

Modelling of Controller Characteristics for a Solar Photo-Voltaic Cell

Anurag Mukherjee*, Abhishri Patil**

*(SENSE, Vellore Institute of Technology, and Vellore.

Email: anurag.mukherjee2019@vitstudent.ac.in)

** (SENSE, Vellore Institute of Technology, and Vellore.

Email: abhishri.patil2019@vitstudent.ac.in)

Abstract:

This paper aims to study the current control part of a voltage source converter interface via the use of a generic proportional and integral controller. The complete system can be modelled as a set of dynamic algebraic equations which can be simultaneously solved using a MATLAB code. Based on this, the control block is modelled in detail. The effect of change in network parameters used to model the circuit and the controller gains is studied on the performance of the time response of the current controller.

Keywords —Circuit stability, voltage-source converters, circuit oscillations, low short circuit systems, electromagnetic simulations, MATLAB, Simulink.

I. INTRODUCTION

Positive sequence models and tools are used to study dynamic simulations. Positive sequence models are the main approach used to evaluate the stability and behaviour of converter interfaced generation system or in interconnection queue. However, these mathematical models have limitations in accurately characterizing their stability under low short circuit grid conditions. These limitations are due to the nature of the mechanism of positive sequence simulations. To have complete observation of the possible oscillatory behaviour, the model is formulated in three ways:

- Representation of the dynamics of the inner current controller loop with the use of generic proportional- integral controller, which will give a reasonably high gain. Further, a small-time constant is used to represent the faster dynamics.
- Introduction of an algebraic network

iteration step between subsequent time steps in order to ensure that the current injection from the voltage source interface is within the specified current limits.

- Introduction of a high value of gain in the phased locked loop circuit, which can cause the converter con- troller to become unstable in a low short circuit situation. Under such conditions, the slightest change in injected currents can cause a large change in the feedback voltage

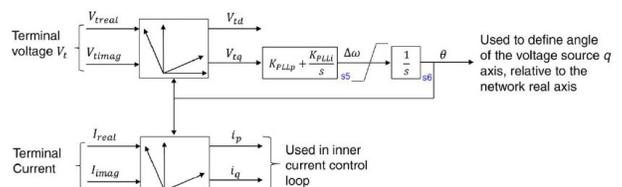


Fig. 1 Positive sequence PLL implementation(e.g. [1])

$$(I_{acmd} - i_a(s))(K_{in} + (K_{li}/s)) = S_o(S)$$

As S_o is a transfer function, taking the inverse Laplace transform on both sides, we get:

$$\mathcal{L}^{-1}(I_{qcmd} - i_q(s))(K_{ip} + (K_{li}/s)) = \mathcal{L}^{-1}S_o(S)$$

$$\mathcal{L}^{-1}(I_{qcmd} - i_q(s)) + \mathcal{L}^{-1}(K_{li}/s)(I_{qcmd} - i_q(s)) = S_o(t)$$

$$(I_{qcmd} - i_q(t))K_{ip} + K_{li} \int (I_{qcmd} - i_q(t)) dt = S_o(t)$$

Differentiating both sides with respect to time t , we get:

$$\frac{dS_o}{dt} = (I_{qcmd} - i_q)K_{li}$$

The intermediate loop current (i_q') is given by,

$$i_q' = S_o + (I_{qcmd} - i_q)K_{ip}$$

where I_{qcmd} the input current command provided as a constants term and i_q is the terminal current used in the inner current control loop which passes through the block S_o as a variable. Now in the second block we have, the product of addition of input current in the block (i.e., $I_{pcmd} - I_q$ and the controller block is S_l . Thus, we get:

$$(I_{pcmd} - i_p(s))(K_{ip} + (K_{li}/s)) = S_1(S)$$

As S_l is a transfer function, taking the inverse Laplace transform on both sides, we get:

$$\mathcal{L}^{-1}(I_{pcmd} - i_p(s))(K_{ip} + (K_{li}/s)) = \mathcal{L}^{-1}S_1(S)$$

$$\mathcal{L}^{-1}(I_{pcmd} - i_p(s)) + \mathcal{L}^{-1}(K_{li}/s)(I_{pcmd} - i_p(s)) = S_1(t)$$

$$(I_{pcmd} - i_p(t))K_{ip} + K_{li} \int (I_{pcmd} - i_p(t)) dt = S_1(t)$$

Differentiating both sides with respect to time t , we get:

$$\frac{dS_1}{dt} = (I_{pcmd} - i_p)K_{li}$$

The intermediate loop current (i_p') at the interconnection of the converter of the circuit is given by:

$$i_p' = S_1 + (I_{pcmd} - i_p)K_{ip}$$

where I_{pcmd} the input current command and provided as constants terms and i_p is the terminal current used in the inner current control loop which passes through the block S_l as a variable.

Also, I_{pcmd} and K_{li} are constants provided as the input current command and the latter one is the one that we get as the inner current loop parameters. The above equations suffice us with differential algebraic system of equation. The total generated voltage of the inverter is obtained through Kirchhoff's Voltage Law (KVL) which is stated as: "In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop." Applying KVL to the load circuit, we get:

$$E_d = V_{td} + i_p.r_e - i_q'.x_e$$

$$E_q = V_{tq} + i_q.r_e + i_p'.x_e$$

Where r_e and x_e are the resistance and inductance paired together in the controller system and V_{td} and V_{tq} are termed as terminal voltage across the controller system. Now, for the S_2 block, the state variable can be defined as a product of E_d and the corresponding controller block.

$$E_d(s) \left[\frac{1}{1 + sT_e} \right] = S_2(s)$$

$$E_d(s) = S_2(s) + s.T_e.S_2(s)$$

Taking inverse Laplace Transform on both sides, we get:

$$\mathcal{L}^{-1}E_d(s) = \mathcal{L}^{-1}S_2(s) + \mathcal{L}^{-1}s.T_e.S_2(s)$$

$$E_d(t) = S_2(t) + T_e \frac{dS_2(t)}{dt}$$

$$\frac{dS_2}{dt} = \frac{1}{T_e} [E_d - S_2]$$

Similarly, for the S_3 block,

$$E_q(s) \left[\frac{1}{1 + sT_e} \right] = S_3(s)$$

$$E_q(s) = S_3(s) + s.T_e.S_3(s)$$

Taking inverse Laplace Transform on both sides, we get:

$$\mathcal{L}^{-1}E_q(s) = \mathcal{L}^{-1}S_3(s) + \mathcal{L}^{-1}s.T_e.S_3(s)$$

$$E_q(t) = S_3(t) + T_e \frac{dS_3(t)}{dt}$$

$$\frac{dS_3}{dt} = \frac{1}{T_e} [E_q - S_3]$$

where T_e is the constant term, we get through inverse Laplace Transform of S_2 and S_3 as well as the other terms as shown previously.

Thus, we have modelled four ordinary differential equations and four algebraic equations as listed below:

$$\frac{dS_o}{dt} = (I_{qcmd} - i_q)K_{li}$$

$$\frac{dS_1}{dt} = (I_{pcmd} - i_p)K_{li}$$

$$\frac{dS_2}{dt} = \frac{1}{T_e} [E_d - S_2]$$

$$\frac{dS_3}{dt} = \frac{1}{T_e} [E_q - S_3]$$

$$i'_q = S_o + (I_{qcmd} - i_q)K_{ip}$$

$$i'_p = S_1 + (I_{pcmd} - i_p)K_{ip}$$

$$E_d = V_{td} + i_p.r_e - i'_q.x_e$$

$$E_q = V_{tq} + i_q.r_e + i'_p.x_e$$

the above set of equations forms a set of Differential Algebraic Equations (DAEs)(e.g. [5], [6].

A differential-algebraic equation (DAE) is an equation involving an ordinary differential equations and algebraic equations. DAEs are a generalization of an ordinary differential equations (ODEs) for which there is a very rich literature for both mathematical theory and numerical solution. While the standard-form ODE can be written as a

DAE, the more general DAE form admits problems that can be quite different from a standard-form ODE. Finding the solution of the above set of DAEs can be done through some numerical approaches. Numerical approaches for the solution of DAEs can be divided into roughly two classes: Direct Discretization of the given system and methods that involve a reformulation i.e., index reduction and Numerical methods which can be used to solve differential algebraic systems including Runge-Kutta methods of a special type.

C. Qualitative analysis of the model

Equilibriums are not always stable. Since stable and unstable equilibrium play quite different roles in the dynamics of a system, it is useful to be able to classify equilibrium points based on their stability. Suppose that we have a set of autonomous ordinary differential equations, written in vector form:

$$\dot{x} = f(x)$$

Suppose that x^* is an equilibrium point. By definition, $f(x^*) = 0$. Now suppose that we take a multivariate Taylor expansion of the right-hand side of our differential equation. We arrive at:

$$x = f(x^*) + \left(\frac{\partial f}{\partial x} \right) (x - x^*) + \dots$$

or

$$x = \left(\frac{\partial f}{\partial x} \right) (x - x^*) + \dots$$

The partial derivative in the above equation is to be interpreted as a Jacobian matrix. In equilibrium, a state system does not always exist because a negative feedback circuit has a system action which is dependent upon some early state and not on the current state. Therefore, the system may assume a resting or an active state. The system causes miss or exceed either before the carrying capacity or before equilibrium target. Therefore, studying equilibrium is important to analyse the behaviour of model. Here we have system of linear ordinary differential equations in positive sequence converter model and therefore first we must linearize this system and find the eigen value of Jacobian matrix at

equilibrium points. To find the equilibrium point of given system of linear equation we equate equations the following equations to zero:

$$\frac{dS_o}{dt} = \frac{(i'_q - S_o) \cdot K_{li}}{K_{ip}} = 0$$

$$\frac{dS_1}{dt} = \frac{(i'_p - S_1) \cdot K_{li}}{K_{ip}} = 0$$

$$\frac{dS_2}{dt} = \frac{1}{T_e} [E_d - S_2] = 0$$

$$\frac{dS_3}{dt} = \frac{1}{T_e} [E_q - S_3] = 0$$

The above four equation give the equilibrium points i'_q, i'_p, E_d and E_q . The Jacobian matrix of this system of linear equations can be represented as:

$$J[S_0, S_1, S_2, S_3] = \begin{bmatrix} \frac{-K_{li}}{K_{ip}} & 0 & 0 & 0 \\ 0 & \frac{-K_{li}}{K_{ip}} & 0 & 0 \\ 0 & 0 & \frac{-1}{T_e} & 0 \\ 0 & 0 & 0 & \frac{-1}{T_e} \end{bmatrix}$$

We can say that since, in this model, as the value of K_{li}, K_{ip} and T_e are positive then all the Eigen values are real, distinct and are less than 0. Thus, the system of linear equations modelled from the positive sequence converter model is asymptotically stable.

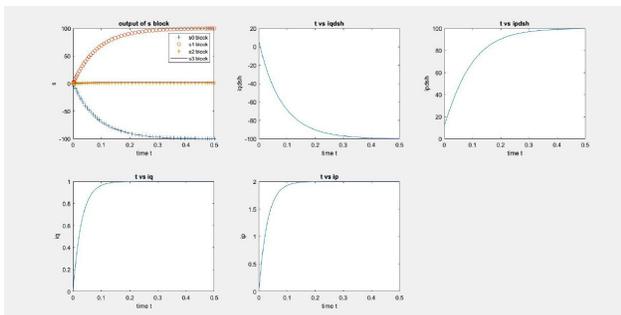


Fig 4. Numerical Solution - I: $x_e=0.001, K_{ip}=6, K_{ip}=70$

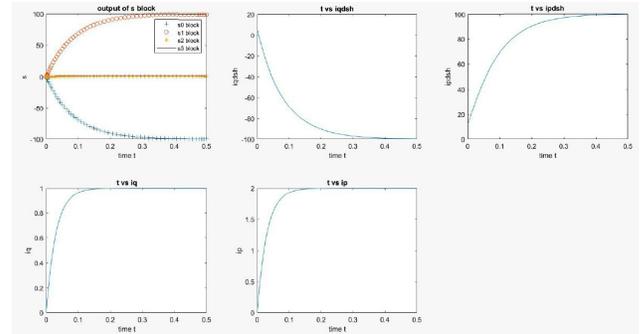


Fig 5. Numerical Solution - II: $x_e=0.001, K_{ip}=20, K_{ip}=120$

III. NUMERICAL SOLUTIONS

MATLAB's standard solver for ordinary differential equations, function ode45, is used to solve for the solutions. The ODE-45 is a sixth stage, fifth order Runge-Kutta method and solves only first order ODE. Thus, we first convert obtained DAE's in ODE format, then convert that into a set of first order ODE's. From the modelled graph in figure 4, we can see that the value of input current becomes stable with time. On increasing the proportional constant K_{ip} , integral constant K_{li} and inductance x_e , the settling times for currents i_p and i_q reduce as shown in figure 5.

IV. SIMULINK DESIGN OF THE MODEL

To verify the results obtained on solving the equations in MATLAB, the entire model was also remade in Simulink.

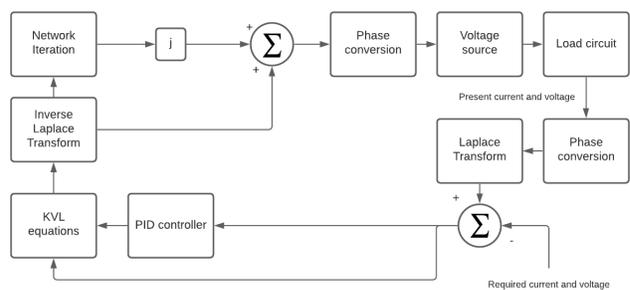


Fig. 6. Derived Block Diagram of the Controller

The Simulink Model was created based on the block diagram derived.

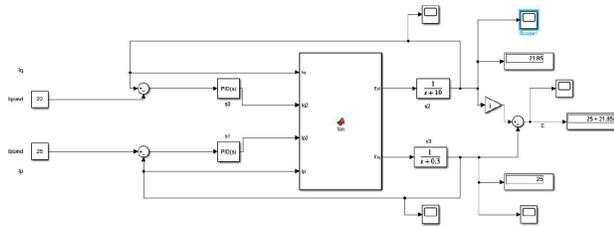


Fig. 7. Simulink Model of the Controller

The comparator gives the difference between the magnitude of required current, and present circuit current. This acts as input to the PID controllers, which outputs the weighted sum of the input signal and the integral of the input signal. On applying KVL equations derived from the load circuit, the MATLAB function, gives the voltage variable E in terms of error current, transfer functions (S_2, S_3) and fed back into the PIC controller, creating a feedback control loop. The input parameters for the load circuit are given by the sum of outputs from S_2 and S_3 . The following graphs are obtained for i_p and i_q , respectively.

As the assumed value of I_{pcmd} is 25, the input current i_p first increases and then stabilizes at 25 (i.e., becomes constant at 25).

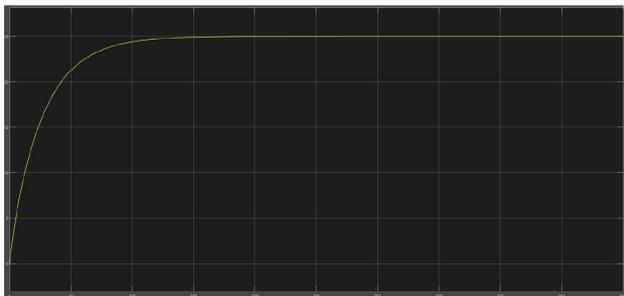


Fig. 8. i_p vs t , $I_{pcmd} = 25$, $x_e = 0.001$, transfer function = $1/s+0.3$

As the assumed value of I_{qcmd} is 22, the input current i_q first increases and then stabilizes at 22 (i.e., becomes constant at 22).

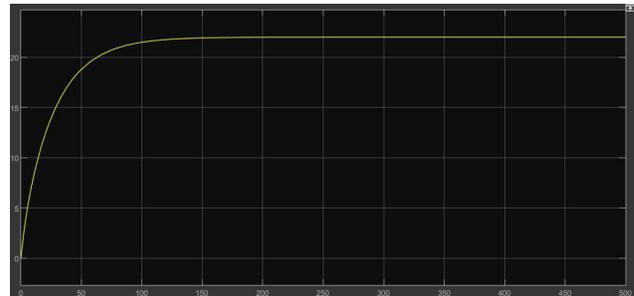


Fig. 9. i_q vs t , $I_{qcmd} = 22$, $x_e = 0.001$, transfer function = $1/s+0.3$

The ideal objective of the circuit is to have i_p and i_q gradually reach the values of I_{pcmd} and I_{qcmd} . However, in real life, i_p and i_q asymptotically reach the limits of I_{pcmd} and I_{qcmd} . We can conclude that the settling time predicted by the MATLAB ODE solves is close to the same output given by the Simulink Block. Similar to our MATLAB proposition, increasing the value of x_e to 0.1 decreases the settling time i.e., the graph will settle faster to the steady condition as can be observed in figures 10 and 11.



Fig. 10. i_p vs t , $x_e = 0.01$, transfer function = $1/s+0.3$



Fig. 11. i_q vs t , $x_e = 0.01$, transfer function = $1/s+0.3$

However, increasing x_e also introduces sudden oscillatory motion in the behaviour of the model. This is more prominent when the denominator of the transfer functions of the S_2 and S_3 blocks approach s as observed in figure 12.



Fig. 12. Increased oscillatory behaviour in i_p vs t

These oscillations can be smoothed over by increasing the value of the constant in the denominator as shown in figure 13.



Fig. 13. Smoothed plot of i_p vs t when denominator is $(s+0.9)$

However, increasing this value too much may result in the settling time also increasing manifolds as demonstrated in figure 14.

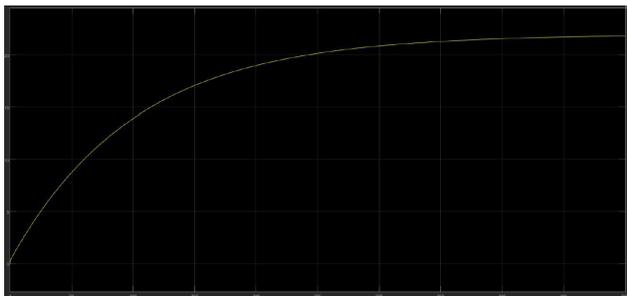


Fig. 14. Increased settling time of i_p vs t with denominator as $(s+10)$

Thus, a balance must be struck between the values of x_e and the transfer function to mediate between unwanted oscillations and settling time. The rate of the current settling down and reaching its limiting value also depends on load circuit parameters such as inductance and the control loop variables such as the proportionality constant K , which defines PID controller behaviour.

The following study was done to check the PID outputs and E_d behaviours with different input current values. We went ahead with running through a fixed array of inputs at uniform intervals. Having assumed V as 100, x_e and r_e as 10, and I_{cmd} as 25, we ran two simulations ranging from I , being 20 to 30 at intervals of 1 and then 24 to 26 at intervals of 0.2. For simplicity's sake, we considered I_p and I_q to be equal. Assuming, $I_{cmd} = 25$, $r_e = 10$, $x_e = 10$, $V = 100$, we get

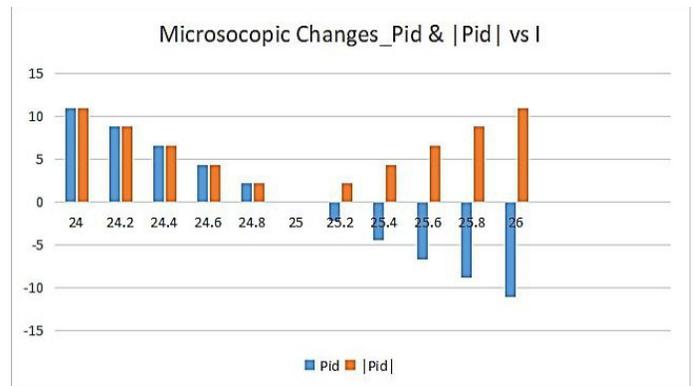


Fig. 15. PID output vs I

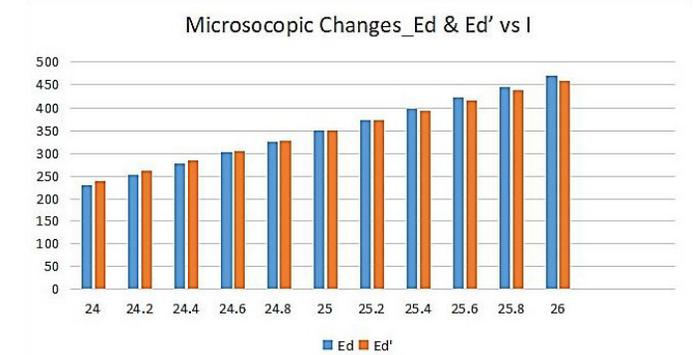


Fig. 16. E_d vs I

As was expected, the output from the PID controller is symmetrical, and their magnitudes form an inverse bell curve around the central minima of 25. E_d forms an unsymmetrical curve, with the left-hand side of the local minima being on an average lower than the right.

V. CONCLUSIONS

As given in the aim, we have designed and simulated a controller for a voltage source converter interface and analysed the time-lapse generated due to the input current and the required current, by implementing the mathematical model in a Simulink block. Furthermore, we have analysed behaviour of the model, and its response to varied parameters.

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REFERENCES

- [1] Deepak Ramasubramanian, Wenzong Wang, PouyanPourbeik, EvangelosFarantatos, AnishGaikwad, SachinSoni, Vladimir Chadliev, Positive sequence voltage source converter mathematical model for use in low short circuit systems. IET Generation, Transmission andDistribution Vol. 14 (1), pp. 87-97
- [2] Ramasubramaniam, D., Yu, Z., Ayyannar, R., et al.: 'Converter model for representingconverter interface generation in large-scale grid simulations', IEEE Trans. Power syst., 2017,32(1), pp.765-773
- [3] Peter Kunkel and Volker Mehrmann : Differential-algebraic Equations: analysis and numerical solution, European Mathematical Society, 2006
- [4] Close, C.M., Frederick, D.K., & Newell, J.C., Modeling and Analysis of Dynamic Systems, Wiley, 2001
- [5] Charles K. Alexander, Matthew N. O. Sadiku: Fundamentals of Electrical Circuit, McGrawHill, 2019
- [6] Banerjee, Dean (2006), PLL Performance, Simulation and Design Handbook (4th ed.), National Semiconductor, archived from the original on 2 December 2012, retrieved 4 December 2012
- [7] T. Jing and A. S. Maklakov, "A Review of Voltage Source Converters for Energy Applications," 2018 International Ural Conference on Green Energy (UralCon), 2018, pp. 275- 281