin queuing model is formulated under following assumption:

- Arrivals are described by poisson probability distribution and come from an finite polulation.
- Process with parameter $\lambda(1+\epsilon)$, where 'ϵ' represents the percentage chane in number of customers, the customers was observed + 60% or 130% then $\epsilon = 0.6$ or $\epsilon = 1.3$ respectively.
- Service time have an exponential with rate parameter μ in the M/M/1 queue, where 1/ μ is the mean service time.
- A single server customers one at a time from the front of the queue, according to a first-come, first-server discipline.
- All arrivals times and services times are assumed to be independent of one another.

System of differential difference equation model given by:

$$\frac{d}{dt}P_0(t) = -\lambda(1+\epsilon)P_0(t) + \mu P_1(t) \quad \dots (1)$$

$$\frac{d}{dt}P_N(t) = \lambda(1+\epsilon)P_N - 1(t) - \mu P_N(t) \quad \dots (2)$$

$$0 = -\lambda(1+\epsilon)P_0 + \mu P_1 0 \dots (3)$$

$$0 = \lambda(1+\epsilon)P_N - 1 - \mu P_N \dots (4)$$

3.STEADY STATESOLUTION:

The probability of n customers in the system,

P_n\{n customers in the system\}$=\left(\frac{\lambda(1+\varepsilon)}{\mu}\right)^n P_0$; $1\le n \le N-1$ …(5)

The probability that system is full given by:

$$P_N\{\text{system is full}\} = \left(\frac{\lambda(1+\varepsilon)}{\mu}\right)^N P_0 \ldots (6)$$

Using condition of normality $\sum_{n=0}^{N} P_n=1$

$$P_0\{\text{system is empty}\} = \left\{1 + \sum_{n=1}^{N}\left[\frac{\lambda(1+\varepsilon)}{\mu}\right]^n\right\}^{-1}$$

$$P_0=\frac{1-\left[\frac{\lambda(1+\varepsilon)}{\mu}\right]}{1-\left[\frac{\lambda(1+\varepsilon)}{\mu}\right]^{N+1}} \ldots (7)$$

4.PERFORMANCE MEASURES:

1. ExpectedSystemSize(L_S)

$L_s = \{\sum_{n=1}^{N} nP_n\}$

$L_s = \left\{\sum_{n=1}^{N} n\left[\frac{\lambda(1+\varepsilon)}{\mu}\right]^n P_0\right\}$

2. Expected queue length (L_q)

$L_q= \{\sum_{n=1}^{N} (n-1)P_n\}$

$L_q = \left\{\sum_{n=1}^{N}(n-1)\left[\frac{\lambda(1+\varepsilon)}{\mu}\right]^n P_0\right\}$

5.NUMERICAL ILLUSTRATION:

Presented bynumericalillustrationoftheabovemodel.

Variationin$L_s and L_q$withrespecttoλ

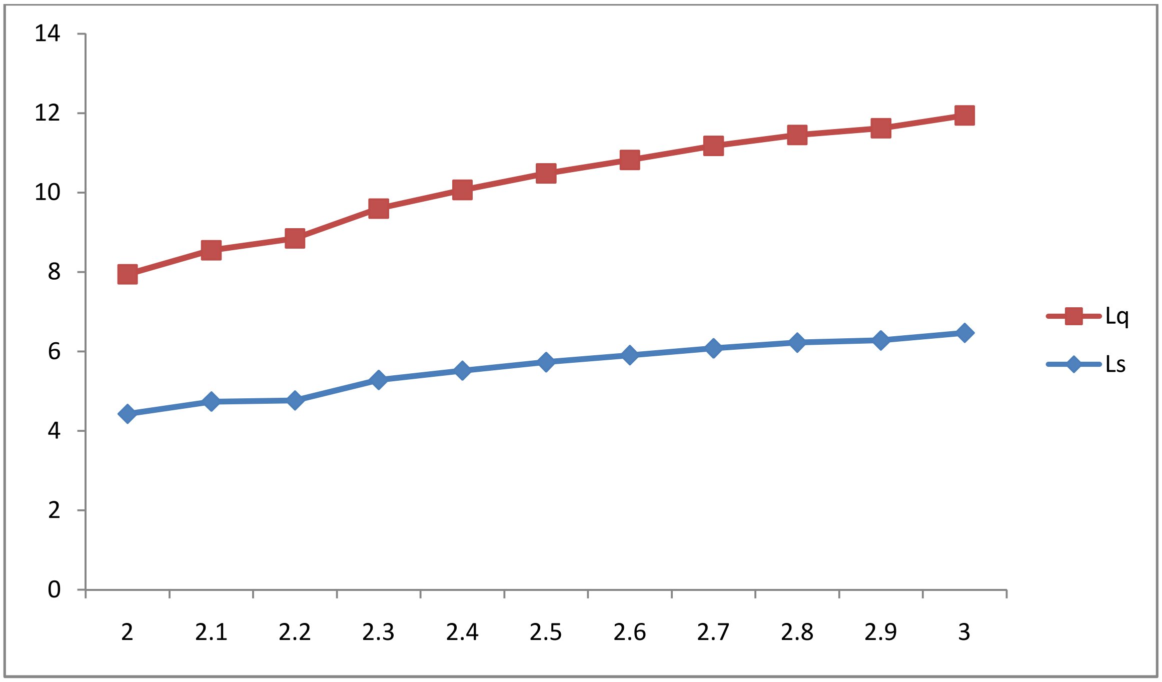
Take value :N=8,μ=3,ε =0.6

Table -1

Averagerateof arrival (λ)	ExpectedSystemSize (Ls)	ExpectedQueueLength (Lq)
2	4.42780	3.51280
2.1	4.73672	3.80560
2.2	4.76672	4.07548
2.3	5.28259	4.311329
2.4	5.51504	4.55036
2.5	5.73050	4.74958
2.6	5.90547	4.91286
2.7	6.07785	5.09507
2.8	6.22276	5.22924
2.9	6.27797	5.34166
3	6.46617	5.47502

Above table show that increasing arrival rate expected system size and length of queue,bothincreases.

Following graph explain the Phenomenon:



6. FINANCIAL ANALYSIS:

Financial analysis of the model is discussed by developing total expected cost, expected revenue and expected profit represented by (TEC, TER & TEP) functions.

Where, $P_0 = \dfrac{1 - \left[\frac{\lambda(1+\varepsilon)}{\mu}\right]}{1 - \left[\frac{\lambda(1+\varepsilon)}{\mu}\right]^{N+1}}$

$$TEC = \left\{ C_s\mu + C_n \sum_{n=1}^{N} n \left[\frac{\lambda(1+\varepsilon)}{\mu}\right]^n P_0 + C_L\lambda \left[\frac{\lambda(1+\varepsilon)}{\mu}\right]^N P_0 \right\}$$

$$TER = R \times \mu \times (1 - P_0)$$

$$TEP = TER - TEC$$

Where,
C_s= Cost per service per unit time
C_h= holding cost per unit per unit time
C_L= Cost associated to each lost unit per unit time
R= Revenue earned per unit per unit time

Table-2

N = 8, $\mu = 3$, $\varepsilon = 0.6$, $C_s = 13$, $C_h = 2$, $C_L = 15$, R = 200

Average rate of arrival (λ)	Total Expected Cost (TEC)	Total Expected Revenue (TER)	Total Expected Profit (TEP)
2	52.1097	549.21	497.101
2.1	53.7443	559.44	505.696
2.2	55.8763	567.66	511.784
2.3	57.1346	574.32	517.186
2.4	58.8498	579.6	520.751
2.5	60.5745	583.8	523.226
2.6	62.2733	587.1	524.827
2.7	63.9597	589.74	525.780
2.8	65.6760	591.78	526.104
2.9	67.2245	593.4	526.176
3	69.0562	594.72	525.664

The above table shows that the total expected profit increases as the mean increases at some level, access reaches a maximum value and begins to decreases. This service fees are fixed, so fees increases after a certain level when the service is overloaded, costs grow faster than revenues.

7. CONCLUSION:

This paper describe single server queuing model with incentivized entry. This M/M/1/N modelis based on expected system size, queue length, probability of having any number of customer in the system, if the system hypothetical vacancy and other relevant operational indicators. Total expected cost, profit and revenue insight of the paper. The model can be implemented to measure the overall performance of the system numerically.
By this model, it allows you to plan effective strategies you can also view the measured financial aspects of your business. The additional optimize of service rate and system size can be realized and system you can read transitive model. The system can also be examined heterogeneously service. The model analysis limited to finite capacity. Further, infinite capacity case of the model can also be studied.

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