RESEARCH ARTICLE OPEN ACCESS

Prominence of Combinatorics and Graph Theory in the Advancement of Science and Technology

V.N.Srinivasa Rao Repalle*, Andualem Benti**, Abebe Legesse Tera***, I.V.Venkateswara Rao****

*(Associate Professor,Department of Mathematics,Wollega University,Ethiopia
Email: rvnrepallef@gmail.com)
** (Lecturer,Department of Mathematics, Wollega University, Ethiopia
Email: danu.jara@gmail.com)
***(Lecturer,Department of Mathematics, Wollega University, Ethiopia
Email: *legesseabebe430@gmail.com*)
****(Senior Assistant Professor of Mathematics, P.B. Siddhartha College of Arts and Science, Vijayawada, India
Email: ivvrao@pbsiddhartha.ac.in)

--************************************----------------------------------

Abstract:

Now a days the world is running behind the technology especially with in the area of computing and information technology. Graph theory is delightful playground for solving such problems of science and technology. A graph can be used to represent almost any physical situation involving discrete objects and a relationship among them. This article presents how the mathematics is back bone to these recent advanced technologies.Because of its inherent simplicity, combinatorics and graph theory have a very wide range of applications in engineering and technology. This article presents the problems of science, technology and socially related like social media problems, air traffic problems, job sequencing problems.

Keywords: Graph Theory, Combinatorics, Traffic flow Problems, Tournaments, Job sequencing.

AMS Subject Classification2010: 05Cxx, 05C20, 05C15, 00A06, 62P30.

--************************************----------------------------------

1. INTRODUCTION

Graph theory is a delightful playground for the exploration of proof techniques in discrete mathematics, and its results have applications in many areas of the computing, social and natural sciences. Euler (1707-1782) became the Father of Graph Theory. Graph theory is considered to have begun in 1736 with the publication of Euler's solution to the Konigsberg bridge problem. After this *Francis Guthrie (1890)* postulated the celebrated four-color conjecture came into prominence and has been flawed ever since.

A model may help to explain a system and to study the effects of different components, and to make predictions about behavior. A mathematical model is a description of a system using mathematical concepts and language. Mathematical models are used in the natural sciences and engineering disciplines as well as in the social sciences.

It is no coincidence that graph theory has been independently discovered many times, since it may quite properly be regarded as an area of applied mathematics.Indeed, the earliest recorded mention of the subject occurs in the works of Euler, and although the original problem he was considering might be regarded as a somewhat playful puzzle, it did arise from the physical world. Subsequent rediscoveries of graph theory by Kirchhoff and Cayley also had their roots in the physical world. Kirchhoff s investigations of electric networks led to his development of the basic concepts and theorems concerning trees in graphs, while Cayley considered trees arising from the enumeration of organic chemical isomers. Another puzzle approach to graphs was proposed by Hamilton. After this, the celebrated four-color conjecture came into prominence and has been notorious ever since. In the present century, there have already been a great many rediscoveries of graph theory. In recent days, VN SrinivasaRaoReaplle et.al defined, a new concept of fuzzy chromatic polynomial which led to the application of traffic light problems at busy junctions [7, 9, 10]

In this article we have given basic definitions that are crucial part of graph theory in preliminaries. These definitions are very easy to understand and provide clear idea of different types of graphs. All the necessary terminologies of graph theory are covered by these definitions. Later various mathematical models of graph theory have been identified and divided as per their fields like Engineering and Technology, Science and real-world applications.

2. PRELIMANIRIES

In this article we attain the all basic definitions from Johan Clark and D. A. Holton [1], Harary.F [2] Narasingh Deo [3], and West.D.B [8].Here all the graphs are finite and simple unless otherwise mentioned.

Definition2.1.A graph G = (V, E) consists of a set of objects V == {v_1,v_2,…..v_n} called vertices, and another set E ={e_1,e_2,……e_n}, whose elements are called edges, such that each edge e_k is identified with an unordered pair (v_i,v_j) of vertices. The vertices v_i,v_j associated with edge e_k are called the end vertices of e_k. The most common representation of a graph is by means of a diagram, in which the vertices are represented as points and each edge as a line segment joining its end vertices. Often this diagram itself is referred to as the graph.

Example 2.1. The following fig1 is an example a graph

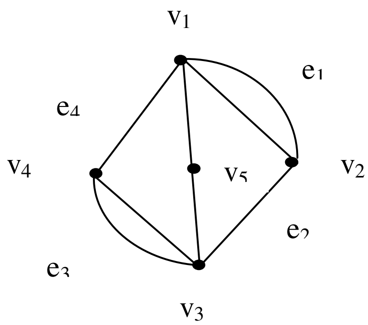

Fig.1: A graph with 5 vertices and 8 edges

Definition 2.2**.** Two vertices u, v in a graph G =(V,E) are said to be adjacent if they are join an edge $\overline{uv}$.

Definition 2.3. An edge 'e' of a graph G is said to be incident with the vertex v if v is an end vertex of e.

Definition 2.4. A graph G = (V, E) is said to be k-regular if all the vertices of G are of degree k , where k is a positive integer.

Definition 2.5. A complete graph is a simple graph in which each pair of distinct vertices is joined by an edge .A complete graph on n vertices is denoted by K_n.

Definition 2.6. Let G be a graph .If the vertex set V of G can be partitioned in to two non-empty disjoint subsets X and Y in such a way that each edge of G has one end vertex in x and another vertex in Y then G is called bipartite graph. The partition V=X $\cup$ Y is called a bipartition of G.

Definition 2.7. A complete bipartite graph is a simple bipartite graph with bipartite V=X $\cup$ Y, in which every vertex in X joined to every vertex of Y. If X has m vertices and Y has n vertices then the complete bipartite graph denoted by $K_{m,n}$.

Example 2.2. Fig2 represents a complete bipartite graph on 6 vertices

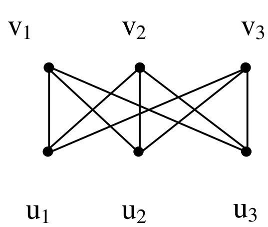

Fig 2. Complete Bipartite Graph $K_{3, 3}$.

Definition 2.8. An alternating sequence of vertices and edges $v_0 e_1 v_1 e_2 v_2 ......v_{n-1} e_n v_n$ such that the end vertices of an edge e_i are v_{i-1} and v_i for i=1, 2, 3...n is called a walk.

Definition 2.9. A trail is a walk $v_0 e_1 v_1 e_2 v_2 ......v_{n-1} e_n v_n$ whose edges are distinct.

Definition 2.10. A path is a trail $v_0 e_1 v_1 e_2 v_2 ......v_{n-1} e_n v_n$ with distinct vertices.

Definition 2.11.A graph G is said to be connected if there is a path between every pair of vertices or else it contains a single vertex. Otherwise it is called disconnected graph

Definition: 2.12. A weighted graph is a graph in which each edge *e* has been assigned a real number w(*e*), called the weight of *e*.

Definition 2.13. A connected graph without cycles is called a tree. A tree with n vertices denoted by T_n .

Definition 2.14. A spanning tree of a graph G is a sub graph of G which contains all the vertices of G that is a tree.

Definition 2.15. A graph G is said to be planar if it can be drawn on plane without intersecting its edges, otherwise G is said to be non-planar.

Definition 2.16. A coloring of a graph G issuch that the adjacent vertices have different colors is called a proper coloringof the graph.

Definition 2.17.A graph G is called k-colorable if we can assign one of k colors to each vertex to achieve a proper coloring.

Definition 2.18.A graph G is said to have chromatic number k, if G is k-colorable but not (k-1)-colorable and is denoted by $\chi(G)$=k.

Example 2.3. Fig 3 represents a graph with label distinct colors to adjacent vertices. That is $\chi(G) = 3$.

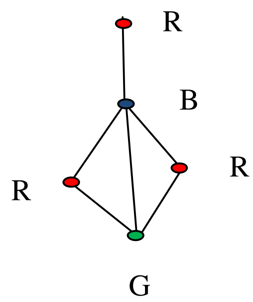

Fig 3.A graph with chromatic number three

Definition 2.19. The minimum number k for which there is a k –edge coloring of G is called the edge chromatic number of G and is denoted by χ'(G).It is also called as chromatic index.
Definition 2.20. A closed walk in a graph contains all the edges of the graph, and then the walk is called an Euler tour and the graph an Euler graph.
Definition 2.21.A Hamiltonian circuit in a connected graph is defined as a closed walk that traverses every vertex of G exactly once, except of course the starting vertex, at which the walk also terminates.
Definition 2.22. A graph is connected if every pair of points is joined by a path.
Definition 2.23.A directed graph G = (V, A) consists of two finite sets V, the vertex set, a nonempty set of elements called the vertices of G and A, the arc set, a (possibly empty) set of elements called the arcs of G, such that each arc *a* in A is assigned an ordered pair of vertices (u, v).
Definition 2.24. A digraph G is said to be strongly connected if for any pair of vertices *u* and *v* in G there is a directed path from *u* to *v*.
Definition 2.25. A graph G is called orientable if it has a strongly connected orientation.
Definition 2.26. A tournament is a digraph with no loops in which any two distinct vertices are joined by exactly one arc. It is the orientation of complete graph.
Definition 2.27. The out degree of *v* is the number of arcs of G that have *v* as its tail.
Definition 2.28. A directed Hamiltonian cycle in a digraph G is a directed cycle which includes every vertex of G. If G contains such a cycle then G is called Hamiltonian.
Result 2.1. [Four Colour Conjecture]: Every planar graph can be properly colored four colors.

3. GRAPH THEORY MODELS of ENGINEERING and TECHNOLOGY

Graphs are considered as an excellent modeling tool which is used to model many types of relations amongst any physical situation. This section explores different concepts involved in graph theory models in engineering and technology. These applications are presented especially to project the model of graph theory and to demonstrate its objective and importance in computer science and engineering

3.1 Civil Engineering

The Konigsberg bridge problem is perhaps the best-known example in graph theory. It was a long-standing problem until solved by Leonhard Euler in 1736, by means of a graph. Euler wrote the first paper ever in graph theory and thus became the originator of the theory of graphs as well as of the rest of topology. The problem is depicted in fig 4

Two islands, C and D, formed by the Pregel River in Konigsberg were connected to each other and to the banks A and B with seven bridges, as shown in Fig. 3.1. The problem was to start at any of the four land areas of the city, A, B, C, or D, walks over each of the seven bridges exactly once, and returns to the starting point. Euler represented this situation by means of a graph, as shown in FIG5. The vertices represent the land areas and the edges represent the bridges. The Konigsberg bridge problem is the same as the problem of drawing FIGs without lifting the pen from the paper and without retracing a line. We all have been confronted with such problems at one time or another.

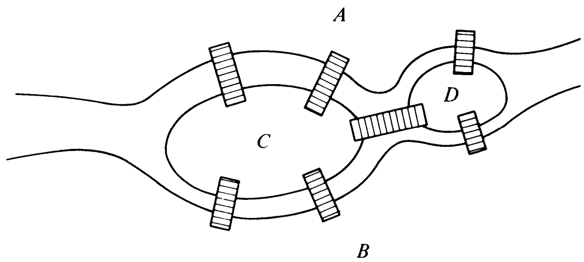

Fig.4: Konigsberg bridge problem.

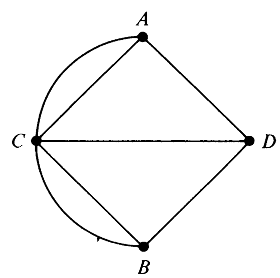

Fig 5: Graph of Konigsberg bridge problem.

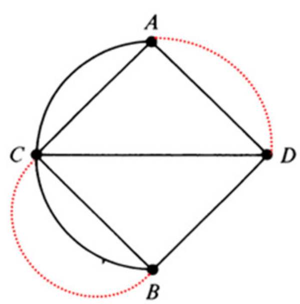

Fig6: Solution to the Konigsberg bridge problem.

Theorem: 3.1[3].A given connected graph G is an Euler graph if and only if all vertices of G are of even degree.

The solution for the Konigsberg bridge problem is getting an Euler graph. From theorem we know that the graph became Euler all the vertices are of even degree. But from the FIG 5 all the vertices are not of even degree.We want to make all the vertices are of even degree .It is possible by adding new edges at alternative set of vertices as shown in the FIG 6 which gives the graph of Konigsberg bridge became an Euler graph. Hence after so many years later by Euler solution they constructed new bridges gives the solution to the Konigsberg bridge problem.

3.2. Computer Science:

3.2.1. Image processing

Image Analysis is the methodology by which information from images is extracted. Image analysis is mainly performed on digital image processing techniques. The image processing techniques can be improved using a graph theoretic approach. The model of graphs in image processing is: to find edge boundaries using graph search algorithms in segmentation.

To calculate the alignment of the picture

- Finding mathematical constraints such as entropy by using minimum spanning tree.
- Finding distance transforms of the pixels and calculates the distance between the interior pixels by using shortest path algorithms.

3.2.2. Operating system

A graph is a data structure of finite set of pairs, called edges or vertices. Many practical problems can be solved with the help of graph in the field of operating system such as job scheduling and resource allocation problems. For example graph coloring concept can be applied in job scheduling problems of CPU, jobs are assumed as vertices of the graph and there will be an edge between two jobs that cannot be executed simultaneously and there will be one to one relationship between feasible scheduling of graphs.

3.2.3. Website designing

Website designing can be modeled as a graph, where the web pages are represented by vertices and the hyper links between them are represented by edges in the graph. This concept is known as web graph. Further functional areas of graphs are in web community are the vertices represent classes of objects, and each vertex representing one type of objects, and each vertex representing a type of object is connected to every vertex representing other kind of objects. In graph theory such a graph is called a complete bipartite graph. There are many advantages of using graph representation in website development.

4. SCIENCE

Graph Models placed a vital role in science too.

4.1. Physics-Network System

Graph theory has wide application in the field of networking. To analyze the graph theory application in networking two areas are considered: graph based representation and network theory. Graph based representation has many advantages such as it gives different point of view; it makes problem much easier and provide more accurate definition. Whereas network theory provide a set of techniques for analyzing a graph and applying network theory using a graph representation. The term graph and network are equal. Both refer to a type of structure in which there exists vertices (i.e. nodes, dots) and edges (i.e. links, lines). There are numerous types of graphs and networks which yield more or less structure. These twoterms can be differentiating on the basis of their utility. The term graph is used in mathematics whereas the term network is used in physics.

4.1.1. Purpose of a Network in physics

Physicists are concerned with modeling real-world structures with networks. Physicist's define algorithms that compress the information in a network to more simple values Graph theoretic concepts are applied in several fundamental issues in network such as connectivity, data gathering, routing, mobility, energy efficiency, topology control, traffic analysis, finding shortest path and load balancing.

4.2. Graph Theory in Operations Research

Graph theory is a very natural and powerful tool in combinatorial operations research.A graph can be used as a model for a network of pipelines through which some commodity is transported from one place to another. The general problem in such a transport network is to maximize the flow or minimize the cost of a prescribed flow. This is an operations-research problem and can be solved by linear programming, but the graph theoretic approach has been found to be computationally more efficient by using Maximum Flow Minimum Cut theorem.

4.3. Chemistry

Graphs are used to model molecule structures for computer processing. Here atoms can be considered as vertices of a graph the bonds that connects them are represented as edges between them. This structures are created based on the properties of compounds and are taken for analysis and processing. This can be used to study the structure of molecules and to check similarity level between molecules.

5. REAL WORLD PROBLEMS

5.1. GPS Navigation Systems

Navigation systems such as the Google Maps, which can give directions to reach from one place to another use shortest path algorithms. They take your location to be the source node and your destination as the destination node on the graph. A city can be represented as a graph by taking landmarks as nodes and the roads as the edges that connect the nodes in the graph. Using BFS/DFS algorithms shortest route is generated which is used to give directions for real time navigation.

5.2. Face Book

It treats each user profile as a node on the graph and two nodes are said to be connected if they are each other's friends. In fact, apply BFS on the face book graph and you will find that any two people are connected with each other by at most five nodes in between. To say, that you can reach any random person in the world by traversing 6 nodes.

5.3. Dodecahedron Problem

A game invented by Sir William Hamilton in 1859 uses a regular solid dodecahedron whose 20 vertices are labeled with the names of famous cities. The player is challenged to travel "around the world" by finding a closed circuit along the edges which passes through each vertex exactly once In graphical terms, the object of the game is to find a spanning cycle in the graph of the dodecahedron, shown in FIG 7. The points of the graph are marked 1,2, • • •, 20 so that the existence of a spanning cycle is evident.

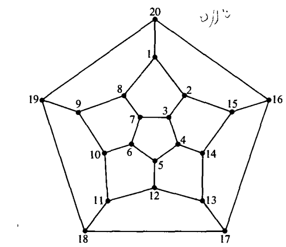

Fig7.Around the World.

5.4. Seating Problem

Nine members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbors at each lunch. How many days can this arrangement last?

This situation can be represented by a graph with nine vertices such that each vertex represents a member, and an edge joining two vertices represents the relationship of sitting next to each otheras in FIG 8 shows two possible seating arrangement are I 2 3 4 5 6 7 8 9 I (solid lines), and 1 3 5 2 7 4 9 6 8 1 (dashed lines). It can be shown by graph-theoretic considerations that there are only two more arrangements possible. They are I 5 7 3 9 2 8 4 6 I and 1 7 95 8 3 6 2 4 1. In general it can be shown that for n people the number of such possible arrangements is (n-1)/2 if n is odd, and (n-2)/2 if n is even.

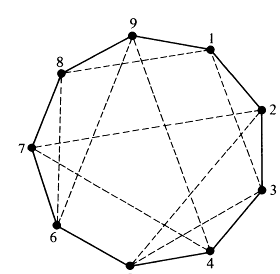

Fig 8: Arrangements at a dinner table.

5.5. Travelling-Salesman Problem

A salesman is required to visit a number of cities during a trip. Given the distances between the cities, in what order should he travel so as to visit every city precisely once and return home, with the minimum mileage traveled?

In our problem, if each of the cities has a road to every other city, we have a complete weighted graph. This graph has numerous Hamiltonian circuits, and we are to pick the one that has the smallest sum of distances (weights). The total number of different Hamiltonian circuits in a complete graph of n vertices can be shown to be (n - 1)!/2. This follows from the fact that starting from any vertex we have (n – 1) edges to choose from the first vertex, (n – 2) from the second, (n – 3) from the third, and so on. These being independent choices, we get (n - 1)! possible number of choices. This number is, however, divided by 2, because each Hamiltonian circuit has been counted twice. Theoretically, the problem of the traveling salesman can always be solved by enumerating all (n - 1)!/2 Hamiltonian circuits, calculating the distance traveled in each, and then picking the shortest one.

5.6. The Chinese Postman Problem

Before starting on his delivery route, a postman must pick up his letters at the post office, then he must deliver letters along each street on his route, and finally he must return to the post office to return all undelivered letters.

We construct a weighted graph G where each edge represents a street in the postman's route, each vertex represents a junction of streets and the weight assigned to each edge represents the length of the street between junctions. The solution to the Chinese postman problem is to identify the Euler tour with possible least weight

5.7. Tournaments

How we can model round-robin tournaments in any game in which draws are not allowed (Such as Tennis).

In round-robin tournaments, we use directed graph as mathematical model to record the results of the game. Consider the players as nodes and join two players *a* and *b* by an arc from *a* to *b* if *a* has beaten *b*. In this digraph the vertex representing a player has maximum out degree won the match. Here we consider an example with five players *v, w, x, y* and *z*. There round-robin tournament results are given by the following digraph, FIG 9

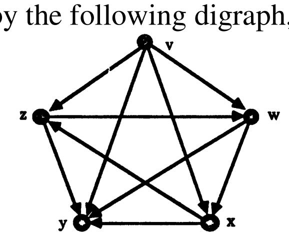

Fig 9: A tournament of five players

Here *v* has beaten every other competitor; *y* has lost each match, each of the other competitors have won two and lost two matches. The vertex *v* has maximum out degree 4. Every other vertex can be reached from v by a directed path. Hence *v* won the tournament.

5.8. Job sequencing Problem

If *n* jobs are assigned to a machine, how to minimize the total machine adjustment time in a job sequencing problem.

The solution for the Job sequencing problem is construct the digraph and obtain the directed Hamiltonian path with least possible units of adjustment time. Let $j_1, j_2, \ldots\ldots j_n$ be the *n* jobs have to be processed on one machine. After each job, the machine must be adjusted to fit to fit the requirements of the next job. Let t_{ij} be the time of adaption from job j_i to job j_j. Now we have to find a sequencing of the jobs that minimize the total machine adjustment time. By using Redei's Theorem [4]

Step: 1 Construct a digraph G with vertices $v_1, v_2, \ldots\ldots v_n$ such that $(v_i, v_j) \epsilon A$ iff $t_{ij} \leq t_{ji}$. By definition G contains a spanning tournament.

Step2: Find a directed Hamiltonian path of G and sequence the jobs accordingly.

As an example, suppose that six jobs J_1, J_2, J_3, J_4, J_5 and J_6 and that time adjustment matrix is given by the fig 10

	J_1	J_2	J_3	J_4	J_5	J_6
J_1	0	5	3	4	2	1
J_2	1	0	1	2	3	2
J_3	2	5	0	1	2	3
J_4	1	4	4	0	1	2
J_5	1	3	4	5	0	5
J_6	4	4	2	3	1	0

Fig10: Time adjustment matrix

The sequence $J_1 \rightarrow J_2 \rightarrow J_3 \rightarrow J_4 \rightarrow J_5 \rightarrow J_6$ requires 13 units of adjustment time. To find the better sequence, construct the digraph G as in step1 which shows in fig 11

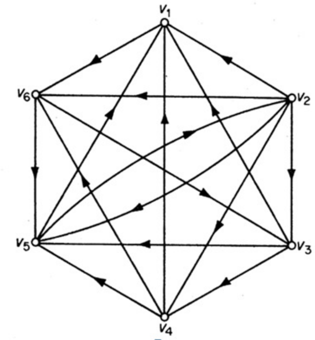

Fig 11: Digraph of job sequencing

$(v_1, v_6, v_3, v_4, v_5, v_2)$ is a directed Hamiltonian pathof G and yields the sequence $J_1 \rightarrow J_6 \rightarrow J_3 \rightarrow J_4 \rightarrow J_5 \rightarrow J_2$ which requires only eight units of adjustment time.

Hence the solution for the Job sequencing problem is construct the digraph and obtain the directed Hamiltonian path with least possible units of adjustment time.

5.9. Examination Scheduling

How to find an examination schedule for the students without conflict their courses.

This examination schedule problems can be modeled by graph taking each course as a vertex and connect the vertices by an edge if the student taking both the courses. Here we can use vertex coloring as a mathematical modeling of graph theory to solve the conflicts.

The graph in fig12 represents the exam scheduling problem. Each vertex stands for a course. An edge between two vertices indicates that a student is taking both courses and therefore the exams cannot be scheduled at the same time. Each exam time slot is associated with a color. A schedule that creates no conflicts for any student corresponds to a coloring of the vertices such that no adjacent vertices receive the same color.

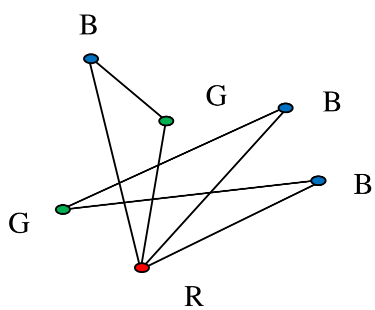

Fig 12: Examination Schedule Problem

5.10. Traffic Flow

In many cities, traffic jams are a big problem. There have been many attempts to try to improve road design and traffic flow using detailed empirical measurements of traffic flow, analysis of traffic accidents, and redesign of accident "black spots", revision of traffic laws and methods of enforcement.

One-way street assignment is often used by cities to help alleviate traffic flow problems. Given a street map of a city one may ask whether or not it is possible to make each street on the map a one-way street in such a way that one can still drive from many part of the city to any other part obeying the one-way rules. We may rephrase this question using a graph and an associated digraph as follows.

First construct a graph G in which each vertex represents a street intersection. Join two vertices x and y of G by an edge if it is possible to travel between x and y without passing through any other intersection. If it is possible to make such a trip between x and y in several ways then there should be an edge corresponding to each of these ways. The resulting graph G gives us, in effect, a street map of the city. Latter we assign direction to each edge of G it becomes a directed graph.

The solution for the traffic flow problem is we have to find a directed graph which is strongly connected orientable such that it may have no bridges. That is, we can find such a digraph D in which every vertex x is reachable from every other vertex y.

5.11. Air Fares

An airlines company has branches in each of six cities $C_1, C_2, \ldots, C_6$. The airfare for a direct flight from C_i to C_j is given by the $(i,j)^{th}$ entry of the following matrix (where ∞ indicates that there is no direct flight.

$$\begin{bmatrix} 0 & 50 & \infty & 40 & 25 & 10 \\ 50 & 0 & 15 & 20 & \infty & 25 \\ \infty & 15 & 0 & 10 & 20 & \infty \\ 40 & 20 & 10 & 0 & 10 & 25 \\ 25 & \infty & 20 & 10 & 0 & 55 \\ 10 & 25 & \infty & 25 & 55 & 0 \end{bmatrix}$$

Fig13: Matrix

The company is interested in computing a table of cheapest fares between pairs of cities. (Even if there is a direct flight between two cities this may not be the cheapest route.) We can first represent the situation by a weighted graph, attached to the edges according to the airfares, as in fig 14.

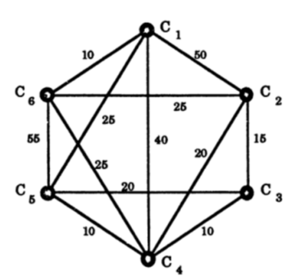

Fig14 The weighted graph of airfares for direct flights between six cities.

The weighted graph of airfares for direct flights between six cities. The problem can then be solved by determining shortest path by using either BFS/DFS algorithm.

5.12. Mobile Network

The mathematical modeling of mobile networking problem is to construct a graph with regions as nodes colored with four colors by using four color conjectures and if two nodes are connected by a line if they can't be the same color .This is known as node coloring algorithm. The algorithm can be used to efficiently plan towers and channels in a mobile network, a very popular method in use by mobile service providers today. As can be seen in the map below, borders wander making it a difficult problem to analyze a map. Instead of using a sophisticated map with many wandering boundaries, it becomes a simpler problem if we use node coloring algorithm. Wireless Service providers employ node coloring to make an extremely complex network map much more manageable.

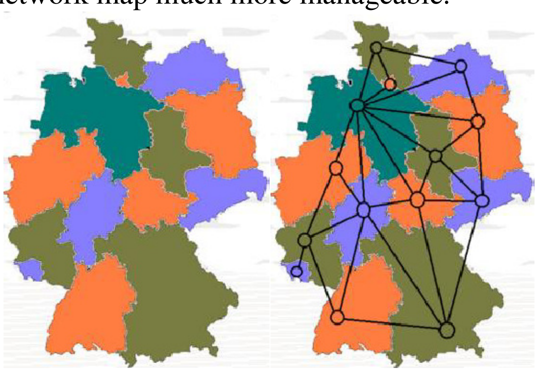

Fig 15(a) A Map with complex wandering boundaries

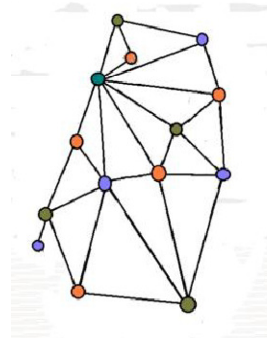

Fig 15(b). The simplified network version of the map derived by node coloring

6. CONCLUSION

This article is deliberate to benefit the students of mathematics and computer science to gain depth knowledge on graph theory and its significance with other areas like Science and real world problems. This article focused on the various mathematical models of Engineering and Technology, science as well as real world applications by using graph theory as a mathematical model. Each model can evidently explore the construction of graph which helps to recognize the application and further development of the model leads to future expansion of research using graph theory as a model and may help to the researchers.

7. REFERENCES

[1] Johan Clark and D. A. Holton, "First look at Graph Theory", *world scientific publishing company USA,* 1991.

[2] Harary F, "Graph Theory", *Addison-Wesley Publishing Company, USA*, 1972.

[3] Narasingh Deo, "Graph theory with applications to Engineering and Computer Science", *Printice Hall of India*, 2001.

[4] M. K. Kuan, "Graphic programming using odd or even points". *Chinese Math.*, pp.273-277, 1962.

[5] R.V.N.SrinivasaRao and J.V.Rao, "Contribution to 1-Quasi total graphs", *Asian journal of Mathematics and statistics*, vol.6 (1), pp.52-56, 2013.

[6] S.G.Shirinivas et.al, "Applications of graph theory in computer science an overview", *International Journal of Engineering Science and Technology*, Vol. 2(9), pp.4610-4621, 2010.

[7] V.N. Srinivasa Rao Repalle and Mamo Abebe, "Fuzzy Chromatic Polynomial of Fuzzy Graphs with Crisp and Fuzzy Vertices Using α-Cuts", Advances *in Fuzzy systems*, Hindwai, Article ID5213020, 11 pages, 2019. https://doi.org/10.1155/2019/5213020s

[8] West.D.B, "Introduction to Graph Theory 2nd Edition" *Pearson Education*, 2002.

[9] V.N. Srinivasa Rao Repalle and Mamo Abebe, "Strong Fuzzy Chromatic Polynomial (SFCP) of Fuzzy Graphs and Some Fuzzy Graph Structures with Applications",*Pure and Applied Mathematics Journal*,Volume9(1),pp.16-25,2020, doi: 10.11648/j.pamj.20200901.13
[10] V.N. Srinivasa Rao Repalle et.al, "Algebraic Properties of Fuzzy Chromatic Polynomials", *IOSR Journal of Mathematics ,Volume 15, Issue 6 Ser. VI , PP 49-52,2019.* doi: 10.9790/5728-1506064952

