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Optimum Tilt Angle for Mounting A Solar Panel in Onitsha and its Environment

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ABSTRACT

The study investigated the monthly based, seasonal based and annual based optimum tilt angles for mounting a solar panel in Onitsha, Anambra North senatorial district. The calculations were based upon the data of monthly mean daily global solar radiation obtained from NASA Langley Research Center Atmospheric Science Data Center; over a period of 22 years (July 1983 - June 2005). The global solar radiation on the tilted surface was estimated using the isotropic and anisotropic models. Study showed that there was no considerable difference between the two models in the determination of the optimum angles. The annual based and seasonal based optimum tilt angles for the zone (**Latitude**: +6.14 (6°08'24"N) and **Longitude**: +6.78 (6°46'48"E) were determined to be 12.74 degree and 25.5, 0 (zero) degree for the dry and wet seasons respectively. It was found that the loss in the amount of collected energy was around 0.78% if the angle of tilt was adjusted seasonally and 4.05% if annual adjustment was used instead of monthly adjustment. We also found that by adjusting this panel monthly, 6% of the total surface area was saved and as such saving cost.

Key words**:** Global Solar Radiation, Optimum Tilt Angle, Onitsha and Solar Panel**.**

INTRODUCTION

Solar collector is a device that collects solar energy from the sun. Depending on the purpose for which the system is designed, a collector can be a flat plate collector or a PV panel. A PV panel converts radiant energy from the sun to electrical energy. One major problem of a PV system is the high cost of implementation due to high cost of the panel. To reduce the cost, we tend to maximize the solar energy received by this panel and as such increase the power output.

When a PV model is perpendicular to the sunlight, the total solar power received by the surface is equal to that of the sunlight. Since the position of the sun on the sky is always changing, the angle between a fix panel and the sun changes as well. Conversely, the total solar power received by a fixed panel is not equal to that emitted by the sun. The total solar radiation received on a flat surface depends on some factors: the latitude of the site location, the clearness index, the tilt angle of the surface, the day of the year and the time of the day. Among all these factors, the only one we have control over is the tilt angle. Getting the optimum tilt angle for a site will help reduce the cost of implementing the project. The available solar radiation mostly within our reach is that of horizontal surfaces. That of tilted surfaces can only be estimated using different models from the corresponding horizontal surfaces. The solar radiation on tilted surfaces is composed of direct (beam) radiation, diffused radiation and ground reflected radiation. The ground albedo value of 0.2 for poor reflecting surfaces and 0.7 value for highly reflecting surfaces like snow area was assumed by Liu and Jordan (1962). All the models used the same methods for calculating beam and ground reflected radiation, but different methods for diffuse radiation. While some models assume diffuse radiation to be the same in all direction over the sky dome (isotropic model), others sees it to be anisotropically distributed round the circumsolar region and isotropically distributed from the rest of the sky dome (Anisotropic Model).

In the northern hemisphere, the panel is optimal when facing south [Ahmad and Tiwari, (2009)]. A. El-Sebaii, *et al.,* (2010) in their study stated that a solar panel not only will be tilted to face south if in the northern hemisphere, but must be to the latitude angle of the location. Gh. A. Kamali[1] I. Moradi[2] and A. Khalili[3] (2005) compared the results of eight widely used models for estimating solar radiation on tilted surfaces with measurements from Karaj, Iran for south and west-facing surfaces inclined at an angle 45^o and 40^o respectively. M. Jamil Ahmad and G. N. Tiwari concluded that monthly based and seasonal based optimum tilt are different for different station; while the annual based optimum tilt is approximately equal to the latitude of the station. In their work, the loss of energy when using the yearly average fixed angle relative to the monthly average tilt is around 13.4% for Delhi station. Gh. A. Kamali *et al*., (2005) in their work concluded that Reindl*et al*., (1990) model showed the best agreement with the measured tilted data.

MATERIALS AND METHOD

The radiation data for this work was obtained from NASA Langley Research Center, Atmospheric Science Data Center; over a period of 22 years (July 1983 - June 2005). We analyzed the data by calculating the monthly average from the daily average. The total global solar radiation is made off the direct, the diffused and the ground reflected solar radiation. At hand are several models that can be used for calculating solar radiation on tilted surfaces; all the models settles with the beam and reflected radiation while differ somewhat on diffused radiation. Whereas some see it as isotropic, others see it as been anisotropic. To calculate the solar radiation on tilted surface the following equations were used:

$$\hat{H}_T = \hat{H}_B + \hat{H}_D + \hat{H}_R \qquad (1)$$

Miguael *et al.,* (2001) model was used to estimate the monthly average daily beam and diffused components on a horizontal surface from the measured monthly average daily global radiation as shown below.

$$\frac{Hd}{H} = 0.952 \qquad \text{if } K_T = 0.13$$

$$\frac{Hd}{H} = 0.868 + 1.335K_T - 5.782K_T^2 + 3.721K_T^3 \quad \text{if } 0.13 < K_T \leq 0.80 \qquad (2)$$

$$\frac{Hd}{H} = 0.141 \text{ if } K_T > 0.80$$

Where, K_T is between 0 to 1. The value is 1 when the cloud is very clear in the noon day and 0 when is totally dark.

H_b can be calculated as follows

$$H_b = H - H_d \qquad (3)$$

The monthly average daily beam radiation received on a tilted surface can be expressed as

$$\hat{H}_B = (H - H_d)\,\hat{R}_b = H_b\hat{R}_b \qquad (4)$$

For surfaces in the northern hemisphere sloped towards the equator, $\hat{R}_b$ is given by Liu and Jordan (1962) as

$$\hat{R}_b = \frac{\cos(\emptyset-\beta)\cos\delta\cos\omega s+\left(\frac{\pi}{180}\right)\omega s\sin(\emptyset-\beta)\sin\delta}{\cos\emptyset\cos\delta\sin\omega s+\left(\frac{\pi}{180}\right)\omega s\sin\emptyset\sin\delta} \quad (5)$$

Where

$$\omega_s = \min\{\cos^{-1}(-\tan\emptyset\tan\delta), \cos^{-1}[-\tan(\emptyset-\beta)\tan\delta]\} \quad (6)$$

"min" means the smaller of the two terms in the bracket.

$$\delta = 23.45\sin\left[\frac{360}{365}(284+n)\right] \text{ in degress} \quad (7)$$

Where the value of n used is that of the middle of the month

Assuming isotropic reflection, the daily ground reflected radiation is written as

$$\hat{H}_R = \hat{H}_\rho (1 - \cos\beta)/2 \quad (8)$$

Where; ρ has a value of 0.2.

The sky-diffuse radiation on a tilted surface is $\hat{H}_d = H_d\hat{R}_d$ (9)

To evaluate $\hat{R}_d$, the isotropic models used were Liu and Jordan (1962) and Koronakis (1986), while the anisotropic models used were Hay (1979), Reindl *et al.,* (1990) and Skartvetit and Olseth (1986). The table1 below shows the mathematical arrangement for these models.

Table 1: List of ratio of average daily diffused radiation on a tilted to that on horizontal surfaces for different models.

Model	Year	Abbreviation	$\hat{R}_d$
Liu and Jordan	1962	*L&J*	$= [1+\cos\beta]/2$
Koronakis	1986	*Kn*	$= \frac{1}{3}[2+\cos\beta]$
Hay	1979	*Ha*	$= \frac{\hat{H}b}{\hat{H}o}\hat{R}b + \left(1-\frac{\hat{H}b}{\hat{H}o}\right)\left[\left(\frac{1+\cos\beta}{2}\right)\right]$
Skartvetit and Olseth	1986	*S&O*	$= \frac{\hat{H}b}{\hat{H}o}\hat{R}b + \Omega\cos\beta + \left(1-\frac{\hat{H}b}{\hat{H}o} - \Omega\right)\left[\left(\frac{1+\cos\beta}{2}\right)\right], \quad Where\ \Omega = \left\{max\left[0,\left(0.3-2\frac{\hat{H}b}{\hat{H}o}\right)\right]\right\}$
Reindl *et al.,*	1990	*Re*	$= \frac{\hat{H}b}{\hat{H}o}\hat{R}b + \left(1-\frac{\hat{H}b}{\hat{H}o}\right)\left[\left(\frac{1+\cos\beta}{2}\right)\right][1+\sqrt{\hat{H}_{b/\hat{H}}}\sin^3(\beta/2)]$

The total global solar radiation on a tilted surface is then,

$$\hat{H}_T = (H - H_d)R_b + H\rho(1-\cos\beta)/2 + H_dR_d \quad (10)$$

Equation 1 to 10 with the value of $\hat{R}_d$ in table 1, was used to determine the total global solar radiation for each model on a different tilt angle for every month. MS-EXCEL was used to plot the graph of global solar radiation on tilted surface ($\hat{H}_T$) against the corresponding tilt angle (β), with the value of β ranging from 0 to 90 degree. A second order polynomial equations was developed to fit the curves generated and the turning point of the curve gives the maximum solar radiation with the corresponding tilt angle for the month (optimum tilt angle).Thus optimum tilt angle for each month was computed. In addition, the seasonal optimal based tilt angle was calculated by finding the average value of the tilt angles for each season (dry and wet seasons). The annual based optimal tilt angle was calculated by finding the average value of the tilt angles for all months of the year.

The general equation for different models is

$Y = AX^2 + BX + C$

Where A, B and C assumed different values for different models

Y = Global solar radiation

X = Tilt angle

RESULTS AND DISCUSSION

Table 2, below shows the values of optimum tilt angle for each month of the year and the corresponding solar radiation. It was used to obtain the graph of optimum tilt angle versus month for all the months in Fig. 1. The graph shows a clear picture of how the optimum tilt varies with month in a year. From the table, the optimum tilt angle for January is 36.8 degree, 25.58 degree for February and 6.03 degree for March. These values remain zero from April to September before increasing to 15.75, 29.31 and 39.47 degree in October November and December respectively. Fig. 1, below shows a clear picture of how the optimal tilt angle varied from January to December. From this fig. 1, we saw that optimum tilt angle remained zero throughout the raining season, but varied from month to month during the dry season. The yearly average optimum tilt angle was obtained to be 12.74 degree by finding the average value of tilt angle for all months of the year.

Table 2: Summary of the Optimum Tilt Angles and Maximum Global Solar Radiation for each month.

Month	**Average Optimum Tilt Angle for all the models, β_{opt} (^{o})**	**Average Maximum Global, Solar Radiaton, for all the models H_{Tmax} (KWh/m^2/day)**
January	36.8002	7.0499
February	25.5804	6.5471
March	6.0289	5.7529
April	0.0000	5.2900
May	0.0000	4.9700
June	0.0000	4.5900
July	0.0000	4.2000
August	0.0000	3.9700
September	0.0000	4.2300

October	15.7498	4.6993
November	29.3069	5.6665
December	39.4727	6.7978
Average	12.7422	5.3136

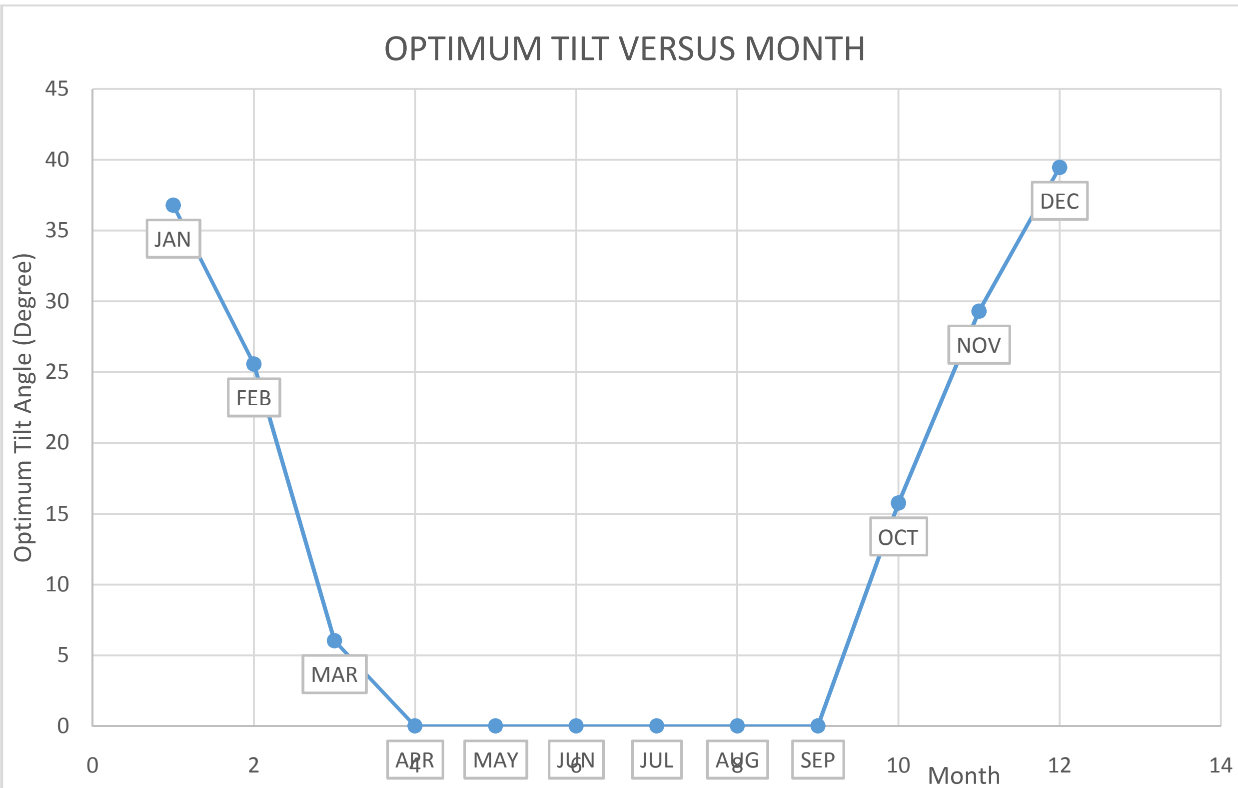

Fig. 1. The Graph of Optimum Tilt Angle β (°) versus Months of the year

The seasonal optimum tilt angles were shown in table 3 to be 25.48 degree in dry season and 0 (zero) degree in wet season. These values were obtained by first finding the seasonal average for each model in each season and then the average for all the models. This required that that panel's tilt angle should be adjusted two times a year, first to zero degree in wet season around April and later to 25.5 degree in dry season around October.

Table 3. The Optimum Tilt Angle of Different Models for Different Seasons and year

Model	Optimum Tilt Angle for Dry Season (°)	Optimum Tilt Angle for Wet Season (°)	Optimum Tilt Angle for a Year (°)
Liu and Jordan	23.6664	0.0000	11.8332
Koronakis	25.4498	0.0000	12.7249
Hay	25.9389	0.0000	12.9695
Skartveit and Olseth	25.9389	0.0000	12.9695
Reindl *et al.,*	26.4281	0.0000	13.2141
Average	25.4844	0.0000	12.7422

Table 4: The Total Global Solar Radiation for Monthly, Seasonal, Yearly and Horizontal Tilt for each Month

Month	Global Solar Radiation for the Monthly βm_{opt} (KWh/m^2/day)	Global Solar Radiation for the Seasonal βs_{opt} (KWh/m^2/day)	Global Solar Radiation for the yearly βy_{opt} (KWh/m^2/day)	Global Solar Radiation for Horizontal Surfaces (KWh/m^2/day)
January	7.0499	6.9439	6.5754	5.9500
February	6.5471	6.5462	6.4282	6.0700
March	5.7529	5.5649	5.7304	5.7000
April	5.2900	5.2900	5.0987	5.2900
May	4.9700	4.9700	4.6534	4.9700
June	4.5900	4.5900	4.2662	4.5900
July	4.2000	4.2000	3.9659	4.2000
August	3.9700	3.9700	3.8351	3.9700
September	4.2300	4.2300	4.1829	4.2300
October	4.6993	4.6627	4.6951	4.5900
November	5.6665	5.6591	5.5107	5.1800
December	6.7978	6.6375	6.2379	5.6000
Total Sum	63.7635	63.2643	61.1799	60.3400

Seasonal adjustment according to table 4 and fig. 2 gives better performances than yearly adjustment while the monthly adjustment gives a better result as expected. When monthly average tilt angle was used for each month, the annual collected energy by the panel was 63.7635 kwh/m^2/day, 63.2643 kwh/m^2/day when seasonal average was used for each season and 61.1799 kwh/m^2/day when yearly average tilt angle was used for the whole year. We also see that by placing the panel horizontally for all the year, the total annual collected energy will be 60.34 kwh/m^2/day. The energy loss if the panel is adjusted seasonally instead of monthly is about 0.7829% while about 4.0518% of the energy is loss for yearly adjustment. By adjusting the angle monthly we gain 5.6737% of the total energy received if the panel is placed horizontally. This as a result will reduce the high cost of the project by reducing the surface area of the panel by 6%. Again seasonal adjustment will help to improve the performance of the system in every season if compared with the yearly adjustment.

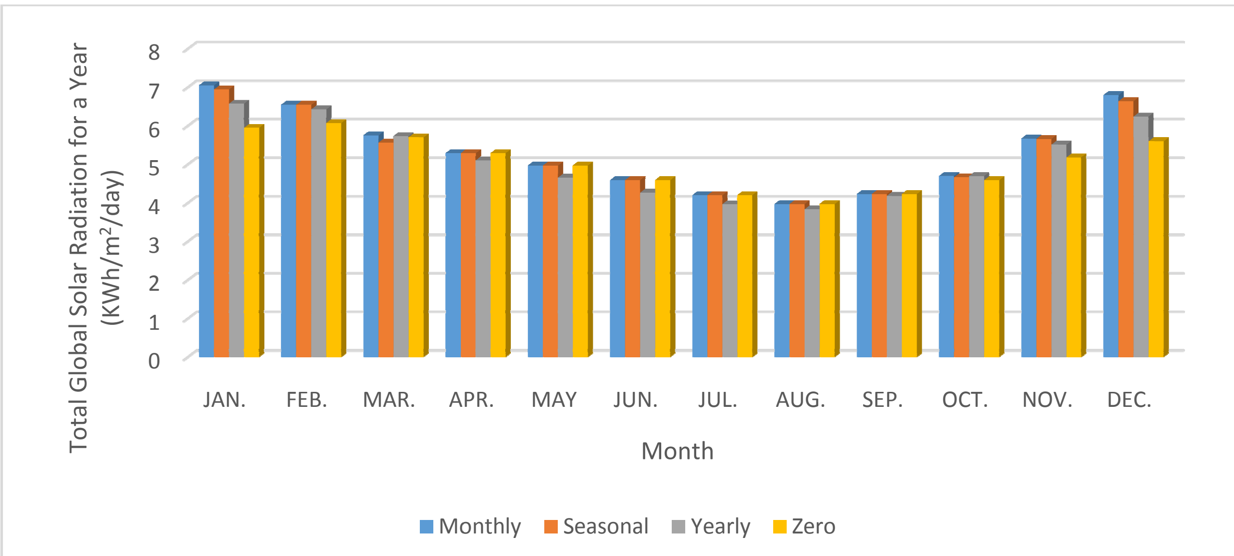

Fig. 2: The Average Monthly Total Solar Radiation for Monthly, Seasonal, Yearly and Zero Tilt for each Month.

Fig. 3 shows that the worst months for poor solar radiation were during the wet season as expected. This made the panel's performance/generation expected to be very poor during this season, therefore, it is expected that panel be placed to obtain maximum solar energy for the season during these period.

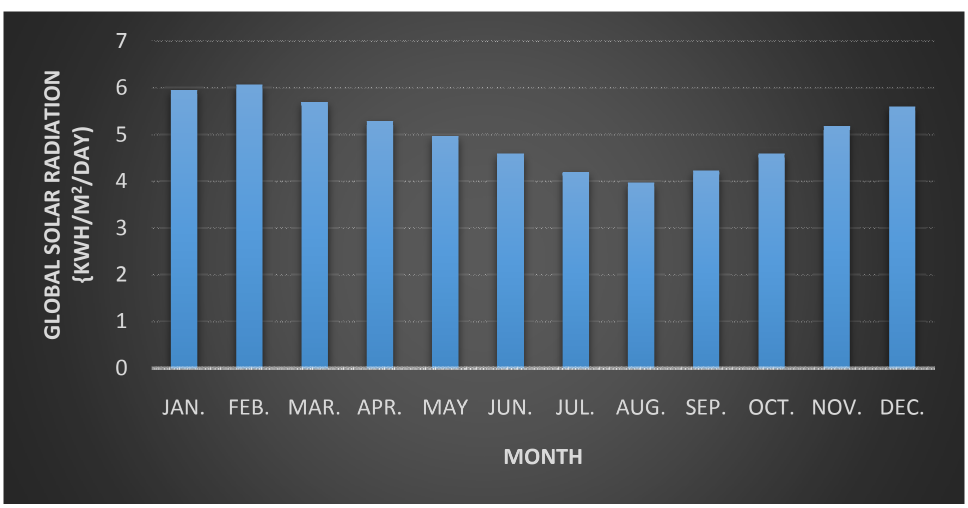

Fig. 3: The average global solar radiation for each month of the year on a horizontal surface.

CONCLUSIONS

The monthly based optimum tilt angle for Onitsha and its environment decreased from 36.8 degree in January to 0 (zero) degree in April and remained constant at this value for six months of the wet season before it started increasing again in October up to December. In a situation where energy requirement is seasonally based, the seasonal optimum tilt angle should be used.

The annual based optimum tilt angle is 12.74 degree, while the seasonal based optimum tilts are 0 (zero) degree and 25.48 degree for wet and dry seasons respectively. Six percent of the total surface area for the panel and cost will be saved by adjusting the panel to its monthly optimum tilt angle every month.

About 0.7829% of the total energy collected annually will be lost if the panel is adjusted seasonally instead of monthly and 4.0518% loss, if yearly adjustment is used instead of monthly adjustment. By adjusting the angle monthly we gain 5.6737% of the total energy received if the panel is placed horizontally.

Nomenclature

SYMBOL	DESCRIPTION
$\hat{H}_T$	Monthly average daily total radiation on a tilted surface
$\hat{H}_B$	Monthly average daily beam radiation on a tilted surface
$\hat{H}_D$	Monthly average daily diffuse radiation on a tilted surface
$\hat{H}_R$	Ground reflected radiation on a tilted surface
K_T	The monthly clearness index
H	Monthly average daily global radiation on a horizontal surface
H_d	Monthly average daily diffuse radiation on a horizontal surface
H_b	Monthly average daily beam radiation on a horizontal surface
$\hat{R}_b$	Ratio of the monthly average daily beam radiation on a tilted surface to that on a horizontal surface
ω_s	The sunset hour angle (in degrees) for the tilted surface for the mean day of the month.
Ø	The latitude of the location
δ	The declination angle
β	Tilt angle
$\hat{R}_d$	The ratio of the average daily diffused radiation on a tilted surface to that on horizontal surface
ρ	The ground albedo

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