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Evaluation of Averaging Techniques for Solving Multi-Objective Optimization (MOO) Problems

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Abstract

The paper evaluates the newly proposed averaging techniques for solving multi-objective optimization problems. Arithmetic mean, Harmonic mean, Geometric mean and many more statistical measures have been used for scalarizing the multiple objective functions. It was noticed that the formulation of these techniques is not logical and appropriate. The numerical examples solved using these techniques were also inferior. Sen's Multi-Objective Optimization (MOO) [1] technique and its two modifications have been suggested for comparative analysis and clarifications.

Keywords: Quadratic mean; Arithmetic mean; Identric mean; Logarithmic Mean; Geometric mean.

1. Introduction

The purpose of multi-objective optimization is to search an appropriate solution of the problems with multiple conflicting objectives . Several techniques of mean, median, geometric mean, harmonic mean, optimal mean, optimal median etc. [2]--------[19] have been proposed for solving the multi-objective optimization problems during the recent past. These techniques are used to scalarize the multiple objective functions to formulate the single objective function. Optimization of the multi-objective function is assumed to generate the most acceptable solution. The proposed techniques have been evaluated on following points:

(i) Formulation of Multi-Objective function
(ii) Suitability of the numerical examples, and
(iii) Interpretation of the results

2. Formulation of multi-objective function

2.1 Averaging Techniques

The mathematical form of multi-objective optimization is described as:

Optimize Z= [Max. Z_1, Max. Z_2 ......Max. Z_r Min. Z_{r+1}.......Min. Z_s]

Subject to:

AX = b and X≥ 0

The individual optima are obtained by optimizing each objective separately as:

$Z_{\text{optima}} = [\Theta_1, \Theta_2 \ldots\ldots\ldots \Theta_s]$

The multi-objective function is formulated by weighting the individual objective functions with the inverse of Arithmetic Mean, Geometric Mean, Harmonic Mean and Optimal Arithmetic , Geometric, Harmonic Mean etc as explained below:

Maximize, $$Z = \frac{\sum_{j=1}^{r} Zj}{\text{AM}\Theta_j, GM\Theta_j, HM\Theta_j \ldots\ldots} - \frac{\sum_{j=r+1}^{s} Zj}{\text{AM}\Theta_{r+1}, \text{GM}\Theta_{r+1}, \text{HA}\Theta_{r+1} \ldots\ldots}$$

Subject to:

AX = b and X≥ 0

The AM, GM and HM are Arithmetic, Geometric and Harmonic Means of absolute optimal Values of different objective functions. New averaging method has also been used to formulate the multi-objective function. The deviations of maximization and minimization objective functions have been weighted by inverse values of various averages [19].

2.2 Sen's MOO technique

Sen's MOO technique and its two modifications have also been presented here for the comparative analysis. The Sen's multi-objective function was formulated as detailed below:

Maximize, $$Z = \frac{\sum_{j=1}^{r} Zj}{\Theta_j,\ Sqr\Theta_j,\ Log\Theta_j\ .} - \frac{\sum_{j=r+1}^{s} Zj}{\Theta_{r+1}, \text{Sqr}\Theta_{r+1}, \text{Log}\Theta_{r+1}}$$

Subject to:

$AX = b$, $\Theta \neq 0$ and $X \geq 0$

Where Ө, Sqr Ө and Log Ө are the absolute values of individual optima, Square root and log values of Ө.

3. Problems in formulation of multi-objective function

i. The objective functions may be of different dimensions. The estimation of Mean, Harmonic Mean, Geometric Mean etc. of Individual optima of different dimensions seems illogical [20]. However, the Sen's Multi-Objective Function is dimensionless.

ii. In presence of high deviations in the individual optima, the Multi-Objective Optimized solution may be biased towards the higher optima. This problem is resolved by weighting each objective function by self optima in the Sen's Multi-Objective Function.

Example: The example used in the study [19] is reproduced here as below:

$$\text{Max.}Z_1 = X_1 + 2X_2$$

$$\text{Max.}Z_2 = X_1$$

$$\text{Min.}Z_3 = -2X_1 - 3X_2$$

$$\text{Min.}Z_4 = \quad - X_2$$

Subject to

$$6X_1 + 8X_2 \leq 47$$
$$X_1 + X_2 \geq 3$$
$$X_1 \leq 4$$
$$X_2 \leq 3$$
$$X_1, X_2 \geq 0$$

4. Individual optimization

The above example was solved using Linear Programming for achieving all the objective functions individually. The results are presented in Table 1.

Table 1: Solution of Individual Optimization

Particulars	Individual Optimization			
	Max. Z_1	Max. Z_2	Min. Z_3	Min. Z_4
X_1, X_2	3.8333, 3	4, 0	3.8333, 3	3.8333, 3
Z_1	9.8333	4	9.8333	9.8333
Z_2	3.8333	4	3.8333	3.8333
Z_3	-16.6666	-8	-16.6666	-16.6666
Z_4	-3	0	-3	-3

Out of four objective functions, three objective functions have unique solution and only second objective function has different solution. There are no conflicts amongst objective functions first, third and fourth. Hence, the above example is not appropriate for the application of multi-objective optimization techniques. This has also been noticed in other research studies [2]-------[19].

5. Multi-objective optimization

The given example was solved by various multi-objective optimization techniques using Contra harmonic mean, Quadratic mean, Newman Sandor mean, Arithmetic mean, Identric mean, Heronian mean, Arithmetic-geometric mean, Logarithmic mean, Geometric, Geometric-harmonic mean and Harmonic mean [19]. The example was further solved by Sen's MOO technique and its modified techniques. The solutions of these techniques are presented in Table 2. It is very clear from the Table that all the fourteen techniques have unique solution.

Table 2: Solution of Average Methods of Multi-Objective Programming.

Technique	X_1, X_2	Multi-Objective Function Z^*		Value of Objective Functions			
				Z_1	Z_2	Z_3	Z_4
Sen's MOO	3.8333, 3	3.9534		9.8333	3.8333	-16.6666	-3
Sen's Sqr.	3.8333, 3	10.8670		9.8333	3.8333	-16.6666	-3
Sen's log	3.8333, 3	36.20122		9.8333	3.8333	-16.6666	-3
		Av.	**New Av.**				

		Method	method				
Contra harmonic Mean	3.8333, 3	3.0263	4.0473	9.8333	3.8333	-16.6666	-3
Quadratic Mean	3.8333, 3	3.4631	4.5857	9.8333	3.8333	-16.6666	-3
Newman Sandor Mean	3.8333, 3	3.7879	4.9762	9.8333	3.8333	-16.6666	-3
Arithmetic Mean	3.8333, 3	3.9761	5.1948	9.8333	3.8333	-16.6666	-3
Identric Mean	3.8333, 3	4.2402	5.4717	9.8333	3.8333	-16.6666	-3
Heronian Mean	3.8333, 3	4.2459	5.4750	9.8333	3.8333	-16.6666	-3
Arithmetic-geometric Mean	3.8333, 3	4.4164	5.6366	9.8333	3.8333	-16.6666	-3
Logarithmic Mean	3.8333, 3	4.5750	5.7911	9.8333	3.8333	-16.6666	-3
Geometric Mean	3.8333, 3	4.9604	6.1372	9.8333	3.8333	-16.6666	-3
Geometric-harmonic Mean	3.8333, 3	5.5917	6.6823	9.8333	3.8333	-16.6666	-3
Harmonic Mean	3.8333, 3	6.2715	7.2504	9.8333	3.8333	-16.6666	-3

The values of real variables are $X_1 = 3.8333$ and $X_2 = 3$ are same for all the optimization techniques. The achievements of all the four real objective functions Z_1, Z_2, Z_3 and Z_4 are also same. The achievements of objective functions under multi-objective optimization solutions have not been interpreted correctly in many studies [2]----------[19]. The solution was highlighted on the basis of the values of multi-objective functions only. The values of multi-objective functions are not all the same. This is due to difference in the formulation of each multi-objective function. Several studies [2]----[19] have concluded that the certain average techniques are superior over Sen's MOO technique in spite of same values of decision variables. It is by chance that few values of multi-objective functions of certain techniques are lesser than Sen's MOO technique for this example. However, both the values of Sen's modified multi-objective functions are the highest with same values of real variables. Should it be concluded that all the eleven methods reported here are inferior to Sen's modified technique?

6. Conclusion

The study reveals that several new averaging techniques proposed for solving multi-objective optimization problems have not been formulated appropriately. The examples used to verify the techniques were not suitable. The results have also not been interpreted correctly. It is clear from Sen's MOO technique that these techniques should not be rated by the values of multi-objective functions.

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