

A New Interpretation for Quantum Mechanics and Its Related Tests Based on Schrodinger Equation and the Law of the Conservation of Energy

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Abstract

In this paper, authors propose a new theoretical particle model for quantum mechanics based on the frame of de Broglie wave-particle duality, Schrödinger Equation and the Law of Conservation of Energy. Based on this model, a group of Schrödinger random equations are used to describe the motion of particle. It is found the wave function in Schrödinger equation is not only probability function and also it is displacement function. And also authors use the group of equations to investigate the following experiments: Single Electron Interference Experiment, Double Slit Experiment, Bell's Inequality Experiments and Schrödinger's Cat Thought Experiment. The investigations answer the questions and solve the mysteries in the experiments so many years for people about these experiments and the Copenhagen interpretation. Finally, authors use the group of Schrödinger random equations to investigate quantum entanglement phenomena and conclude that the quantum entanglement exists in the microscopic world but it is only a random physical phenomenon.

Keywords:

Interpretation of Quantum Mechanics, Schrödinger Equation, the group of Schrödinger Random Equations, Wave-Particle Duality, Wave Function, Wave Function Collapse, Wave Packet Expansion, Single Electron Interference Experiment, Double Slit Experiment, Quantum Eraser Experiment, Delayed Choice Experiment, Bell's Inequality, Schrodinger's Cat Thought Experiment, Quantum Entanglement, Quantum Superposition, Copenhagen interpretation.

1. Introduction

As we know that quantum mechanics was founded about 80 years ago and has been used to investigate the microscopic world, [1] – [16]. From its establishment to now, the development of quantum mechanics has not only laid the solid theoretical foundation for the development of the many other branches of contemporary science and technology, but also it has obtained many vast achievements in applications of these branches. Even if so, a few questions and puzzles still have been accompanying quantum mechanics, especially about the Copenhagen interpretation of quantum mechanics and Single Slit Interference Experiment and Double Slit Experiment and other experiments, [17] – [24], which is usually thought to be an orthodox interpretation.

When we go back to review the Copenhagen interpretation, it is not too difficult to find that there are some fundamental theoretical questions left to be unanswered, or some interpretations for quantum mechanics make people confused. These unanswered questions and interpretations make quantum mechanics appear to be incomplete and caused some well-known scholars, including Einstein, Schrödinger and Weinberg, made sharp criticisms on the Copenhagen interpretation, [13], [14]. For example, one of the pioneers of quantum mechanics, Schrödinger proposed Schrödinger's Cat Thought Experiment to disagree the statement of the quantum superposition from the Copenhagen interpretation.

So far, during over 80 years, in order to well understand the questions of quantum mechanics or make the Copenhagen interpretation complete, people have proposed a variety of interpretations. For example, Multiple World Interpretation, Quantum Decoherence Interpretation and so on. However, these proposed interpretations could not completely and reasonably explain the unanswered questions in the Copenhagen interpretation and the related experiments, for example, the Double Slit Experiment, refer to Reference [25] – [28].

In this paper, authors propose a new theoretical quantum or particle model for quantum mechanics based on the frame of de Broglie wave-particle duality, Schrödinger Equation and the Law of Conservation of Energy. Based on this model, a group of Schrödinger random equations are built and used to describe the motion of particle. By using the group of Schrödinger random equations, it is found the wave function, $\Psi(q_j, t)$, is not only the probability function interpreted by the Copenhagen interpretation, and also it is a displacement function of which is used to describe the motion of quantum or particle. In addition to the above ones, authors use the newly developed model and the group of Schrödinger random equations to explain the wave function collapse from the Copenhagen interpretation and the wave packet expansion. And also, authors investigate the following experiments: Single Electron Interference Experiment, Double Slit Experiment, Bell's Inequality Experiments and Schrödinger's Cat Thought Experiment. It shows that the this new theoretical quantum or particle model can correctly and logically solve the mysteries from the experiments for people so many years about. It means these the experiments support the new particle model proposed in this paper directly or indirectly. Finally, authors use this model to theoretically investigate quantum entanglement phenomena. The investigation indicates that the quantum entanglement can exist, but it only is random phenomena of quantum or particle.

2. Physical Model of Quantum

Since de Broglie proposed electron possessing wave-particle duality in 1923, [9], the people have realized the particles in the microscopic world possess wave-particle duality. This conclusion has been proved by many experiments. However, how to understand and explain wave-particle duality of a particle is different for different people, [23]. For example, the Copenhagen interpretation is to use the complementary principle to explain wave-particle duality based on the Double Slit Experiment. This explanation has governed the academia for many years, but it gradually shows its limitations, as some conclusions of which are very difficult to accept by people and some strange points of view could be easily derived from this principal, like a state of quantum superposition. For example, the very famous scientist Schrödinger proposed Schrödinger's Cat' Thought Experiment to disagree this principle.

In this paper, authors propose a physical model for a moving quantum or particle, the below only term of particle is used, as follow:

- 1) A moving particle possesses the wave-particle duality proposed by de Broglie and satisfies Plank Law. A free particle corresponds to a plan wave and its mathematical expression is as follows:

$$\psi(x,t) = e^{i(px-Et)/\hbar} \quad (1)$$

$$E = h\nu \quad (2)$$

$$p = \frac{h}{\lambda} \quad (3)$$

In the above formulas,

$\psi(x,t)$ is wave function from de Broglie wave, but it satisfies Schrödinger Equation;

t is time;

x is coordinate;

h is Plank constant;

ν is frequency of wave;

p is momentum;

E is particle's energy;

λ is wave length;

i is a complex number, $i = \sqrt{-1}$.

- 2) The motion of the particle can be described by Schrödinger's Time-Dependent Equation,

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_i, t) + V(q_j, t) \Psi(q_j, t) = i\hbar \frac{\partial \Psi(q_j, t)}{\partial t} \quad (4)$$

In the above equation,

m is a particle mass;

$V(x, t)$ is potential energy function;

∇^2 is the Laplacian operator;

q_j is general coordinates, $q_j = x, y, z$;

$\Psi(q_j, t)$ is a wave function. But it is noticed that $\Psi(q_j, t)$ is used to denote the complete state wave function and lower $\psi(q_j)$ is used to represent the state function with the time dependence removed.

- 3) The motion of the particle is caused by external energy, E_n , through stimulating. E_n is a quantized and random variable decided by its random frequency ν_n , that is, E_n satisfies Plank Law. The direction and amount of stimulating particle are random variables, that is,

$$E_1 = h\nu_1;$$

$$E_2 = h\nu_2;$$

$$E_3 = h\nu_3;$$

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$$E_n = h\nu_n;$$

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In the above equations,

$$n = 1, 2, 3, \Lambda;$$

ν_j is random frequency and $\nu_j \neq \nu_k$, where j and k are natural numbers, $j = 1, 2, 3, \Lambda$

$$k = 1, 2, 3, \Lambda$$

E_n is quantized and random external energy.

- 4) During the particle moving, for different transient time interval, $t_1, t_2, t_3, \Lambda, t_n, \Lambda$, the external energy E_n possesses different values,

$$E_1 = h\nu_1, 0 \leq t \leq t_1;$$

$$E_2 = h\nu_2, t_1 \leq t \leq t_2;$$

$$E_3 = h\nu_3, t_2 \leq t \leq t_3; \tag{5}$$

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$$E_n = h\nu_n, t_{n-1} \leq t \leq t_n$$

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In the above equation,
 t_n is time and $n = 1, 2, 3, \Lambda$

5) During the motion of the particle, the Law of Conservation of Energy is always satisfied.

Based the above five assumptions, we can write mathematical expression for this model.

3. Mathematical Description of Physical Model

3.1 Group of Schrödinger’s Equations

We know that a moving particle can be described by Schrödinger’s Equation, Equation (4) in quantum mechanics. When securitizing each item in Equation (4), we can notice that the left side of Equation (4) is the expression of energy summation from the Hamiltonian function, potential energy + kinetic energy. According to our assumption 5), the Law of Conservation of Energy is satisfied during the motion of the particle. So, we treat that Schrödinger’s Equation is the equation of conservation of energy. According to the Law of Conservation of Energy, the moving particle possessing energy is equal to the particle’s absorbing from external energy.

Based on this realization, we think the right side of Equation (4), $h \frac{\partial \Psi(q_j, t)}{\partial t}$, is the item of the external energy. According to our assumption 4), the external energy is a quantized and random variable, that is, this external energy satisfies Plank Law:

$$E_1 = h\nu_1, E_2 = h\nu_2, E_3 = h\nu_3, \Lambda, E_n = h\nu_n \Lambda$$

Let the right side of Schrödinger’s Equation be equal to random external energy, then we have that

$$h \frac{\partial \Psi_j(q_j, t)}{\partial t} = E_j = h\nu_j \quad (6)$$

$$\Rightarrow \frac{\partial \Psi_j(q_j, t)}{\partial t} = \nu_j \quad (7)$$

$$\Rightarrow \Psi_j(q_j, t) = \int \nu_j dt + F_j(q_j)$$

$$\Rightarrow \Psi_j(q_j, t) = \nu_j t + F_j(q_j) = F_j(q_j) \left[1 + \frac{\nu_j t}{F_j(q_j)} \right]$$

where $F_j(q_j)$ is a function of q_j . This equation tells us that $\Psi_j(q_j, t)$ is a linear function of t .

According to Equation (4) and Equation (7), for different random frequency, we should have different Schrödinger's Equations. That is to say, we can use a group of Schrödinger's Equation to describe a moving particle during its moving. This group of Schrödinger's Equations is as follows:

$$\begin{aligned} -\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= ih\nu_1 & 0 \leq t \leq t_1 \\ -\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= ih\nu_2 & t_1 \leq t \leq t_2 \\ -\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= ih\nu_3 & t_2 \leq t \leq t_3 \end{aligned} \quad (8)$$

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$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ih\nu_n \quad t_{n-1} \leq t \leq t_n$$

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Equation (8) is a mathematical expression of the model. It is a group of three dimensional and random Schrödinger's Equations with time state. It tells us that during transient time interval, $t_{n-1} \leq t \leq t_n$, the motion of the particle is certain motion and can be described by Schrödinger equation. Since the particle is stimulated by a quantized and random external energy, E_n , after

this transient time interval, the motion of the particle will be transferred to another certain motion. This new certain motion of the particle also can be described by another Schrödinger equation. But this transferring from one transient time interval to another is random. During the motion of the particle, the motion of the particle is described by a group of Schrödinger random equations. That is to say, in a very long time period, the motion of the particle is a random motion. However, in a transient time interval, the motion of the particle is a certain motion. The random motion of the particle is composed of a series of certain motions of the particle, as the particle absorbs random external energy. From now on, Equation (8) is called the group of Schrödinger Random Equations in this paper.

Next we discuss the characteristics of the group of Schrödinger Random Equations.

3.2 Characteristics of Group of Schrödinger Random Equations

According to the group of Schrödinger Random Equations, this group of equations possesses the following characteristic:

- a) It possesses the same harmonic equation.

It should be noticed that the left side of the group of Schrödinger Random Equations possess the same expression,

$$-\frac{h^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t)=0 \quad (9)$$

Mathematically, it is a wave equation. This means if the random external energy is zero, the particle moves in free state and its motion can be described by the wave equation. Physically, it is mathematical description of wave-particle duality of a particle. That is, a particle with a mass, m , moves in wave equation.

- b) The group of Schrödinger Random Equations possesses the characteristics of probability.

The group of Schrödinger's Equations is a group of random equations. Although it consists of many certain equations, however, transferring from one certain equation to another is random. For one Schrödinger's Equation, the characteristics of probability do not appear, but for a group of Schrödinger Random Equations, the characteristics of probability will appear.

- c) The group of Schrödinger Random Equations is a group of wave equations.

Any equation in this group of equations possesses all characteristics of wave. For example, any two wave equations will have the same natural frequency. When their phase difference satisfies the condition of interference, they will interfere. For example,

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_n \tag{A}$$

will interfere

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_{n+1} \tag{B}$$

d) If one more external energy function, $E_w = \hbar v$, is added to a moving particle, for example, aobserver, detector andmeasuring device, then group of Schrödinger Random Equations will become that

$$\begin{aligned} -\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= i(\hbar v_1 + \hbar v) & 0 \leq t \leq t_1 \\ -\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= i(\hbar v_2 + \hbar v) & t_1 \leq t \leq t_2 \\ -\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= i(\hbar v_3 + \hbar v) & t_2 \leq t \leq t_3 \end{aligned} \tag{8.1}$$

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$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i(\hbar v_n + \hbar v) \quad t_{n-1} \leq t \leq t_n$$

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Here we discuss this group of equationsfor three cases as follows:

Case 1

When $v \gg v_n$, $E_w = hv \gg E_n = hv_n$ the group of Schrödinger Random Equations will no longer be a random group of equations and becomes only one wave equation. Under the condition, the characteristics of probability will be lost.

$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ihv \quad 0 \leq t \leq t_1$$

$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ihv \quad t_1 \leq t \leq t_2$$

$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ihv \quad t_2 \leq t \leq t_3$$

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$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ihv \quad t_{n-1} \leq t \leq t_n$$

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$$\Rightarrow -\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ihv \quad 0 \leq t \leq t_n \quad (8.2)$$

Case 2

When $v \ll v_n$, then $E_w = hv \ll E_n = hv_n$, the group of Schrödinger Random Equations will still be a group of random equations.

$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ihv_1 \quad 0 \leq t \leq t_1$$

$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ihv_2 \quad t_1 \leq t \leq t_2$$

$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = ihv_3 \quad t_2 \leq t \leq t_3 \quad (8.3)$$

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$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar v_n \quad t_{n-1} \leq t \leq t_n$$

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Case 3

When $v \approx v_n$, then $E_w = \hbar v \approx E_n = \hbar v_n$ the group of Schrödinger’s Equations will be affected by $E_w = \hbar v$, the characteristics of probability will be weakened.

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar(v_1 + v) \quad 0 \leq t \leq t_1$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar(v_2 + v) \quad t_1 \leq t \leq t_2$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar(v_3 + v) \quad t_2 \leq t \leq t_3 \tag{8.4}$$

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$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar(v_n + v) \quad t_{n-1} \leq t \leq t_n$$

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4. Physical Meanings of Wave Function, Wave Function Collapse and Wave Packet Expansion

4.1 Physical Meanings of Wave Function

It is very important to obtain an analysis solution of the group of Schrödinger Random Equations, through the analysis solution we can deeply understand the wave function and the motion of the particle.

Without losing of generality, we solve the following equation,

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar v_n \quad t_{n-1} \leq t \leq t_n \tag{10}$$

The harmonic equation of (10) is Equation (9) and as follows

$$-\frac{h^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = 0$$

In order to well understand the physical meaning of $\Psi(q_j, t)$, we simplify Equation (9) into one dimensional equation by using x to replace q_j . Then we have that

$$-\frac{h^2}{2m} \nabla^2 \Psi(x, t) + V(x, t) \Psi(x, t) = 0$$

$$\Rightarrow -\frac{h^2}{2m} \frac{\partial^2 \Psi(x, t)}{\partial x^2} + V(x, t) \Psi(x, t) = 0$$

$$\Rightarrow \frac{1}{\Psi(x, t)} \frac{\partial^2 \Psi(x, t)}{\partial x^2} = \frac{2m}{h^2} V(x, t)$$

$$\Rightarrow \frac{\partial \ln \Psi(x, t)}{\partial x} = \frac{2m}{h^2} V(x, t)$$

$$\Rightarrow \ln \Psi(x, t) = \int \left[\frac{2m}{h^2} V(x, t) + C_1 \right] dx$$

$$\Rightarrow \Psi(x, t) = e^{\int \left[\frac{2m}{h^2} V(x, t) + C_1 \right] dx} = C e^{\int \left[\frac{2m}{h^2} V(x, t) \right] dx}$$

$$\Rightarrow \Psi(x, t) = C e^{\int \left[\frac{2m}{h^2} V(x, t) \right] dx}$$

If let potential energy function $V(x, t) = -\Delta V(x, t) = V_1(x, t) - V_2(x, t)$ and assume $V_1(x, t) < V_2(x, t)$, then we have

$$\Psi(x, t) = C e^{-\int \left[\frac{2m}{h^2} \Delta V(x, t) \right] dx} \tag{10-1}$$

Equation (10-1) is a solution of harmonic equation, Equation (9). And also, it is a solution of the harmonic equation of the group of Schrödinger Random Equations, too. Apparently, $\Psi(x, t)$ is not a random function and it is a certain and decreased displacement function.

In the group of Schrödinger Random Equations, a typical Schrödinger's Equation can be wrote as follows:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(x,t)+V(x,t)\Psi(x,t)=i\hbar\nu_n \quad t_{n-1}\leq t\leq t_n \quad (11)$$

Let $\Psi_{\text{harmonic}}(x,t)=Ce^{-\int[\frac{2m}{\hbar^2}\Delta V(x,t)]dx}$ and $\Psi_{\text{particular}}(x,t,i\hbar\nu_n)$ is a particular solution of

Equation (11). Here $\Psi_{\text{particular}}(x,t,\nu_n)$ includes random frequency ν_n , then we have a general solution of Equation (11) based on differential equation theory as follows:

$$\Psi(x,t)=Ce^{-\int[\frac{2m}{\hbar^2}\Delta V(x,t)]dx}+\Psi_{\text{particular}}(x,t,i\hbar\nu_n), \quad t_{n-1}\leq t\leq t_n \quad (12)$$

Where C is an integral constant and will be determined by initial conditions and boundary conditions, $\Delta V(x,t)=V_1(x,t)-V_2(x,t)$ is potential energy difference between two points. ν_n is random frequency of the external energy, $n=1,2,3\Lambda$.

From Equation (12), $\Psi(x,t)$ includes a random variable, ν_n . This ν_n makes $\Psi(x,t)$ random transferring from transient time internal $t=t_{n-1}-t_n$ to another transient time internal $t=t_n-t_{n+1}$. So, $\Psi(x,t)$ is a random function during $t\neq t_n-t_{n-1}$. However, we can see that $\Psi(x,t)$ is also a displacement function from Equation (10-1) during $t_{n-1}\leq t\leq t_n$. That is to say, the wave function $\Psi(x,t)$ is a **displacement function with probability characteristics**.

4.2 Wave Function Collapse and Wave Packet Expansion

The wave function collapse is a very important and argued problem in the Copenhagen interpretation for over 80 years. However, if we use the group of Schrödinger Random Equations to discuss the wave function collapse, this problem can be well explained.

First of all, according to the proposed particle model by authors, the reason of the motion of the particle is to absorb random external energy and the motion of the particle is described by the group of Schrödinger Random Equations. During the motion a particle, there are many wave

functions, $\Psi_1, \Psi_2, \Psi_3, \dots, \Psi_n, \dots, n=1,2,3,\dots$. These wave functions are certain displacement functions with probability characteristics for their own transient time intervals $t = t_{n-1} - t_n$ and $\Psi_j \neq \Psi_k$. When measuring a moving particle, we can only measure one certain Schrödinger Equation at one time. For example, in transient time interval, $t = t_{n-1} - t_n$, we can measure Ψ_{n-1} . However, in next transient time interval, $t = t_n - t_{n+1}$, we can only measure Ψ_n . This explains the so called wave function collapse.

In fact, after we understand the difference between a group of Schrödinger Random Equations and a Schrödinger equation, we can know that there is not existence of the wave function collapse. Using the wave function collapse in the Copenhagen interpretation is not accurate term to describe the measuring wave function.

If we check in Equation (12),

$$\Psi(x,t) = Ce^{-\int [\frac{2m}{\hbar^2} \Delta V(x,t)] dx} + \Psi_{particular}(x,t, i\hbar v_n)$$

then we find that during the motion of the particle, there are many wave functions, $\Psi_1(x,t), \Psi_2(x,t), \Psi_3(x,t), \dots, \Psi_n(x,t), \dots, n=1,2,3,\dots$. These wave functions are certain displacement functions with probability characteristics for their own transient time intervals. Since there are different random frequencies, we have that $\Psi_j(x,t) \neq \Psi_k(x,t)$. So, we can form a series of $\Psi_1(x,t), \Psi_2(x,t), \Psi_3(x,t), \dots, \Psi_n(x,t), \dots$. In this series, $\Psi_j(x,t) \neq \Psi_k(x,t)$. So, the Wave Packet Expansion is well explained.

5. Interpretation for Single Electron Interference Experiment

This important experiment involves single electron interference, [22] –[43]. When sending an electron through a double-slit apparatus one at a time, this single electron appears on the screen. However, an interference pattern emerges when these electrons are allowed to build up one by one.

We explain this interference pattern emerging on the screen as follows:

This demonstrates the wave-particle duality, that is, the electron is measured as a single pulse at a single position, while the many electrons describe the probability of wave functions the electron at a specific place on the screen. Notice: this diagram and picture are from the internet.

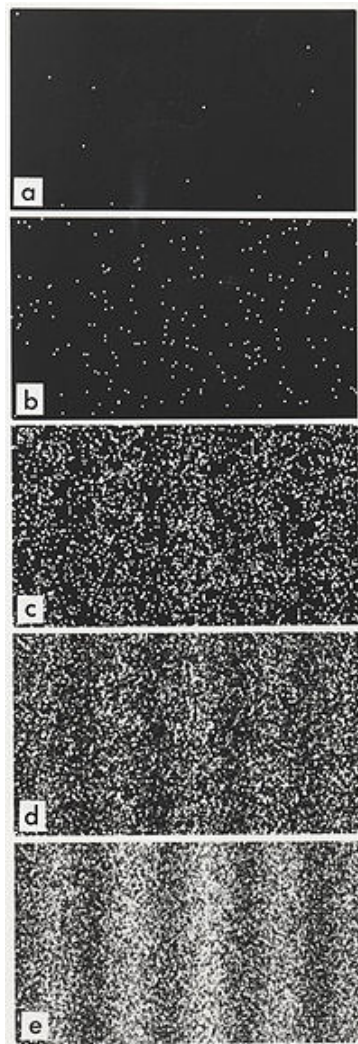


Figure 1 Single Slit Electron Interference

According to the group of Schrödinger Random Equations, Equation (8), we have that

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar v_1 \quad 0 \leq t \leq t_1 \quad \longleftarrow \text{First equation}$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar v_2 \quad t_1 \leq t \leq t_2 \quad \longleftarrow \text{Second equation}$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar v_3 \quad t_2 \leq t \leq t_3 \quad \longleftarrow \text{Third equation}$$

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$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i\hbar v_n \quad t_{n-1} \leq t \leq t_n \quad \longleftarrow \text{Nth equation}$$

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This group of equations possesses the same harmonic equation, that is, they possess the same natural frequency. However, they have the different initial phases. When the first electron is sent out, suppose this electron is from the first equation. The first equation is a certain wave function and will be shown on the screen by a single pulse. If sending the second electron, suppose this electron is from the second equation. Since the characteristics of probability of the group of Schrödinger Random Equations, then there is a probability of the first electron interfering with the second electron on the screen. Gradually, as the numbers of sending out electrons are increased, the probability of electrons interfering with another electron will increase. When the phase difference of n the electron and $(n-1)$ the electron satisfy the conditions of interference, the interfering of electrons will take place and the interference pattern emerges on the screen.

6. Interpretation for The Double Slit Experiment With Electrons

The double slit experiment has become a classic experiment, because the experiment with photons is a demonstration of the wave-particle duality, [27] – [29]. More important, it displays the fundamentally probabilistic nature of quantum mechanics. In 1927, Davisson and Germer demonstrated that electrons show the same behavior. In 1961, when Claus Jönsson of the University of Tübingen performed it with electron beams. In 1974, the Italian physicists Pier Giorgio Merli, Gian Franco Missiroli, and Giulio Pozzi repeated the experiment using single electrons and biprism (instead of slits), showing that each electron interferes with itself as predicted by quantum theory. Below we use electrons as examples to investigate the Double Slit Experiment.

Suppose the diagram and interference pattern are shown in Figure 2 and Figure 3. Notice: this diagram and picture are from the internet.

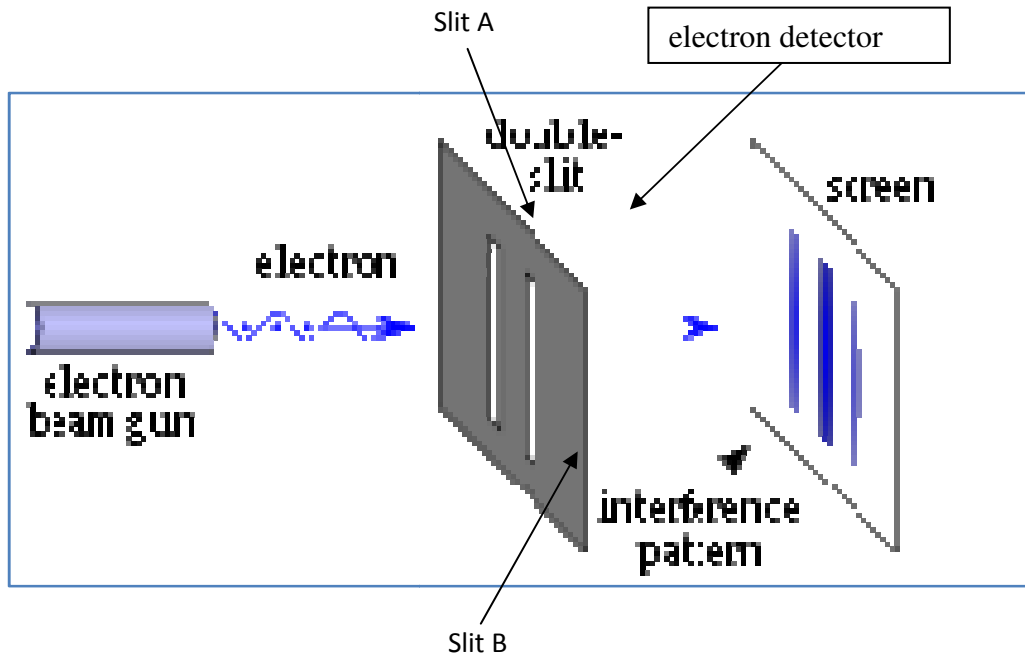


Figure 2 Double Slit Experiment Diagram

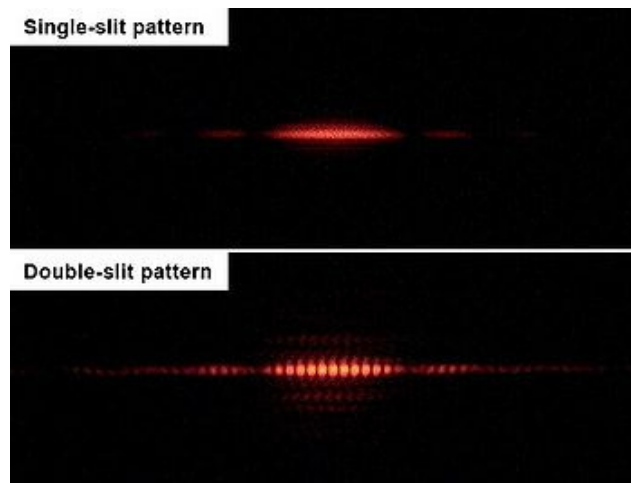


Figure 3 Electrons Interference Pattern

In the experiment, an electron beam gun sends electrons passing through the slits, Slit A and Slit B, shown in Figure 2.

If sending electrons through a Slit A or Slit B at a time, then this will result in single particles appearing on the screen. In fact, this is Single Electron Interference Experiment. About the explanation for this experiment, see Section 5.

If sending electrons through a Slit A and Slit B, the wave nature of electrons cause the electrons passing through the two slits to interfere on the screen.

According our model and the group of Schrödinger Random Equations This group of equations possess the same harmonic equation, that is, they possess the same natural frequency. When the phase difference of n the electron and $(n - 1)$ th electron satisfy the conditions of interference, the interfering of electrons will take place and the interference pattern emerges on the screen, generating bright and dark bands on the screen,

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_1 \quad 0 \leq t \leq t_1$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_2 \quad t_1 \leq t \leq t_2$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_3 \quad t_2 \leq t \leq t_3$$

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$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_{n-1} \quad t_{n-2} \leq t \leq t_{n-1} \quad \text{(N-1)th equation}$$

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_n \quad t_{n-1} \leq t \leq t_n \quad \text{Nth equation}$$

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If electrons detectors are positioned at Slit A or Slit B, showing through which slit an electron goes, the interference pattern will disappear. Traditionally, according to the Copenhagen interpretation, this which-way experiment illustrates the complementarity principle that electrons can behave as either particles or waves, but cannot be observed as both at the same time. However, this interpretation can not be accepted by many people. Now, we use the group of Schrödinger Random Equations.

For example, suppose that an electrons detector is positioned after Slit A. When an electron goes through Slit A and is detected by the detector, then it is like an external energy is added to this electron.

If numerous electrons go through Slit A and are detected by the detector, then it is equal to an external energy being added to the electrons. It should be noticed that the detecting frequency from the detector, ν , is a constant. This is because during the period of detecting, the detector always uses a constant frequency to detect electrons. Suppose the external energy function from detector, $E_{\text{detector}} = h\nu$, is added to electrons, then the group of Schrödinger Random Equations will become that

$$\begin{aligned}
 -\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t) &= i(\hbar\nu_1+h\nu) & 0\leq t\leq t_1 \\
 -\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t) &= i(\hbar\nu_2+h\nu) & t_1\leq t\leq t_2 \\
 -\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t) &= i(\hbar\nu_3+h\nu) & t_2\leq t\leq t_3
 \end{aligned} \tag{C}$$

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$$-\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t) = i(\hbar\nu_n+h\nu) \quad t_{n-1}\leq t\leq t_n$$

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As ν_n is random variable, in order to illustrate this problem we only consider one case for $E_{\text{detector}} = h\nu$. For other cases, see Section 3.2. If $E_{\text{detector}} = h\nu$ is high, that is, $h\nu \gg \hbar\nu_n$, then above the group of Schrödinger Random Equations become a wave equation with a constant frequency:

$$\begin{aligned}
 -\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t) &= i\hbar\nu & 0\leq t\leq t_1 \\
 -\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t) &= i\hbar\nu & t_1\leq t\leq t_2 \\
 -\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t) &= i\hbar\nu & t_2\leq t\leq t_3
 \end{aligned}$$

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$$-\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t) = i\hbar\nu \quad t_{n-1}\leq t\leq t_n$$

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⇒

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t)=i\hbar\nu \quad 0\leq t\leq t_n \quad (D)$$

Equation (D) tells us after the numerous electrons go through Slit A and are detected by the detector, all these electrons will possess a common and constant frequency. According to Equation (12), the wave function will no longer possess the characteristics of the probability of the group of Schrödinger Random Equations. So this why the interference pattern on the screen disappear after the electrons detector is positioned after Slit A. Under this circumstance, the electrons will keep the wave-particle duality. However, the characteristics of probability of the group will lose.

However, if the electrons detector is removed at Slit A, the external energy function from detector, $E_{\text{detector}} = \hbar\nu$, is removed from electrons. So the group of Schrödinger Random Equations will recover its original forms, Equation (8),

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t)=i\hbar\nu_1 \quad 0\leq t\leq t_1$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t)=i\hbar\nu_2 \quad t_1\leq t\leq t_2$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t)=i\hbar\nu_3 \quad t_2\leq t\leq t_3$$

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$$-\frac{\hbar^2}{2m}\nabla^2\Psi(q_j,t)+V(q_j,t)\Psi(q_j,t)=i\hbar\nu_n \quad t_{n-1}\leq t\leq t_n$$

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At this moment, the interference pattern will appear again on the screen.

Similarly, the above explanations are suitable to explain the Quantum Eraser Experiment and the Delayed Choice Experiment.

7 Interpretation for Bell's Inequality and Its Experimental Test;

A Bell's inequality experiment is a real-world physics experiment designed to test the theory of quantum mechanics in relation to Einstein's concept of local realism. According to Bell's theorem, if nature actually operates in accord with any theory of local hidden variables, then the results of a Bell test will be constrained in a particular, quantifiable way. If the results of an experiment violate Bell's inequality, local hidden variables can be ruled out as their cause. To date, many types of Bell test have been performed in physics laboratories. All Bell tests have supported the theory of quantum mechanics and not the hypothesis of local hidden variables, [44] – [54].

Why all Bell experiments support the theory of quantum mechanics and not the hypothesis of local hidden variables? According to the newly proposed model in this paper, the group of Schrödinger Random Equations can be used in describing the motion of a particle. This model shows that there are not the local hidden variables in the group of Schrödinger Random Equations. This group consists only of many certain wave functions. The waves in the group are not only probability function and also they are displacement functions. Authors think that all Bell's inequality experiments indirectly support the model proposed in this paper and the group of Schrödinger Random Equations.

8. Interpretation for Schrödinger's Cat Thought Experiment

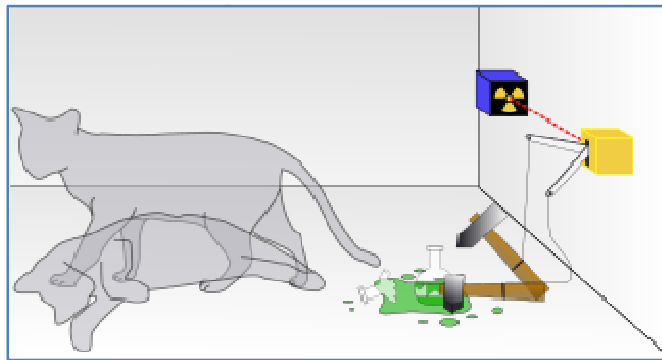


Figure 4 Schrödinger's Cat Experiment

Schrödinger's cat is a thought experiment, devised by Erwin Schrödinger in 1935. The experiment presents a hypothetical cat that may be simultaneously both alive and dead, a state known as a quantum superposition. According to Schrödinger, the Copenhagen interpretation

implies that the cat remains both alive and dead until the state has been observed. Schrödinger did not agree the idea of dead-and-alive cats as a serious possibility. On the contrary, he used the example to illustrate the absurdity of the existing view of quantum mechanics, [55] – [57].

In the Double Slit Experiment for Electron, if electrons detector is positioned at the slit, showing through which slit an electron goes, the interference pattern will disappear. Traditionally, people think this which-way experiment illustrates the complementarity principle of the Copenhagen interpretation that electrons can behave as either particles or waves, but cannot be observed as both at the same time.

However, according to our explanation in Section 6 for the Double Slit Experiment for Electron, the group of Schrödinger Random Equations can be used to explain well why the interference pattern disappear if electron detectors are positioned at the slits. Under this circumstance, the electrons will keep the wave-particle duality, but can be observed as both at the same time. However, the characteristics of probability of the group will lose. But the interference pattern will appear again if electron detector is removed at the slit. The reason is that due to the energy from the detector interferes with the group of Schrödinger Random Equations so that the interference pattern disappears. It is not that electrons can behave as either particles or waves or but cannot be observed as both at the same time. We think a state known as a quantum superposition does not exist in quantum mechanics. So, the Schrödinger's cat can not be simultaneously both alive and dead, only in alive or dead.

Notice: Figure 4 is from the internet.

9. Interpretation and Predication of Quantum Entanglement.

Quantum entanglement is physical phenomenon and was the subject of a 1935 paper by Albert Einstein, Boris Podolsky, and Nathan Rosen, and Erwin Schrödinger shortly thereafter. This phenomenon occurs when a pair or group of particles is generated, interact, or share spatial proximity in a way such that the quantum state of each particle of the pair or group cannot be described independently of the state of the others, even when the particles are separated by a large distance. Quantum entanglement has been demonstrated experimentally with photons, neutrinos, electrons, molecules as large as buckyballs and even small diamonds.

Now we use the model proposed in this paper to investigate this physical phenomenon.

According to the model in this paper, any moving particle can be described by the group of Schrödinger Random Equations,

$$\begin{aligned}
 -\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= i\hbar v_1 & 0 \leq t \leq t_1 \\
 -\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= i\hbar v_2 & t_1 \leq t \leq t_2 \\
 -\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) &= i\hbar v_3 & t_2 \leq t \leq t_3
 \end{aligned} \tag{8}$$

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$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_n \quad t_{n-1} \leq t \leq t_n$$

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Suppose two identical moving particles, labeled A and B, are in the state of entanglement and in the same state of motion at moment t_{n-1} . Because the particles should follow the group of Schrödinger Random Equations, their motions should keep the same during transient time interval, $t_{n-1} \leq t \leq t_n$. Mathematically, the equation of motion is as follows:

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_n \quad t_{n-1} \leq t \leq t_n \quad (E)$$

However, after transient time interval from $t_{n-1} \leq t \leq t_n$ into $t_n \leq t \leq t_{n+1}$, the motion of these two particles will be interrupted by two random external energy with different values, $E_j = \hbar v_j$, and $E_k = \hbar v_k$. For example, Particle A is interrupted by $E_j = \hbar v_j$ and Particle B is interrupted by $E_k = \hbar v_k$. So Equation (E) becomes two different and random equations of motions,

Particle A

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_j \quad t_n \leq t \leq t_{n+1} \quad (F)$$

Particle B

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(q_j, t) + V(q_j, t) \Psi(q_j, t) = i \hbar v_k \quad t_n \leq t \leq t_{n+1} \quad (G)$$

Apparently, although they have the same harmonic equation, Equation (F) and Equation (G) are different. So we can say that quantum entanglement indeed exists, but it is a kind of random physical phenomenon of quantum. This conclusion is supported by scientists from the University of Glasgow reported taking the first ever photo of a strong form of quantum entanglement known as Bell entanglement on 13 July 2019, [58].

10. Conclusions

In this paper, the following conclusions are obtained:

- a. It is found the wave function, $\Psi(q_j, t)$, is not only probability function and also is displacement function;
- b. The Copenhagen interpretation about the wave function collapse is not very accurate and usually causes people misunderstanding about quantum mechanics. Using the group of Schrödinger Random Equations can explain well the so called “the wave function collapse”. In fact, the phenomenon of the wave function collapse does not exist. It should be called the measurement of wave function;
- c. The wave packet expansion is generated by the wave functions with different values at the different times and displacements, $\Psi_1(q_1, t)$, $\Psi_2(q_2, t)$, $\Psi_3(q_3, t)$, $\Lambda \Psi_n(q_n, t)$, Λ , $n = 1, 2, 3, \Lambda$. The wave function is not only probability function and also is a decreased function of displacement and time;
- d. The group of Schrödinger Random Equations is used to investigate the Single Electron Interference Experiment and the Double Slit Experiment With Electrons. The investigations indicate that the two experiments can be explained well by the group of Schrödinger Random Equations. Especially, in the Double Slit Experiment for Electron, if electrons detector is positioned at the slit, the interference pattern will disappear, this mystery for people so many years is explained. Under the circumstance of the electrons detector being positioned at the slit, the electrons will keep the wave-particle duality. The complementarity principle based on this experiment is not true. A state known as a quantum superposition does not exist in quantum mechanics and the microscopic world.
- e. Authors investigate Schrödinger's Cat Thought Experiment. The investigation indicates that the Schrödinger's cat can not be simultaneously both alive and dead, only in alive or dead.
- f. Authors investigate Bell's inequality experiments. The investigation indicates that according to the newly proposed model in this paper, the group of Schrödinger Random Equations can be used in describing the motion of a particle and there are not the local hidden variables in quantum mechanics. All Bell's inequality experiments indirectly support indirectly the model proposed in this paper.
- g. Authors use the group of Schrödinger Random Equations to investigate quantum entanglement phenomena and find that the quantum entanglement may exist but it is a random phenomena.

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