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Impacts of a New Spatial Variable on a Black Hole Metric Solution

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Abstract:

Mathematical formulations are often correlated with the truth; however, there are a few cases where mathematics is forcefully used to predict something. As a result, fiction is introduced.“The particle problem in the General Theory of Relativity”,one of the historical papers by Einstein and Rosen in 1935 used a simple spatial variable in the blackhole solutions to omit the concept of singularity and infinity. As a result, the concept of spacetime portal—a topological configuration with two asymptotically flat regions connected through a bridge—was mathematically backed up. In this paper, I have introduced a similar kind of variable $u^x = r - 2M$. Referencing this variable, I have logically stated that $u^2 = r - 2M$ introduced in Einstein and Rosen’s paper might not necessarily correlate to reality. Such introduction of variable creates different fiction possibilities in theoretical physics and those possibilities are discussed in this paper.

Keywords **—Black hole, General Theory of Relativity, Spacetime portal, topological configuration.**

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I. INTRODUCTION

The wormhole is a special topological configuration that connects two distant spatial points of the universe via a bridge. It is probably one of the most fascinating ideas in physics since everyone is exhilarated by the fictional idea of time travel. [1] The idea of wormhole first appeared on a paper “Comments on Einstein Theory of Gravity” by Ludwig Flamm in 1916, a year after General Theory of Relativity was published. Later in 1935, [2] Albert Einstein and Nathan Rosen proposed an atomistic theory of matter and electricity excluding singularity which provided a mathematical framework for wormholes. In that paper, Einstein and Rosen introduced a variable $u^2 = r - 2M$ in the Schwarzschild metric that got rid of the concept of singularity. We will discuss briefly how that variable was introduced and then we will analyze the introduction of a similar variable that disobeys the idea of Einstein and Rosen’s paper.

II. DEVELOPMENT OF $u^2 = r - 2M$ VARIABLE

[3] When Einstein formulated his Gravitational field equation at the end of 1915, he needed to approximate the solution so that Arthur Eddington could test the deflection of starlight due to the gravitational field of the sun to prove General relativity true. It was Karl Schwarzschild who formulated a metric which was a quite exact and non-trivial solution in a spherically symmetric coordinate system.

The simplest form of Schwarzschild metric involves familiar variables like space, time, and mass variables as:

$$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{1}{(1 - \frac{2M}{r})}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi)$$

In this equation, the coast signature $(+ - - -)$ has been used. Since it is a solution in the spherical coordinate system, θ means the latitude and φ refers to the co-longitude. The diagonal elements of the metric tensor are: $g_{11} = 1 - \frac{2M}{r}$, $g_{22} = \frac{1}{g_{11}} = \frac{1}{1-\frac{2M}{r}}$, $g_{33} = r^2$, and $g_{44} = r^2 \sin^2 \theta$. Here, $M = Gm$. G is gravitational constant and M is a Mass parameter. [4]The theoretical black hole's solution that we deal with has the event horizon specifically called 'bifurcated killing horizon', which is a smooth space-like manifold with a union of two null hypersurfaces that intersect transversally to form a bifurcate null surface.

Noticing the singularity at the event horizon i.e. $r = 2m$ turned out to be a perplexing discrepancy to Schwarzschild. It can also be seen that the metric has intrinsic curvature singularity at $r = 0$. [3] So, Schwarzschild replaced the r coordinate with a preferable r' coordinate such that $r' = (r^3 - (2M)^3)^{\frac{1}{3}}$. This was a commendable approach of Schwarzschild to have used a simplified solution by introducing r' where it is just the cube root of the result gained by subtracting the $(2M)^3$from the total space-time volume enclosed by r. Now, the singularity is contained at the origin.

Singularity is the biggest adversary of physics and mathematics. To eradicate the problem of singularity, Einstein and Rosen, in the same 1935 paper, suggested an introduction of a plain variable that would change the future of physics and science fiction.

With the introduction of the new variable $u^2 = r - 2m$, the Schwarzschild metric becomes:

$$ds^2 = \left(\frac{u^2}{u^2 + 2m}\right) dt^2 - 4(u^2 + 2m)du^2 - (u^2 + 2M)^2(d\theta^2 + \sin^2\theta\, d\varphi)$$

It can be seen that new $g_{\mu\nu}$ has been obtained and the coefficient of dt^2 becomes zero when $u = 0$. Now, this new metric can take any value for u, and hence, there is no singularity or infinity at any finite point. This solution gives a field free from a singularity that contains the electrically neutral elementary particle. This erudite move did not just solve the vexing problem of singularity, but also put forth a new possibility. It can be noted that u varies from $+infinity$ to $-infinity$, r varies from $+infinity$ to $2m$ and again from $2m$ to $+infinity$. Hence, two similar sheets are connected to by hyperplane at$r = 2m$ or $u = 0$.

[5]We can also consider that a two-sided Schwarzschild solution gives two asymptotically flat regions, each having an identical black hole. In that case, those black holes are entangled and those bifurcated horizons touch at the center of the origin.

Similarly, there is another metric that was formulated including the electromagnetic field in the General theory of relativity that incorporated inhomogeneous and homogeneous Maxwell equations. [6]Reissner-Nordström metric is the solution of General Relativity which describes the space-time around a spherically symmetric charged body with mass m and charge q as:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)dt^2 - \frac{1}{(1-\frac{2m}{r}+Q^2/r^2)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Here, Q is an integration constant $(Q = \frac{q^2 G}{4\pi\varepsilon_o c^4})$ gained while finding g_{11} and g_{22} , denoted as electric charge. When we assign $q = 0$, the Reissner–Nordströmmetric transforms into the Schwarzschild metric. In the 1935 paper, Einstein discussed the consequence of introducing the new variable u in this solution as in the previous Schwarzschild metric. [6]Since M and Q are two integration constants that are independent of each other, mass is not determined by charge. We try to assume that there is no mass in the field and assign $M = 0$, so that, we can conceive a solution without singularity.

So, we get the following solution:

$$ds^2 = (1 - Q^2/r^2)\, dt^2 - \frac{1}{(1-Q^2/r^2)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

When we plug in a new variable $u^2 = r^2 - \frac{Q^2}{2}$, we get:

$$ds^2 = (\frac{2u^2}{(2u^2 + Q^2)})\, dt^2 - du^2 - (u^2 + Q^2/2)\,(d\theta^2 + \sin^2\theta d\varphi^2)$$

This metric represents an elementary electrical particle without a mass and this solution is free from the singularity. This solution is valid for every point in the 2 asymptotical sheets where the charge signifies the Einstein-Rosen Bridge.

Similarly, there are other famous metrics like [7] Morris-Throne metric (involving gravitational red-shift function and shape function), [8]Kerr metric (dealing with space-time near an uncharged spherically symmetric rotating black hole) , [9]Vacuum Static Brane-Dickes wormholes solution (dealing with torsion), [10]Kruskal-Szekeres Metric(a maximally extended Schwarzschild solution) and many other elegant solutions. However, all of them hold a congruency because they all use the u^2-like variable of various forms to eliminate the singularity and claim that the revised metric including " u^2 term" gives two asymptotically flat regions joining each other via a bridge.

III.INTRODUCTION AND CONSEQUENCE OF NEW VARIABLE $\boldsymbol{u^x = r - 2M}$

One intriguing question can prove that introduction of the term u^2 wouldn't necessarily uphold mathematical consonance and claim that it settles the problem of singularity and actuates a new concept-wormhole. What if we use u^3 or u^4 or u^x where x can be any constant? If we take $u^x = r - 2M$, then the Schwarzschild metric becomes:

$$ds^2 = \left(\frac{u^x}{u^x+2M}\right)dt^2 - x^2(u^x + 2M)u^{x-2}du^2 - (u^x + 2M)^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

In this equation, it is noticeable that the value of $g_{\mu\nu}$ changes. When u varies from $+infinity$ to $-infinity$ then there are two possibilities for r. If x is even, then the condition for r is the same as the previous one. But, when x is odd, then there can be negative r as well. This yields a new contradiction in the concept since the integrity of negative r can be cross-interrogated.
Another insight into it is that if we use the fundamental theorem of algebra, we can see that there is a maximum x number of possible solutions for u and there may not be only two asymptotical flat regions, but many regions connecting each other. Presumably, the most fascinating part of this idea is that it results in a concept 'probability of spatial coordinate' since we will not be able to locate the exact point to be reached after the opening of another side of the wormhole.

This concept might introduce a new problem. Since every mathematical solution of u will not be real, there will be an imaginary solution of spatial coordinate u. [11] As we know that imaginary numbers also have some physical reality applications like[12] radio electronics, special relativity, and superluminal speeds, a hypothesis for the possibility of 'spatial solution of different universe' might be put forward.
The Kruskal metric also predicts the possibility of a wormhole connecting two universes, strictly called 'white hole'. [13] The wormhole structure and topology of this model can be modified in such a way that this model can produce an infinite number of universes along with both space and time direction.
The idea of the spatial solution for a different universe is pretty interesting although we are not apprised of the reality. We can still proceed with our view of why the 'different universe' concept is insensible. Since a black hole may not be eternal as suggested by hawking due to [14] the Hawking radiation, the existence of such a universe is not possible.
[15]Another point to be noted is that three alleged big bang universes are either spatially finite or spatially infinite. Spatially infinite black hole universe cannot fit inside the spatially finite big bang universe. The two different spatially infinite big bang universes either have a constant negative curvature or a constant zero curvature. Nevertheless, no black hole universe possesses such a constant curvature and hence can't fit into either of the spatially infinite big bang universes. And joining such arbitrary universes by one bridge is an absurd idea. Thus, it is better to say that the black hole, if it does not end to singularity, takes us to a different point of the same universe.

III. CONCLUSION

The scientific consortium has procured its first-ever picture of a 'black hole', but we are still speculative about what lies behind the event horizon. Our colossal understanding of mathematics genuinely confronts us to endless possibilities, but all of them bear some sort of incongruity or mathematics is forcedly employed to claim that the idea is veracious and convincing. We do not yet precisely know what lies behind it but one can efficaciously resume deriving more plausible equations and correlating them with the probable reality. We have encountered what revolution even a simple change in a variable can bring. Thus a logically strong mathematical variable must be introduced to make a hypothesis of what lies beyond the black hole, otherwise, we have seen what possibilities we have to consider before taking the $u^2 = r - 2M$ as a correct variable for metric solutions to end the concept of singularity.

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