

# Dimensional Synthesis of a Four-Bar Mechanism With Prescribed Timing For Given Precession Points

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## Abstract:

Dimensional synthesis is a part of the kinematic synthesis that requires the mechanism dimensions in such a way that the given mechanism can achieve a given prescribed motion sequence. Among all the three categories of dimensional synthesis, rigid body guidance is considered to be the most challenging task as it requires proper positioning as well as orientation of the body in space as per requirement. In this paper, a dyad based approach is presented for the dimensional synthesis of a four-bar mechanism for rigid body guidance. The approach is justified with the help of a numerical example, which suggests that the method is more powerful and easy to apply on such problems.

**Keywords** —dimensional synthesis, four bar mechanism, prescribed timings, rigid body guidance.

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## I. INTRODUCTION

Dimensional synthesis is a part of the kinematic synthesis in which the goal is to find the mechanism dimensions which is required to achieve a given prescribed mechanism motion sequence [1]. The mechanism dimension calculation involves calculation of the link lengths, relative or absolute rigid-body positions (link positions), and link end coordinates. The mechanism parameters required for the prescribed motion of a mechanism in a sequence include link positions, path points, and displacement angles. While in kinematic analysis the dimensions of the mechanism are known in advance and the resulting motion has to be calculated while in dimensional synthesis, the reverse process is used. the mechanism motion sequence is known, and dimensions are obtained as depicted in figure 1.

In the process of dimensional synthesis, the three subcategories are defined as:

- (1) motion generation,
- (2) path generation, and

- (3) function generation
- In case of motion generation, the dimensions of

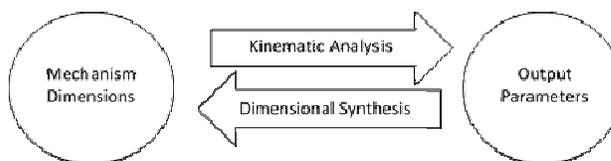


Figure 1: Kinematic analysis vs dimensional synthesis

mechanisms are calculated in order to achieve given rigid-body positions, while in path generation the body is supposed to pass through the prescribed path points. In function generation, mechanism dimensions are required to meet the given crank and follower-link displacement angles to trace an approximate input out relation governed by a given function. The detailed difference and common points on these categories are given in [3] which provides the light on the dimensional synthesis of mechanism and gives the qualitative and

quantitative techniques used for path, motion, and function generation.

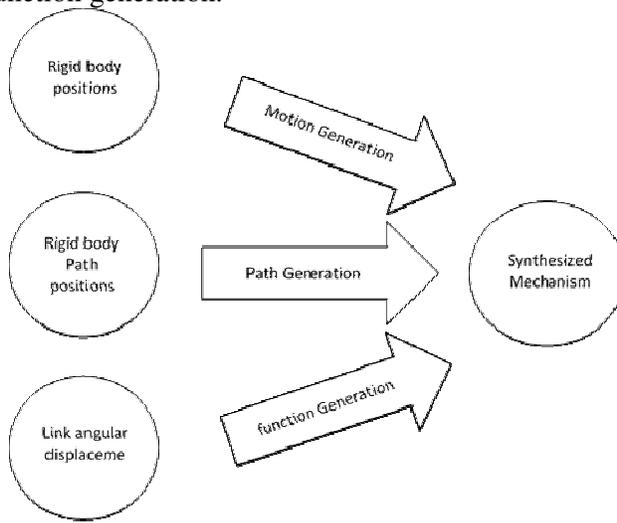


Figure 2 Types of Dimensional synthesis

qualitative methods of the dimensional synthesis are oldest methods and include graphical techniques, these techniques can provide a good amount of information and usually is free of computational efforts [4] while in the second category of quantitative methods which includes mathematical modelling of the mechanism and requires the equations to be solved analytically or numerically using the algorithms and or solution of the algebraic equations [5]. In this section, the quantitative method of dimensional synthesis is used.

**II. PLANAR FOUR-BAR MOTION GENERATION:  
 THREE PRECISION POSITIONS**

Figure 3 shows a planar mechanism (four-bar) which achieves a three coupler positions as shown.

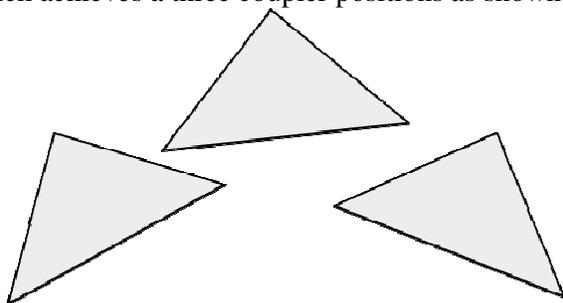


Figure 3 Example of Rigid body guidance

The orientation, location and the position of the tracer which is a coupler can be defined using the coordinates of the given rigid-body point as well as the rigid-body displacement angle. In the case of motion generation, the given mechanism dimensions are required to be calculated in order to achieve precision positions. As an example: A motion generation is given in figure 4 which illustrates the sequence of operation of a wheel loading mechanism of a landing gear. In order to achieve effective operation, the wheel loading mechanism must be fitted inside the frame of the vehicle and should be guided from the frame to a given position so that the wheels can make a proper contact on the ground. The given function is a case of rigid body guidance system and a planar four-bar motion generation problem can be carried out using the equations given below. The given system of equations provides two dyads solution which is simple and effective method. These equations are useful in order to calculate the four-bar mechanism dimensions required and can be used to solve a set of three precision positions. Figure 4 gives us the position of both the dyads of a planar four-bar mechanism in a starting position which is described as Position 1 and the dotted line gives us the displaced position marked as Position j.

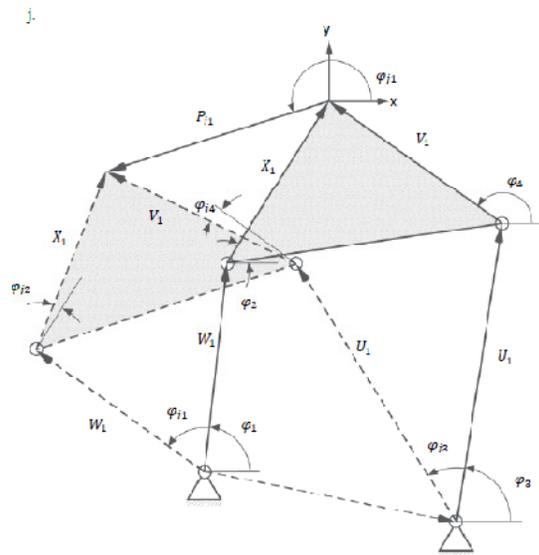


Figure 1 Displaced position of a four-bar mechanism

The figure shows two dyads which include the vector chains W–X and U–V, respectively. The equations can be formed by taking the loop of the vector sum in the starting and displaced positions for each given dyad, these vector-loop equations are also derived in [6, 7]. The equations are derived by taking the counter-clockwise rotation of the vector sum for each given dyad, the vector-loop equations as written for W-X loop taking into account the displacement P and U-V loop with displacement P for the given four-bar mechanism dyads are:

$$W_1 e^{i\varphi 1} + X_1 e^{i\varphi 2} + P_{j1} e^{i\varphi j1} - W_1 e^{i\varphi 1 + \varphi j1} - X_1 e^{i\varphi 2 + \varphi j2} = 0$$

$$U_1 e^{i\varphi 3} + V_1 e^{i\varphi 4} + P_{j1} e^{i\varphi j1} - U_1 e^{i\varphi 3 + \varphi j3} - V_1 e^{i\varphi 4 + \varphi j4} = 0 \quad (1.1)$$

The above equations are factorised for the unknown terms resulting standard-form vector-loop equations given as:

$$W_1 e^{i\varphi 1} (e^{i\varphi j1} - 1) + X_1 e^{i\varphi 2} (e^{i\varphi j2} - 1) = P_{j1} e^{i\varphi}$$

$$U_1 e^{i\varphi 3} (e^{i\varphi j3} - 1) + V_1 e^{i\varphi 4} (e^{i\varphi j4} - 1) = P_{j1} e^{i\varphi} \quad (1.2)$$

On expanding equation 1.2 in to the real and imaginary parts and writing the separate equations for both gives us the following set:

$$W_1 \cos \varphi 1 (\cos \varphi j1 - 1) - W_1 \sin \varphi 1 \sin \varphi j1 + X_1 \cos \varphi 2 (\cos \varphi j2 - 1) - X_1 \sin \varphi 2 \sin \varphi j2 = P_{j1} \cos \varphi$$

$$W_1 \sin \varphi 1 (\cos \varphi j1 - 1) + W_1 \cos \varphi 1 \sin \varphi j1 + X_1 \cos \varphi 2 (\cos \varphi j2 - 1) - X_1 \sin \varphi 2 \sin \varphi j2 = P_{j1} \sin \varphi \quad (1.3)$$

For simplification putting  $W_1 \cos \varphi 1 = W_{1x}$ ,  $W_1 \sin \varphi 1 = W_{1y}$ ,  $X_1 \cos \varphi 2 = X_{1x}$ , and  $X_1 \sin \varphi 2 = X_{1y}$ , Equation 3.3 becomes

$$W_{1x} (\cos \varphi j1 - 1) - W_{1y} \sin \varphi j1 + X_{1x} (\cos \varphi j2 - 1) - X_{1y} \sin \varphi j2 = P_{j1} \cos \varphi$$

$$W_{1y} (\cos \varphi j1 - 1) + W_{1x} \sin \varphi j1 + X_{1y} (\cos \varphi j2 - 1) - X_{1x} \sin \varphi j2 = P_{j1} \sin \varphi \quad (1.4)$$

Similarly, separating the real and imaginary parts for the U-V dyads and specifying as above, the given equation becomes

$$U_{1x} (\cos \varphi j3 - 1) - U_{1y} \sin \varphi j3 + V_{1x} (\cos \varphi j4 - 1) - V_{1y} \sin \varphi j4 = P_{j1} \cos \varphi$$

$$U_{1y} (\cos \varphi j3 - 1) + U_{1x} \sin \varphi j3 + V_{1y} (\cos \varphi j4 - 1) + V_{1x} \sin \varphi j4 = P_{j1} \sin \varphi \quad (1.5)$$

The given equations 1.4 and 1.5 can be solved in matrix form for three precision positions which can be achieved by specifying the displacement position,  $j = 2, 3$ , Equations 1.4 and 1.5 can be expressed in the given matrix form.

$$\begin{bmatrix} (\cos \varphi j1 - 1) & -\sin \varphi j1 & (\cos \varphi j2 - 1) & -\sin \varphi j2 \\ \sin \varphi j1 & (\cos \varphi j1 - 1) & \sin \varphi j2 & (\cos \varphi j2 - 1) \\ (\cos \varphi j1 - 1) & -\sin \varphi j1 & (\cos \varphi j2 - 1) & -\sin \varphi j2 \\ \sin \varphi j1 & (\cos \varphi j1 - 1) & \sin \varphi j2 & (\cos \varphi j2 - 1) \end{bmatrix} \begin{bmatrix} W_{1x} \\ W_{1y} \\ X_{1x} \\ X_{1y} \end{bmatrix} = \begin{bmatrix} P_{21} \cos \varphi \\ P_{21} \sin \varphi \\ P_{31} \cos \varphi \\ P_{31} \sin \varphi \end{bmatrix} \quad (1.6)$$

$$\begin{bmatrix} (\cos \varphi j3 - 1) & -\sin \varphi j3 & (\cos \varphi j4 - 1) & -\sin \varphi j4 \\ \sin \varphi j3 & (\cos \varphi j3 - 1) & \sin \varphi j4 & (\cos \varphi j4 - 1) \\ (\cos \varphi j3 - 1) & -\sin \varphi j3 & (\cos \varphi j4 - 1) & -\sin \varphi j4 \\ \sin \varphi j3 & (\cos \varphi j3 - 1) & \sin \varphi j4 & (\cos \varphi j4 - 1) \end{bmatrix} \begin{bmatrix} U_{1x} \\ U_{1y} \\ V_{1x} \\ V_{1y} \end{bmatrix} = \begin{bmatrix} P_{21} \cos \varphi \\ P_{21} \sin \varphi \\ P_{31} \cos \varphi \\ P_{31} \sin \varphi \end{bmatrix} \quad (1.7)$$

These above equations 1.6 and 1.7 can be solved using any standard mathematical form such as Cramer’s rule to find out the dyad’s components  $W_1$ – $X_1$  and  $U_1$ – $V_1$ , respectively. In Equation 1.6, angles  $\varphi j1$  and  $\varphi j2$  are considered as “free choices” as they can be prescribed as per the required precision positions “free choice” is defined as the variable that can be specified as per the requirement and is based on the person’s own preferences. Similarly, in equation 1.7, given angles  $\varphi j3$  and  $\varphi j4$  are considered as the free choices as they can be prescribed as per the precision positions required by the body.

As the required conditions can be fulfilled by an infinite way of unique combinations of angles  $\varphi j1, \varphi j2, \varphi j3$  and  $\varphi j4$ , the possible number dyad solutions from above equations can be infinite. This makes the solution of such problems very demanding and cumbersome.

The mechanism resulting from the above equations achieves the precision positions, but also performs desired operation in according with prescribed dyad displacement angles. By using the combination of both precision positions as well as the prescribed dyad displacement angles, the resulting mechanism provides the best feasible solution solutions which are free of order defects and corresponding dyad displacements are specified.

### III. NUMERICAL EXAMPLE

The problem-solving method shown above is implemented using an example of wheel loading mechanism as shown in figure 5. The three

positions achieved by the wheel loading mechanism is shown below. It is desired to achieve the three positions as shown. The positions and angular displacements are given in table 1.

TABLE I  
INPUT DATA

Sr.	Precession points	Pj	$\phi_{j4}(0)$	$\phi_{j1}(0)$	$\phi_{j3}(0)$
1	1	0,0	--	--	--
2	2	0.2, 0.7	-50	18	-40
3	3	0.2,1.4	-80	40	-90

Using the above input values equation 6 and 7 are solved in MATLAB software and the results are shown in figure 6.

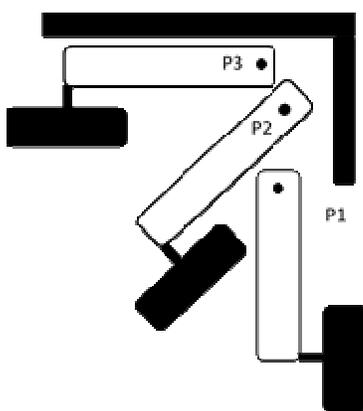


Figure 2 Landing and loading position of the wheel

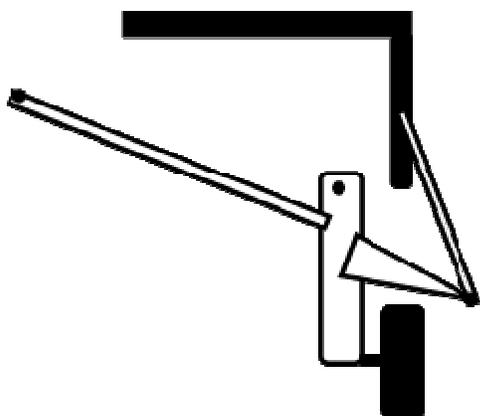


Figure 6 Synthesised mechanism

The link lengths are calculated based on the formulation shown above. The computed link lengths (dyad lengths) are W-X 2.06, U-V 0.60. And the resultant four bar mechanism for the landing wheel design is shown in figure 5.

#### IV. CONCLUSIONS

The resultant wheel precisely follows the loading and unloading positions. The method suggested in this paper is straight forward method and can be used for more complex mechanism such as 6-bar mechanism. This method can easily be extended for more precision points i.e more than three precision points.

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