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$m\omega$-CLOSED SETS IN MICRO TOPOLOGICAL SPACES

SELVARAJ GANESAN

Assistant Professor, PG & Research Department of Mathematics,

Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India.

(Affiliated to Alagappa University, Karaikudi, Tamil Nadu, India)

e-mail : sgsgsgsgsg77@gmail.com; Orcid: 0000-0002-7728-8941

Abstract. The aim of this paper, we offer a new class of sets called $m\omega$-closed sets in Micro topological spaces and we study some of its basic properties. It turns out that this class lies between the class of Micro closed sets and the class of Micro generalized closed sets. As applications of $m\omega$-closed sets, we introduce $T_{m\omega}$-spaces, ${}_gT_{m\omega}$-spaces. Moreover, we obtain certain new characterizations for the $T_{m\omega}$-spaces, ${}_gT_{m\omega}$-spaces. We introduce a new class of continuous maps called $m\omega$-continuous and to discuss some of its properties in terms of $m\omega$-closed.

keywords: $m\omega$-closed sets, $m\omega$-open sets, $T_{m\omega}$-spaces, ${}_gT_{m\omega}$-spaces, $m\omega$-continuous map and $m\omega$-irresolute map.

1. INTRODUCTION

S. Chandrasekar [1] introduced and studied Micro Pre-open, Micro semi open, Micro continuous, Micro pre-continuous and Micro semi-continuous in Micro topological spaces. S. Chandrasekar [2] Micro α-open sets and Micro α-continuity respectively. H. Z. Ibrahim [21, 22] introduced and studied Micro β-open sets and Micro b-open sets in Micro topology. S. Ganesan [20] introduced and studied Micro regular open sets and Micro π-open sets in Micro topological spaces. Recently, S. Ganesan [19] introduced and studied Micro generalized closed sets and Micro generalized continuous in Micro topological spaces. In this paper, we offer a new class of sets called $m\omega$-closed sets in Micro topological spaces and we study some of its basic properties. As applications of $m\omega$-closed sets, we introduce $\mathrm{T}_{m\omega}$-spaces, ${}_{g}\mathrm{T}_{m\omega}$-spaces. Moreover, we obtain certain new characterizations for the $\mathrm{T}_{m\omega}$-spaces, ${}_{g}\mathrm{T}_{m\omega}$-spaces. We introduce a new class of continuous maps called $m\omega$-continuous and to discuss some of its properties in terms of $m\omega$-closed.

2. PRELIMINARIES

Definition 2.1. [23]

If (U, $\tau_R(X)$) is the nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(1) *The nano interior of the set A is defined as the union of all nano open subsets contained in A and it is denoted by nint(A). That is, nint(A) is the largest nano open subset of A.*

(2) *The nano closure of the set A is defined as the intersection of all nano closed sets containing A and it is denoted by ncl(A). That is, ncl(A) is the smallest nano closed set containing A.*

Definition 2.2. [1] *Let (U, $\tau_R(X)$) be a nano topological space. Then, $\mu_R(X) = \{N \cup (\acute{N} \cap \mu) : N, \acute{N} \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.*

Definition 2.3. [1] *The Micro topology $\mu_R(X)$ satisfies the following axioms*

(1) *U, $\phi \in \mu_R(X)$.*

(2) *The union of the elements of any sub-collection of* $\mu_R(X)$ *is in* $\mu_R(X)$.

(3) *The intersection of the elements of any finite sub collection of* $\mu_R(X)$ *is in* $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro topological spaces and The elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4. *$A \subseteq A$ of a space (U, $\tau_R(X)$, $\mu_R(X)$) is called*

(1) *Micro semi-open* [1] *if $A \subseteq$ Mic-cl(Mic-int(A)).*

(2) *Micro α-open* [2] *if $A \subseteq$ Mic-int(Mic-cl(Mic-int(A))).*

The complement of above mentioned micro open sets are called their respective micro closed sets.

The Micro semi-closure of a subset A of U, denoted by Mscl(M) is defined to be the intersection of all Micro semi-closed sets of (U, $\tau_R(X)$) containing A.

The Micro semi-interior of a subset M of U, denoted by Msint(M) is defined to be the union of all Micro semi-open sets of (U, $\tau_R(X)$) containing A.

Theorem 2.5. [1] *Every Micro closed set is Micro semi closed.*

Definition 2.6. [19] *A subset A of a space (U, $\tau_R(X)$, $\mu_R(X)$) is called Micro generalized closed (briefly mg-closed) set if mcl(A) $\subseteq$ T whenever $A \subseteq T$ and T is Micro open in (U, $\tau_R(X)$, $\mu_R(X)$). The complement of mg-closed set is called mg-open set.*

Definition 2.7. *A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) is called*

(1) *Micro-continuous* [1] *if f^{-1}(V) is a Micro closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every Micro closed set V of (L, $\tau'_R(Y)$, $\mu'_R(Y)$).*

(2) *Micro semi-continuous* [1] *if* f^{-1}*(V) is a Micro semi-closed set of (U,* $\tau_R(X)$*,* $\mu_R(X)$*) for every Micro closed set V of (L,* $\tau'_R(Y)$*,* $\mu'_R(Y)$*).*

(3) *Micro* α*-continuous* [2] *if* f^{-1}*(V) is a Micro* α*-closed set of (U,* $\tau_R(X)$*,* $\mu_R(X)$*) for every Micro closed set V of (L,* $\tau'_R(Y)$*,* $\mu'_R(Y)$*).*

(4) *mg-continuous* [19] *if* f^{-1}*(V) is a mg-closed set of (U,* $\tau_R(X)$*,* $\mu_R(X)$*) for every Micro closed set V of (L,* $\tau'_R(Y)$*,* $\mu'_R(Y)$*).*

3. $m\omega$-CLOSED AND $m\omega$-OPEN SETS

Definition 3.1. *A subset A of a space (U,* $\tau_R(X)$*,* $\mu_R(X)$*) is called Micro* $m\omega$ *closed (briefly* $m\omega$*-closed) set if mcl(A)* $\subseteq$ *T whenever A* $\subseteq$ *T and T is Micro semi-open in (U,* $\tau_R(X)$*,* $\mu_R(X)$*). The complement of* $m\omega$*-closed set is called* $m\omega$*-open set.*

Proposition 3.2. *Every Micro closed set is* $m\omega$*-closed but not conversely.*

Proof. Let A be a Micro closed set and T be any Micro open set containing A. Since A is Micro closed, we have mclo(A) = A $\subseteq$ T. Hence A is $m\omega$-closed. □

Example 3.3. *Let U =* $\{1, 2, 3\}$ *with U/ R=* $\{\{1, 2, 3\}\}$ *and X=* $\{2, 3\}$*. The nano topology* $\tau_R(X) = \{\phi, U\}$*. If* $\mu = \{1, 3\}$*. then the micro topology* $\mu_R(X) = \{\phi, \{1, 3\}, U\}$*. Then* $m\omega$*-closed sets are* $\{\phi, \{2\}, \{1, 2\}, \{2, 3\}, U\}$*. Here, H =* $\{1, 2\}$ *is* $m\omega$*-closed set but not Micro closed.*

Proposition 3.4. *Every* $m\omega$*-closed set is mg-closed but not conversely.*

Proof. Let M be an $m\omega$-closed set and T be any Micro open set containing M. Since every Micro open set is Micro semi open, we have mclo(M) $\subseteq$ T. Hence M is mg-closed. □

Example 3.5. *Let U =* $\{1, 2, 3\}$ *with U/ R=* $\{\{1\}, \{2, 3\}\}$ *and X=* $\{1\}$*. The nano topology* $\tau_R(X) = \{\phi, \{1\}, U\}$*. If* $\mu = \{1, 3\}$ *then the micro topology* $\mu_R(X) = \{\phi,$

{1}, {1, 3}, U}. Then mω-closed sets are {ϕ, {2}, {2, 3}, U} and mg-closed sets are {ϕ, {2}, {1, 2}, {2, 3}, U}. Here, H = {1, 2} is mg-closed but not mω-closed.

Remark 3.6. *The following examples show that mω-closed sets are independent of Micro α-closed sets and Micro semi-closed sets.*

Example 3.7. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.5. Then mα-closed sets are {ϕ, {2}, {3}, {2, 3}, U} and Micro semi-closed sets are {ϕ, {2}, {3}, {2, 3}, U}. Here, {3} is mα-closed and Micro semi-closed but not mω-closed.*

Example 3.8. *Let U = {1, 2, 3} with U/ R= {{1}, {2, 3}} and X = {1}. The nano topology $\tau_R(X)$ = {ϕ, {1}, U}. If μ = {2, 3} then the Micro topology $\mu_R(X)$ = {ϕ, {1}, {2, 3}, U}. Then mω-closed sets are {ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, U} ; mα-closed sets are {ϕ, {1}, {2, 3}, U} and Micro semi-closed sets are {ϕ, {1}, {2, 3}, U}. Here, {1, 3} is mω-closed but not mα-closed and not Micro semi-closed.*

Proposition 3.9. *If S and G are mω-closed sets, then S ∪ G is also a mω-closed set in (U, $\tau_R(X)$, $\mu_R(X)$).*

Proof. If S ∪ G ⊆ T and T is Micro semi-open, then S ⊆ T and G ⊆ T. Since S and G are $m\omega$-closed, mcl(S) ⊆ T and mcl(G) ⊆ T and hence mcl(S ∪ G) = mcl(S) ∪ mcl(G) ⊆ T. Thus S ∪ G is $m\omega$-closed set in (U, $\tau_R(X)$, $\mu_R(X)$). □

Remark 3.10. *If K and L are mω-closed sets, then K ∩ L is mω-closed set.*

Example 3.11. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.3. Here, K = {1, 2} and L = {2, 3} are mω-closed sets but K ∩ L = {2} is also mω-closed.*

Proposition 3.12. *If A subset A of (U, $\tau_R(X)$, $\mu_R(X)$) is a mω-closed then mcl(A) − A contains no nonempty Micro closed set in (U, $\tau_R(X)$, $\mu_R(X)$).*

Proof. Suppose that A is $m\omega$-closed. Let S be a Micro closed subset of mcl(A) $-$ A. Then A $\subseteq$ S^c . Since S^c is Micro open and A is $m\omega$-closed, therefore mcl(A) $\subseteq$ S^c . Consequently, S $\subseteq$ $(mcl(A))^c$. We already have S $\subseteq$ mcl(A). Thus S $\subseteq$ mcl(A) $\cap$ $(mcl(A))^c$ $= \phi$. Therefore S is empty. □

Proposition 3.13. *If A subset A of (U, $\tau_R(X)$, $\mu_R(X)$) is a $m\omega$-closed if and only if mcl(A) $-$ A does not contain any nonempty Micro semi-closed set in (U, $\tau_R(X)$, $\mu_R(X)$).*

Proof. Suppose that A is $m\omega$-closed. Let S be a Micro semi-closed subset of mcl(A) $-$ A. Then A $\subseteq$ S^c . Since A is $m\omega$-closed. we have mcl(A) $\subseteq$ S^c . Consequently, S $\subseteq$ $(mcl(A))^c$. We have S $\subseteq$ mcl(A). Hence S $\subseteq$ mcl(A) $\cap$ $(mcl(A))^c$ $= \phi$. Therefore S is empty.

Conversely, suppose that mcl(A) $-$ A does not contain any nonempty Micro semi-closed set in (U, $\tau_R(X)$, $\mu_R(X)$). Let A $\subseteq$ S and that S be Micro semi-open. If mcl(A) $\nsubseteq$ S, then mcl(A) $\cap$ S^c $= \phi$. Since mcl(A) is a Micro closed set and S^c is a Micro semi-closed set of (U, $\tau_R(X)$, $\mu_R(X)$), m-cl(A) $\cap$ S^c is a Micro semi-closed set of (U, $\tau_R(X)$, $\mu_R(X)$). Therefore $\phi \neq$ m-cl(A) $\cap$ S^c $\subseteq$ mcl(A) $-$ A and so mcl(A) $-$ A contains a nonempty Micro semi-closed set, Which is a contradicition to the hypothesis. Thus, A is in $m\omega$-closed. □

Proposition 3.14. *If A is $m\omega$-closed in (U, $\tau_R(X)$, $\mu_R(X)$) and A $\subseteq$ B $\subseteq$ mcl(A), then B is also a $m\omega$-closed in (U, $\tau_R(X)$, $\mu_R(X)$).*

Proof. Let T be a Micro semi open set of (U, $\tau_R(X)$, $\mu_R(X)$) such that B $\subseteq$ T. Then A $\subseteq$ T. Since A is $m\omega$-closed, we get, mcl(A) $\subseteq$ T. Now mcl(B) $\subseteq$ mcl(mcl(A)) = mcl(A) $\subseteq$ T. Therefore, B is also a $m\omega$-closed in (U, $\tau_R(X)$, $\mu_R(X)$). □

Proposition 3.15. *Let $A \subseteq L \subseteq U$ and suppose that A is $m\omega$-closed in (U, $\tau_R(X)$, $\mu_R(X)$). Then A is $m\omega$-closed relative to L.*

Proof. Let A ⊆ L ∩ T, where T is Micro semi open in (U, $\tau_R(X)$, $\mu_R(X)$). Then A ⊆ T and hence mcl(A) ⊆ T. This implies that L ∩ mcl(A) ⊆ L ∩ T. Thus A is $m\omega$-closed relative to L. □

Definition 3.16. *The intersection of all Micro semi open subsets of (U, $\tau_R(X)$, $\mu_R(X)$) containing A is called the Micro-semi kernel of A and denoted by m-sker(A).*

Lemma 3.17. *A subset A of (U, $\tau_R(X)$, $\mu_R(X)$) is $m\omega$-closed if and only if mcl(A) ⊆ m-sker(A).*

Proof. Suppose that A is $m\omega$-closed. Then mcl(A) ⊆ U whenever A ⊆ U and U is Micro semi open. Let x ∈ mcl(A). If x ∉ m-sker(A), then there is a Micro semi open set U containing A such that x ∉ U. Since U is a Micro semi open set containing A, we have x ∉ mcl(A) and this is a contradiction. Therefore, mcl(A) ⊆ m-sker(A).

Conversely, let mcl(A) ⊆ m-sker(A). If U is any Micro semi open set containing A, then mcl(A) ⊆ m-sker(A) ⊆ U. Therefore, A is $m\omega$-closed. □

Definition 3.18. *A subset A of (U, $\tau_R(X)$, $\mu_R(X)$) is said to be $m\omega$-open if A^C is $m\omega$-closed.*

Proposition 3.19. *(1) Every Micro open set is $m\omega$-open set but not conversely.*
(2) Every $m\omega$-open set is mg-open set but not conversely.

Proof. Omitted. □

Proposition 3.20. *A subset A of a Micro topological space U is said to $m\omega$-open if and only if $P \subseteq mint(A)$ whenever $M \supseteq P$ and P is Micro semi closed in U.*

Proof. Suppose that A is $m\omega$-open in U and M $\supseteq$ P, where P is Micro semi closed in U. Then $A^c \subseteq P^c$, where P^c is Micro semi open in U. Hence we get mcl (A^c) $\subseteq$ P^c implies ($mint(A))^c \subseteq P^c$. Thus, we have mint(A) $\supseteq$ P.
conversely, suppose that $A^c \subseteq$ T and T is Micro semi open in U then A $\supseteq$ T^c and T^c is Micro semi closed then by hypothesis mint(A) $\supseteq$ T^c implies ($mint(A))^c \subseteq$ T. Hence mcl (A^c) $\subseteq$ T gives A^c is $m\omega$-closed. □

Proposition 3.21. *In a Micro topological space U, for each $u \in U$, either $\{u\}$ is Micro semi closed or $\{u\}^c$ is $m\omega$-closed in U.*

Proof. Suppose that {u} is not Micro semi closed in U. Then $\{u\}^c$ is not Micro semi open and the only Micro semi open set containing $\{u\}^C$ is the space U itself. That is $\{u\}^c \subseteq$ U. Therefore, mcl ($\{u\}^C$) $\subseteq$ U and so $\{u\}^C$ is $m\omega$-closed. □

4. $\mathbf{T}_{m\omega}$-SPACES

Definition 4.1. *A space $(U, \tau_R(X), \mu_R(X))$ is called*

(1) *$T_{m1/2}$-space if every mg-closed set is Micro closed.*
(2) *$T_{m\omega}$-space if every $m\omega$-closed set in it is Micro closed.*

Example 4.2. *Let $U = \{1, 2, 3\}$ with $U/R = \{\{2\}, \{1, 3\}\}$ and $X = \{1, 2\}$. The nano topology $\tau_R(X) = \{\phi, \{2\}, \{1, 3\}, U\}$. If $\mu = \{3\}$ then the micro topology $\mu_R(X) = \{\phi, \{2\}, \{3\}, \{1, 3\}, \{2, 3\}, U\}$. Then $m\omega$-closed sets are $\{\phi, \{1\}, \{2\}, \{1, 2\}, \{1, 3\}, U\}$. Thus $(U, \tau_R(X), \mu_R(X))$ is a $T_{m\omega}$-space .*

Example 4.3. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.3. Thus $(U, \tau_R(X), \mu_R(X))$ is not a $T_{m\omega}$-space.*

Proposition 4.4. *Every $T_{m1/2}$-space is $T_{m\omega}$-space but not conversely.*

Proof

Follows from Proposition 3.4. □

Example 4.5. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.5. Thus (U, $\tau_R(X)$, $\mu_R(X)$) is not a $T_{m1/2}$-space.*

Definition 4.6. *A space (U, $\tau_R(X)$, $\mu_R(X)$) is called a $T_{m\alpha}$-space if every Micro α-closed set in it is Micro closed.*

Remark 4.7. *$T_{m\omega}$-spaces and $T_{m\alpha}$-spaces are independent.*

Example 4.8. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.7. Thus (U, $\tau_R(X)$, $\mu_R(X)$) is a $T_{m\omega}$-space but not a $T_{m\alpha}$-space.*

Example 4.9. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.8. Thus (U, $\tau_R(X)$, $\mu_R(X)$) is a $T_{m\alpha}$ but not $T_{m\omega}$-space.*

Theorem 4.10. *For a space (U, $\tau_R(X)$, $\mu_R(X)$) the following properties are equivalent:*

(1) *(U, $\tau_R(X)$, $\mu_R(X)$) is a $T_{m\omega}$-space.*
(2) *Every singleton subset of (U, $\tau_R(X)$, $\mu_R(X)$) is either Micro semi closed or Micro open.*

Proof

(i) → (ii). Assume that for some u ∈ U, the set {u} is not a Micro semi closed in (U, $\tau_R(X)$, $\mu_R(X)$). Then the only Micro semi-open set containing $\{u\}^c$ is U and so $\{u\}^c$ is $m\omega$-closed in (U, $\tau_R(X)$, $\mu_R(X)$). By assumption $\{u\}^c$ is Micro closed in (U, $\tau_R(X)$, $\mu_R(X)$) or equivalently {u} is Micro open.

(ii) $\to$ (i). Let M be a $m\omega$-closed subset of (U, $\tau_R(X)$, $\mu_R(X)$) and let u $\in$ mcl(S). By assumption {u} is either Micro semi-closed or Micro open.

Case (a) Suppose that {u} is Micro semi-closed. If u $\notin$ S, then mcl(S) $-$ S contains a nonempty Micro semi-closed set {u}, which is a contradiction to Theorem 3.13. Therefore x $\in$ S.

Case (b) Suppose that {u} is Micro open. Since u $\in$ mcl(S), {u} $\cap$ S $\neq \phi$ and so u $\in$ S. Thus in both case, u $\in$ S and therefore mcl(S) $\subseteq$ S or equivalently S is a Micro closed set of (U, $\tau_R(X)$, $\mu_R(X)$).

5. ${}_g\mathbf{T}_{m\omega}$-SPACES

Definition 5.1. *A space (U, $\tau_R(X)$, $\mu_R(X)$) is called a ${}_gT_{m\omega}$-space if every mg-closed set in it is mω-closed.*

Example 5.2. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.3. Then mg-closed sets are ϕ, {2}, {1, 2}, {2, 3}, U. Thus (U, $\tau_R(X)$, $\mu_R(X)$) is a ${}_gT_{m\omega}$-space and the space (U, $\tau_R(X)$, $\mu_R(X)$) in the Example 3.5, is not a ${}_gT_{m\omega}$-space.*

Proposition 5.3. *Every $T_{m1/2}$-space is ${}_gT_{m\omega}$-space but not conversely.*

Proof

Follows from Proposition 3.2. □

Example 5.4. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 5.2, is a ${}_gT_{m\omega}$-space but not a $T_{m1/2}$-space.*

Remark 5.5. *$T_{m\omega}$-space and ${}_gT_{m\omega}$-space are independent.*

Example 5.6. *The space (U, $\tau_R(X)$, $\mu_R(X)$) in the Example 5.2, is a ${}_gT_{m\omega}$-space but not a $T_{m\omega}$-space and the space (U, $\tau_R(X)$, $\mu_R(X)$) in the Example 3.5, is a $T_{m\omega}$-space but not a ${}_gT_{m\omega}$-space.*

Theorem 5.7. *If (U, $\tau_R(X)$, $\mu_R(X)$) is a ${}_gT_{m\omega}$-space, then every singleton subset of (U, $\tau_R(X)$, $\mu_R(X)$) is either mg-closed or $m\omega$-open.*

Proof

Assume that for some x ∈ X, the set {x} is not a mg-closed in (U, $\tau_R(X)$, $\mu_R(X)$). Then {x} is not a Micro closed set, since every Micro closed set is a mg-closed set. So $\{x\}^c$ is not Micro open and the only Micro open set containing $\{x\}^c$ is X itself. Therefore $\{x\}^c$ is trivially a mg-closed set and by assumption, $\{x\}^c$ is an $m\omega$-closed set or equivalently {x} is $m\omega$-open.

The converse of Theorem 5.7 need not be true as seen from the following example.

Example 5.8. *Let U = {1, 2, 3} with U/ R= {{1, 2, 3}} and X= {2, 3}. The nano topology $\tau_R(X) = \{\phi, U\}$. If $\mu = \{2\}$. then the micro topology $\mu_R(X) = \{\phi, \{2\}, U\}$. Then $m\omega$-closed sets are = {φ, {1, 3}, U} and mg-closed sets are {φ, {1}, {3}, {1, 2}, {1, 3}, {2, 3}, U}. The sets {1} and {3} are mg-closed in (U, $\tau_R(X)$, $\mu_R(X)$) and the set {2} is $m\omega$-open. But the space (U, $\tau_R(X)$, $\mu_R(X)$) is not a ${}_gT_{m\omega}$-space.*

Theorem 5.9. *A space (U, $\tau_R(X)$, $\mu_R(X)$) is $T_{m1/2}$ if and only if it is both $T_{m\omega}$ and ${}_gT_{m\omega}$.*

Proof

Necessity. Follows from Propositions 4.4 and 5.3.

Sufficiency. Assume that (U, $\tau_R(X)$, $\mu_R(X)$) is both $T_{m\omega}$ and ${}_gT_{m\omega}$. Let A be a mg-closed set of (U, $\tau_R(X)$, $\mu_R(X)$). Then A is $m\omega$-closed, since (U, $\tau_R(X)$, $\mu_R(X)$) is a ${}_gT_{m\omega}$. Again since (U, $\tau_R(X)$, $\mu_R(X)$) is a $T_{m\omega}$, A is a Micro closed set in (U, $\tau_R(X)$, $\mu_R(X)$) and so (U, $\tau_R(X)$, $\mu_R(X)$) is a $T_{1/2}$.

6. $m\omega$-CONTINUOUS MAPS

Definition 6.1. *A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) is called Micro ω-continuous (briefly, mω-continuous) if $f^{-1}(V)$ is a mω-closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every Micro closed set V of (L, $\tau'_R(Y)$, $\mu'_R(Y)$).*

Theorem 6.2. *Every Micro continuous is mω-continuous but not conversely.*

Proof. Follows from Proposition 3.2. □

Example 6.3. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.3. Let L = {1, 2, 3} with L/ R= {{1}, {2, 3}} and Y= {1}. The nano topology $\tau'_R(Y)$ = {ϕ, {1}, L}. If μ = {1, 3} then the micro topology $\mu'_R(Y)$ = {ϕ, {1}, {1, 3}, L}. Define f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) be the identity map. Then f is mω-continuous but not Micro continuous, since f^{-1}({2, 3})= {2, 3} is not Micro closed in (U, $\tau_R(X)$, $\mu_R(X)$).*

Theorem 6.4. *Every mω-continuous is mg-continuous but not conversely.*

Proof. Follows from Proposition 3.4. □

Example 6.5. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.5. Let L = {1, 2, 3} with L/ R= {{1}, {2, 3}} and Y= {3}. The nano topology $\tau'_R(Y)$ = {ϕ, {3}, L}. If μ = {1, 3} then the micro topology $\mu'_R(Y)$ = {ϕ, {3}, {1, 3}, L}. Define f : (U, $\tau_R(X)$, $\mu_R(X)$) $\to$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) be the identity map. Then f is mg-continuous but not mω-continuous, since f^{-1}({1, 2})= {1, 2} is not mω-closed in (U, $\tau_R(X)$, $\mu_R(X)$).*

Remark 6.6. *The following examples show that mω-continuity are independent of Micro α-continuity and Micro semi-continuity.*

Example 6.7. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.5. Let L = {1, 2, 3} with L/ R= {{1, 2, 3}} and Y= {1, 2}. The nano topology $\tau'_R(Y)$ = {ϕ, L}. If μ = {1, 2} then the micro topology $\mu'_R(Y)$ = {ϕ, {1, 2}, L}. Define f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) be the identity map. Then f is Micro α-continuity and Micro semi-continuity but not mω-continuity, since f^{-1}({3})= {3} is not mω-closed in (U, $\tau_R(X)$, $\mu_R(X)$).*

Example 6.8. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 3.8. Let L = {1, 2, 3} with L/ R= {{1}, {2, 3}} and Y= {1}. The nano topology $\tau'_R(Y)$ = {ϕ, {1}, L}. If μ = {3} then the micro topology $\mu'_R(Y)$ = {ϕ, {1}, {3}, {1, 3}, L}. Define f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) be the identity map. Then f is mω-continuity but not Micro α-continuity and not Micro semi-continuity, since f^{-1}({1, 2}) = {1, 2} is not Micro α-closed and not Micro semi-closed in (U, $\tau_R(X)$, $\mu_R(X)$).*

Remark 6.9. *The composition of two mω-continuous maps need not be mω-continuous and this is shown from the following example.*

Example 6.10. *Let U, $\tau_R(X)$, μ and $\mu_R(X)$ as in the Example 4.2. Let L = {1, 2, 3} with L/ R= {{2}, {1, 3}} and Y= {2}. The nano topology $\tau'_R(Y)$ = {ϕ, {2}, L}. If μ = {1, 3} then the micro topology $\mu'_R(Y)$ = {ϕ, {2}, {1, 3}, L}. Let Q = {1, 2, 3} with Q/ R= {{3}, {1, 2}, {2, 3}} and Z = {1, 2}. The nano topology $\tau'_R(Z)$ = {ϕ, {1, 2}, Q}. If μ = {1, 3} then the micro topology $\mu'_R(Z)$ = {ϕ, {1}, {1, 2}, {1, 3}, Q}. Then mω-closed sets are = {ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, L}. Define f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) and g : (L, $\tau'_R(Y)$, $\mu'_R(Y)$) $\rightarrow$ (Q, $\tau^*_R(Z)$, $\mu^*_R(Z)$) are the identity map. Clearly f and g are mω-continuous but their g $\circ$ f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (Q, $\tau^*_R(Z)$, $\mu^*_R(Z)$) is not mω-continuous, because V = {2, 3} is Micro closed in (Q, $\tau^*_R(Z)$, $\mu^*_R(Z)$) but (g $\circ$ f $^{-1}$({2, 3})= f^{-1}(g^{-1}({2, 3})) = f^{-1}({2, 3}) = {2, 3}, which is not mω-closed in (U, $\tau_R(X)$, $\mu_R(X)$) .*

Theorem 6.11. *A map f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) is $m\omega$-continuous if and only if f^{-1}(U) is $m\omega$-open in (U, $\tau_R(X)$, $\mu_R(X)$) for every Micro open set U in (L, $\tau'_R(Y)$, $\mu'_R(Y)$).*

Proof. Let f : (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) be $m\omega$-continuous and U be an Micro open set in (L, $\tau'_R(Y)$, $\mu'_R(Y)$). Then U^c is Micro closed in (L, $\tau'_R(Y)$, $\mu'_R(Y)$) and since f is $m\omega$-continuous, $f^{-1}(U^c)$ is $m\omega$-closed in (U, $\tau_R(X)$, $\mu_R(X)$) . But $f^{-1}(U^c) = f^{-1}((U))^c$ and so $f^{-1}(U)$ is $m\omega$-open in (U, $\tau_R(X)$, $\mu_R(X)$).
Conversely, assume that $f^{-1}(U)$ is $m\omega$-open in (U, $\tau_R(X)$, $\mu_R(X)$) for each Micro open set U in (L, $\tau'_R(Y)$, $\mu'_R(Y)$). Let F be a Micro closed set in (L, $\tau'_R(Y)$, $\mu'_R(Y)$). Then F^c is Micro open in (L, $\tau'_R(Y)$, $\mu'_R(Y)$) and by assumption, $f^{-1}(F^c)$ is $m\omega$-open in (U, $\tau_R(X)$, $\mu_R(X)$). Since $f^{-1}(F^c) = f^{-1}((F))^c$, we have $f^{-1}(F)$ is Micro closed in (U, $\tau_R(X)$, $\mu_R(X)$) and so f is $m\omega$-continuous. □
We introduce the following definition

Definition 6.12. *A map f: (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) is called $m\omega$-irresolute if f^{-1}(V) is a $m\omega$-closed set of (U, $\tau_R(X)$, $\mu_R(X)$) for every $m\omega$-closed set V of (L, $\tau'_R(Y)$, $\mu'_R(Y)$).*

Theorem 6.13. *Every $m\omega$-irresolute map is $m\omega$-continuous but not conversely.*

Proof. Let f: (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) be a $m\omega$-irresolute map. Let V be a Micro closed set of (L, $\tau'_R(Y)$, $\mu'_R(Y)$). Then by the Proposition 3.2, V is $m\omega$-closed. Since f is $m\omega$-irresolute, then $f^{-1}(V)$ is a $m\omega$-closed set of (U, $\tau_R(X)$, $\mu_R(X)$). Therefore f is $m\omega$-continuous. □

Example 6.14. *Let U, $\tau_R(X)$, μ, $\mu_R(X)$, L, $\tau'_R(Y)$, μ, $\mu'_R(Y)$ and f as in the Example 6.10. It is clear that $\{2, 3\}$ is $m\omega$-closed set of (L, $\tau'_R(Y)$, $\mu'_R(Y)$) but $f^{-1}(\{2, 3\})$*

=\{2, 3\} is not a $m\omega$-closed set of (U, $\tau_R(X)$, $\mu_R(X)$). Thus f is not $m\omega$-irresolute map. However f is $m\omega$-continuous map.

Theorem 6.15. *Let f: (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) and g : (L, $\tau'_R(Y)$, $\mu'_R(Y)$) $\rightarrow$ (Q, $\tau^*_R(Z)$, $\mu^*_R(Z)$) be any two maps. Then*

(1) *$g \circ f$ is $m\omega$-continuous if g is Micro-continuous and f is $m\omega$-continuous.*
(2) *$g \circ f$ is $m\omega$-irresolute if both f and g are $m\omega$-irresolute.*
(3) *$g \circ f$ is $m\omega$-continuous if g is $m\omega$-continuous and f is $m\omega$-irresolute.*

Proof. (1) Since g is a Micro-continuous from (L, $\tau'_R(Y)$, $\mu'_R(Y)$) $\rightarrow$ (Q, $\tau^*_R(Z)$, $\mu^*_R(Z)$), for any Micro closed set q as a subset of Q, we get $g^{-1}(q) = G$ is a Micro closed set in (L, $\tau'_R(Y)$, $\mu'_R(Y)$). As f is a $m\omega$-continuous map. We get $(g \circ f)^{-1}(q) = f^{-1}(g^{-1}(q)) = f^{-1}(G) = S$ and S is a $m\omega$-closed set in (U, $\tau_R(X)$, $\mu_R(X)$). Hence (g $\circ$ f) is a $m\omega$-continuous map.
(2) Consider two $m\omega$-irresolute maps, f: (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) and g : (L, $\tau'_R(Y)$, $\mu'_R(Y)$) $\rightarrow$ (Q, $\tau^*_R(Z)$, $\mu^*_R(Z)$) is a $m\omega$-irresolute maps. As g is consider to be a $m\omega$-irresolute map, by Definition 6.12, for every $m\omega$-closed set q $\subseteq$ (Q, $\tau^*_R(Z)$, $\mu^*_R(Z)$), $g^{-1}(q) = G$ is a $m\omega$-closed in (L, $\tau'_R(Y)$, $\mu'_R(Y)$). Again since f is $m\omega$-irresolute, $(g \circ f)^{-1}(q) = f^{-1}(g^{-1}(q)) = f^{-1}(G) = S$ and S is a $m\omega$-closed set in (U, $\tau_R(X)$, $\mu_R(X)$). Hence (g $\circ$ f) is a $m\omega$-irresolute map.
(3) Let g be a $m\omega$-continuous map from (L, $\tau'_R(Y)$, $\mu'_R(Y)$) $\rightarrow$ (Q, $\tau^*_R(Z)$, $\mu^*_R(Z)$) and q subset of Q be a Micro closed set. Therefore $g^{-1}(q) = G$ is a $m\omega$ closed set in (L, $\tau'_R(Y)$, $\mu'_R(Y)$). Also since f is $m\omega$-irresolute, we get $(g \circ f)^{-1}(q) = f^{-1}(g^{-1}(q)) = f^{-1}(G) = S$ and S is a $m\omega$-closed set in (U, $\tau_R(X)$, $\mu_R(X)$). Hence (g $\circ$ f) is a $m\omega$-continuous map. □

Theorem 6.16. *Let f: (U, $\tau_R(X)$, $\mu_R(X)$) $\rightarrow$ (L, $\tau'_R(Y)$, $\mu'_R(Y)$) be an $m\omega$-continuous map. If (U, $\tau_R(X)$, $\mu_R(X)$), the domain of f is an $T_{m\omega}$-space, then f is Micro-continuous.*

Proof. Let V be a Micro closed set of (L, $\tau'_R(Y)$, $\mu'_R(Y)$). Then f^{-1}(V) is a $m\omega$-closed set of (U, $\tau_R(X)$, $\mu_R(X)$), since f is $m\omega$-continuous. Since (U, $\tau_R(X)$, $\mu_R(X)$) is an $T_{m\omega}$-space, then f^{-1}(V) is a Micro closed set of (U, $\tau_R(X)$, $\mu_R(X)$). Therefore f is Micro-continuous.

Conclusion

In this paper, we have defined and studied some basic properties of $m\omega$-closed sets, $T_{m\omega}$-spaces, ${}_gT_{m\omega}$-spaces and $m\omega$-continuous map. In future, we have extend this work in various Micro topological fields.

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