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(1,2)*-Ÿ-CLOSED SETS IN BITOPOLOGICAL SPACES

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Abstract:

In this paper, we introduce a new class of sets namely (1,2)*-Ÿ-closed sets in bitopological spaces. This class lies between the class of$\tau_{1,2}$-closed sets and the class of (1,2)*-g-closed sets.

Key words and Phrases: **(1,2)*-Ÿ-closed set, (1,2)*-Ÿ-open set.**

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I. INTRODUCTION

Levine [7] also introduced the notion of g-closed sets and investigated its fundamental properties. This notion was shown to be productive and very useful. Kelly [6] introduced the concepts of bitopological spaces.Recently, Bhattacharya and Lahiri [4], Arya and Nour [3], Sheik John [17] and Rajamani and Viswanathan [9]introduced sg-closed sets, gs-closed sets, ω-closed sets andαgs-closed sets respectively.

In this paper, we introduce a new class of sets namely (1,2)*-Ÿ-closed sets inbitopological spaces. This class lies between the class of $\tau_{1,2}$-closed sets and the class of (1,2)*-g-closed sets.

II.PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2) (briefly, X) will denote bitopological space (briefly, BTPS).

Definition 2.1 Let H be a subset of X. Then H is said to be $\tau_{1,2}$-open [12] if $H = P \cup Q$ where $P \in \tau_1$ and $Q \in \tau_2$.

The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.

Notice that$\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 2.2 [12]Let H be a subset of a bitopological space X. Then

(i) the $\tau_{1,2}$-closure of H, denoted by $\tau_{1,2}$-cl(H), is defined as $\cap$ {F :H$\subseteq$ F and F is $\tau_{1,2}$-closed}.

(ii) the $\tau_{1,2}$-interior of H, denoted by $\tau_{1,2}$-int(H), is defined as $\cup$ {F : F $\subseteq$H and F is $\tau_{1,2}$-open}.

Definition 2.3A subset H of a BTPS X is called:
(i) (1,2)*-semi-open set [11] if H$\subseteq\tau_{1,2}$-cl($\tau_{1,2}$-int(H));
(ii) (1,2)*-preopen set [11] if H$\subseteq\tau_{1,2}$-int($\tau_{1,2}$-cl(H));
(iii) (1,2)*-α-open set [8] if H$\subseteq\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(H)));
(iv) (1,2)*-β-open set [13] (= (1,2)*-semi-preopen [13]) if H $\subseteq\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(H)));
(v) regular (1,2)*-open set [11] if H = $\tau_{1,2}$-int($\tau_{1,2}$-cl(H)).

The complements of the abovementionedopen sets are called their respective closed sets.

The (1,2)*-preclosure [15] (resp. (1,2)*-semi-closure [11], (1,2)*-α-closure [8], (1,2)*-semi-pre-closure [11]) of a subset H of X, denoted by (1,2)*-pcl(H) (resp. (1,2)*-scl(H), (1,2)*-αcl(H), (1,2)*-spcl(H)) is defined to be the intersection of all (1,2)*-preclosed (resp. (1,2)*-semi-closed, (1,2)*-α-closed, (1,2)*-semi-preclosed) sets of X containing H. It is known that (1,2)*-pcl(H) (resp. (1,2)*-scl(H), (1,2)*-αcl(H), (1,2)*-spcl(H)) is a (1,2)*-preclosed (resp. (1,2)*-semi-closed, (1,2)*-α-closed, (1,2)*-semi-preclosed) set. For any subset H of an arbitrarily chosen bitopological space, the (1,2)*-semi-interior [11] (resp. (1,2)*-α-interior [8], (1,2)*-preinterior[15]) of H, denoted by (1,2)*-sint(H) (resp. (1,2)*-αint(H), (1,2)*-pint(H)), is defined to be the union of all (1,2)*-semi-open (resp. (1,2)*-α-open, (1,2)*-preopen) sets of X contained in H.

Definition 2.4A subset H of a BTPS X is called

(i) (1,2)*-generalized closed (briefly,(1,2)*-g-cld) set [16] if $\tau_{1,2}$-cl(H) $\subseteq$ U whenever H$\subseteq$ U and U is $\tau_{1,2}$-open in X.

(ii) (1,2)*-semi-generalized closed (briefly,(1,2)*-sg-cld) set [11] if (1,2)*-scl(H) $\subseteq$ U whenever H$\subseteq$ U and U is (1,2)*-semi-open in X.

(iii) (1,2)*-generalized semi-closed (briefly,(1,2)*-gs-cld) set [11] if (1,2)*-scl(H) $\subseteq$ U whenever H$\subseteq$ U and U is $\tau_{1,2}$-open in X.

(iv) (1,2)*-α-generalized closed (briefly,(1,2)*-αg-cld) set [15] if (1,2)*-αcl(H) $\subseteq$ U whenever H$\subseteq$ U and U is $\tau_{1,2}$-open in X.

(v) (1,2)*-generalized semi-preclosed(briefly,(1,2)*-gsp-cld) set [15] if (1,2)*-spcl(H) $\subseteq$ U whenever H$\subseteq$ U and U is $\tau_{1,2}$-open in X.

(vi) (1,2)*-$\hat{g}$-closed set ((1,2)*-ω-cld)) [5] if $\tau_{1,2}$-cl(H) $\subseteq$ U whenever H$\subseteq$ U and U is (1,2)*-semi-open in X.

(vii) (1,2)*-αgs-closed (briefly, (1,2)*- αgs-cld) set [15] if (1,2)*-αcl(H) $\subseteq$ U whenever H$\subseteq$ U and U is (1,2)*-semi-open in X.

(viii) (1,2)*-g*s-closed (briefly, (1,2)*-g*s-cld) set [11] if (1,2)*-scl(H) $\subseteq$ U whenever H$\subseteq$ U and U is (1,2)*-gs-open in X.

The complements of the abovementioned closed sets are called their respective open sets.

Remark 2.5 The collection of all (1,2)*-g-cld (resp. (1,2)*-ω-cld, (1,2)*-gs-cld, (1,2)*-gsp-cld,(1,2)*-αg-cld,(1,2)*-αgs-cld, (1,2)*-sg-cld, (1,2)*-g*s-cld, (1,2)*-α-cld, (1,2)*-semi-cld) sets is denoted by (1,2)*-GC(X) (resp. (1,2)*-ωC(X), (1,2)*-GSC(X), (1,2)*-GSPC(X), (1,2)*-αgC(X), (1,2)*-αGSC(X),(1,2)*-SGC(X), (1,2)*-G*SC(X), (1,2)*-αC(X), (1,2)*-SC(X)).

The collection of all (1,2)*-g-open (resp. (1,2)*-ω-open, (1,2)*-gs-open, (1,2)*-gsp-open, (1,2)*-αg-open,(1,2)*-αgs-open, (1,2)*-sg-open, (1,2)*-g*s-open, (1,2)*-α-open, (1,2)*-semi-open) sets is denoted by (1,2)*-GO(X) (resp. (1,2)*-ωO(X),(1,2)*-GSO(X), (1,2)*-GSPO(X), (1,2)*-αgO(X), (1,2)*-αGSO(X), (1,2)*-SGO(X), (1,2)*-G*SO(X), (1,2)*-αO(X), (1,2)*-SO(X)).
We denote the power set of X by P(X).

Definition 2.6 [16]Subset H of a BTPS X is said to be (1,2)*-locally closed if H = U $\cap$ F, where U is $\tau_{1,2}$-open and F is $\tau_{1,2}$-closed in X.

Remark 2.7
(1) Every $\tau_{1,2}$-open set is(1,2)*-g*s-open [16].
(2) Every (1,2)*-semi-open set is (1,2)*-g*s-open [11].
(3) Every (1,2)*-g*s-open set is (1,2)*-sg-open [16].
(4) Every (1,2)*-semi-cld set is (1,2)*-gs-cld [16].
(5) Every $\tau_{1,2}$-closed set is (1,2)*-gs-cld [16].

III.(1,2)*-$\ddot{\Upsilon}$-CLOSED SETS IN BITOPOLOGICAL SPACES

We introduce the following definition.
Definition 3.1A subset H of a BTPS X is called a (1,2)*-$\ddot{\Upsilon}$-closed (briefly, (1,2)*-$\ddot{\Upsilon}$-cld) set if $\tau_{1,2}$-cl(H) $\subseteq$ U whenever H$\subseteq$ U and U is(1,2)*-gs-open in X.

Proposition 3.2 Every $\tau_{1,2}$-closed set is (1,2)*-$\ddot{\Upsilon}$-cld.

Proof: If His any $\tau_{1,2}$-closed set in Xand Gis any (1,2)*-gs-open set containing H, then G $\supseteq$H = $\tau_{1,2}$-cl(H). Hence H is (1,2)*-$\ddot{\Upsilon}$-cld.

The converse of Proposition 3.2 need not be true as seen from the following example.

Example 3.3 Let X = {x, y, z}, τ_1 = {ϕ, X, {x, y}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x, y}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {z}} are called $\tau_{1,2}$-closed.Here, H = {x, z} is(1,2)*-$\ddot{\Upsilon}$-cld set but not $\tau_{1,2}$-closed.

Proposition 3.4 Every (1,2)*-$\ddot{\Upsilon}$-cld set is (1,2)*-g*s-cld.

Proof: If H is a (1,2)*-$\ddot{\Upsilon}$-cld subset of X and G is any (1,2)*-gs-open set containing H, then G $\supseteq\tau_{1,2}$-cl(H) $\supseteq$(1,2)*-scl(H). Hence H is (1,2)*-g*s-cld in X.

The converse of Proposition 3.4 need not be true as seen from the following example.

Example 3.5 In Example 3.3,Here, H = {z} is(1,2)*-g*s-cld but not (1,2)*-$\ddot{\Upsilon}$-cld set in X.

Proposition 3.6 Every (1,2)*-$\ddot{\Upsilon}$-cld set is (1,2)*-ω-cld.

Proof: Suppose that H$\subseteq$ G and G is (1,2)*-semi-open in X. Since every (1,2)*-semi-open set is (1,2)*-gs-open and H is (1,2)*-$\ddot{\Upsilon}$-cld, therefore $\tau_{1,2}$-cl(H) $\subseteq$ G. Hence H is (1,2)*-ω-cld in X.

The converse of Proposition 3.6 need not be true as seen from the following example.

Example 3.7 Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X, {y, z}}. Then the sets in {ϕ, {x}, {y, z}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {x}, {y, z}} are called $\tau_{1,2}$-closed.Here, H = {x, z} is (1,2)*-ω-cld but not (1,2)*-Ϋ̂-cld set in X.

Proposition 3.8 Every (1,2)*-g*s-cld set is (1,2)*-sg-cld.

Proof: Suppose that H⊆ G and G is (1,2)*-semi-open in X. Since every (1,2)*-semi-open set is (1,2)*-gs-open and H is (1,2)*-g*s-cld, therefore (1,2)*-scl(H) ⊆ G. Hence H is (1,2)*-sg-cld in X.

The converse of Proposition 3.8 need not be true as seen from the following example.

Example 3.9 Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X, {y, z}}. Then the sets in {ϕ, {x}, {y, z}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {x}, {y, z}} are called $\tau_{1,2}$-closed.Here H = {x, y} is(1,2)*-sg-cldbut not (1,2)*-g*s-cld set in X.

Proposition 3.10 Every (1,2)*-ω-cld set is (1,2)*-αgs-cld.

Proof: If H is a (1,2)*-ω-cld subset of X and G is any (1,2)*-semi-open set containing H, then G ⊇$\tau_{1,2}$-cl(H) ⊇(1,2)*-αcl(H). Hence H is(1,2)*-αgs-cld in X.

The converse of Proposition 3.10 need not be true as seen from the following example.

Example 3.11 Let X = {x, y, z},τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {y, z}} are called $\tau_{1,2}$-closed.Here,H = {y} is (1,2)*-αgs-cld but not (1,2)*-ω-cld set in X.

Proposition 3.12 Every (1,2)*-Ϋ̂-cld set is(1,2)*-g-cld.

Proof: If H is a (1,2)*-Ϋ̂-cld subset of X and G is any $\tau_{1,2}$-open set containing H,since every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G ⊇$\tau_{1,2}$-cl(H). Hence H is (1,2)*-g-cld in X.

The converse of Proposition 3.12 need not be true as seen from the following example.

Example 3.13 Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X, {y, z}}. Then the sets in {ϕ, {x}, {y, z}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {x}, {y, z}} are called $\tau_{1,2}$-closed.Here, H = {x, y} is (1,2)*-g-cld but not (1,2)*-Ϋ̂-cld set in X.

Proposition 3.14 Every (1,2)*-Ϋ̂-cld set is (1,2)*-αgs-cld.

Proof: If H is a (1,2)*-Ϋ̂-cld subset of X and G is any (1,2)*-semi-open set containing H, since every (1,2)*-semi-open set is (1,2)*-gs-open, we have G ⊇$\tau_{1,2}$-cl(H)⊇(1,2)*-αcl(H). Hence H is (1,2)*-αgs-cld in X.

The converse of Proposition 3.14 need not be true as seen from the following example.

Example 3.15 Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X, {y, z}}. Then the sets in {ϕ, {x}, {y, z}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {x}, {y, z}} are called $\tau_{1,2}$-closed.Here,H = {x, z} is (1,2)*-αgs-cld but not (1,2)*-Ϋ̂-cld set in X.

Proposition 3.16 Every (1,2)*-Ϋ̂-cld set is (1,2)*-αg-cld.

Proof: If H is a (1,2)*-Ϋ̂-cld subset of X and G is any $\tau_{1,2}$-open set containing H, since every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G ⊇$\tau_{1,2}$-cl(H) ⊇(1,2)*-αcl(H). Hence H is (1,2)*-αg-cld in X.

The converse of Proposition 3.16 need not be true as seen from the following example.

Example 3.17 Let X = {x, y, z}, τ_1 = {ϕ, X, {z}} and τ_2 = {ϕ, X, {x, y}}. Then the sets in {ϕ, {z}, {x, y}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {z}, {x, y}} are called $\tau_{1,2}$-closed.Here, H = {x, z} is (1,2)*-αg-cld but not (1,2)*-Ϋ-cld set in X.

Proposition 3.18 Every (1,2)*-Ϋ-cld set is (1,2)*-gs-cld.

Proof: If H is a (1,2)*-Ϋ-cld subset of X and G is any $\tau_{1,2}$-open set containing H, since every $\tau_{1,2}$-open set is (1,2)*-gs-open,we have G $\supseteq\tau_{1,2}$-cl(H) $\supseteq$(1,2)*-scl(H). Hence H is (1,2)*-gs-cld in X.

The converse of Proposition 3.18 need not be true as seen from the following example.

Example 3.19 Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {y, z}} are called $\tau_{1,2}$-closed.Here, H = {z} is (1,2)*-gs-cld but not (1,2)*-Ϋ-cld set in X.

Proposition 3.20 Every (1,2)*-Ϋ-cld set is (1,2)*-gsp-cld.

Proof: If H is a (1,2)*-Ϋ-cld subset of X and G is any $\tau_{1,2}$-open set containing H, every $\tau_{1,2}$-open set is (1,2)*-gs-open, we have G $\supseteq\tau_{1,2}$-cl(H) $\supseteq$(1,2)*-spcl(H). Hence H is (1,2)*-gsp-cld in X.

The converse of Proposition 3.20 need not be true as seen from the following example.

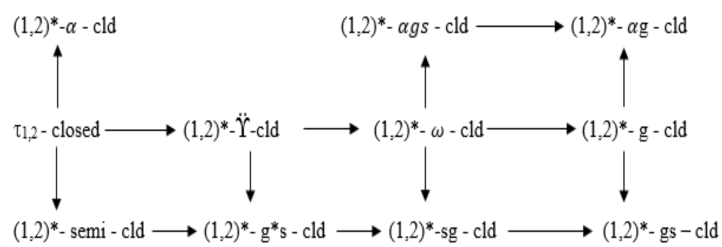
Example 3.21 In Example 3.19,Here,H = {z} is (1,2)*-gsp-cld but not (1,2)*-Ϋ-cld set in X.

Remark 3.22 The following example shows that (1,2)*-Ϋ-cld sets are independent of (1,2)*-α-cld sets and (1,2)*-semi-cld sets.

Example 3.23 Let X = {x, y, z}, τ_1 = {ϕ, X, {x, y}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x, y},X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {z}} are called $\tau_{1,2}$-closed.Here, H = {x, z} is (1,2)*-Ϋ-cld but it is neither (1,2)*-α-cld nor (1,2)*-semi-cld in X.

Example 3.24 Let X = {x, y, z}, τ_1 = {ϕ, X, {x}} and τ_2 = {ϕ, X}. Then the sets in {ϕ, {x}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, X, {y, z}} are called $\tau_{1,2}$-closed.Here, H = {y} is (1,2)*-α-cldas well as(1,2)*-semi-cld in X but it is not(1,2)*-Ϋ-cldin X.

Remark 3.25 From the above discussions and known results in [11,15,16] we obtain thefollowing diagram, where A → B (resp. A ↮ B)representsA implies B but not conversely (resp. A and B are independent of each other).



None of the above implications is reversible as shown in the remaining examples and in the related papers [15,16].

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