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RP-82: Formulation of Some Classes of Solvable Standard Bi-quadratic Congruence of Prime-power Modulus

Prof. B M Roy
Head Department of Mathematics
Jagat Arts, Commerce & I H P Science College, Goregaon
Dist- Gondia, M. S., INDIA. Pin: 441801
(Affiliated to R T M Nagpur University, Nagpur)

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Abstract:

In this paper, some classes of solvable standard bi-quadratic congruence of prime-power modulus are formulated. The established formulae are tested and found true. Examples are solved using established formulae. The formulae work well. Readers' time of calculation is shortened. Formulation is the merit of the paper.

***Keywords* —** Bi-quadratic congruence, Binomial expansion, Chinese Remainder Theorem.

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INTRODUCTION

Any standard fourth degree congruence is called a standard bi-quadratic congruence. It can be written as $x^4 \equiv b\ (mod\ m)$. If $m = p$, a prime positive integer, then the congruence is called a bi-quadratic congruence of prime modulus. If m is a composite integer, then it is called a bi-quadratic congruence of composite modulus.

The value of x that satisfies the congruence is called its solutions. The congruence is called solvable if b is a bi-quadratic residue of m [5]. If there exist a positive integer a such that $b = a^4\ (mod\ m)$, then b is the bi-quadratic residue of m. The author already has formulated some classes of standard bi-quadratic congruence of composite modulus.

LITERATURE-REVIEW

The author referred many books on Number theory and found a little discussion on standard bi-quadratic congruence. But no formulation is found on the said congruence.

The standard bi-quadratic congruence of the type: (1) $x^4 \equiv a^4 (mod\ 4^n)$;

(2) $x^4 \equiv a^4\ (mod\ 4^n.b)$

has been formulated and published in IJETRM, vol-3, issue-1, feb-2019 [3].

Also, the standard bi-quadratic congruence of the type: (1) $x^4 \equiv a^4\ (mod\ \ 4^n.b^m)$;

(2) $x^4 \equiv a^4\ (mod\ \ r.4^n.b^m)$

has been formulated and submitted (accepted) to IJRIAR, in the February issue, 2019 [4].

NEED OF RESEARCH

The literature of mathematics says approximately nothing about the said standard bi-quadratic congruence. Some discussion on general bi-quadratic congruence is found. The bi-quadratic congruence under consideration can be solved by a time-consuming and complicated method, known as Chinese Remainder Theorem (CRT) [1]. Readers do not want to use the CRT for solutions. The author tried his best with sincere effort to formulate some more congruence and presented the result in this paper. This is the need of the research.

PROBLEM-STATEMENT

Here, the problem of study is "To establish a formula of solutions of the standard bi-quadratic congruence:

(1) $x^4 \equiv a^4 \ (mod\ 4.p^n)$;

(2) $x^4 \equiv a^4 \ (mod\ 8.p^n)$; p being a positive prime integer; n any positive integer.

ANALYSIS &RESULT

Consider the congruence: $x^4 \equiv a^4 (mod\ 4p^n); p\ being\ a\ positive\ prime\ integer.$

If $x = 4p^n \pm a$, then $x^4 = (4p^n \pm a)^4$

$$= (4p^n)^4 + 4.3p^4)^4.a + \frac{4.3}{1.2}(4p^n)^2.a^2 + \frac{4.3.2}{1.2.3}(4p^n)^1.a^3 + a^4$$

$$= 4p^n(\dots\dots) + a^4$$

$$\equiv a^4 (mod\ 4p^n).$$

Therefore, $x = 4p^n \pm a$ satisfies the congruence $x^4 \equiv a^4 (mod\ 4p^n)$ and hence it is a solution if the said congruence.

Also, it can be easily seen that $x = 2p^n \pm a$, satisfies the congruence $x^4 \equiv a^4 (mod\ 4p^n)$ and is a solution of it.

Thus, it is seen that $x^4 \equiv a^4\ (mod\ 4p^n)$ has four solutions $x \equiv 4p^n \pm a; 2p^n \pm a\ (mod\ 4p^n)$.

It is also seen that the said congruence has no other solutions. Hence it has exactly four incongruent solutions.

Now consider the next congruence $x^4 \equiv a^4\ (mod\ 8p^n)$.

As in above, it can be easily seen that $x \equiv 4p^n \pm a; 2p^n \pm a\ (mod\ 8p^n)$ are also the solutions of the said congruence.

If $x = 6p^n \pm a; 8p^n \pm a\ (mod\ 8p^n)$ are the four other solutions of $x^4 \equiv a^4 (mod\ 8p^n)$. It is also found that there is no other solution exist. Hence, the congruence $x^4 \equiv a^4 (mod\ 8p^n)$ has exactly eight incongruent solutions.

Sometimes the congruence are given in the form:$x^4 \equiv b\ (mod\ 4p^n)$ OR $x^4 \equiv b\ (mod\ 8p^n)$.

In such cases, it can be written as:$x^4 \equiv b + k.4p^n = a^4\ (mod\ 4p^n)$.

Similarly the second congruence can also be written [2].

ILLUSTRATIONS

Consider the congruence $x^4 \equiv 125\ (mod\ 500)$

It can be written as $x^4 \equiv 125 + 500 = 625 = 5^4\ (mod\ 4.5^3)$

It is of the type $x^4 \equiv a^4\ (mod\ 4.p^n)\ with\ \ a = 5,\ p = 5, n = 3$.

It has exactly four incongruent solutions given by $x \equiv 2p^n \pm a; 4p^n \pm a\ (mod\ 4.p^n)$.

$$\equiv 2.5^3 \pm 5; 4.5^3 \pm 5\ (mod\ 4.5^3)$$

$$\equiv 250 \pm 5; 500 \pm 5\ (mod\ 500)$$

$$\equiv 245, 255; 495, 5\ (mod\ 500).$$

These are the solutions.

Consider the congruence $x^4 \equiv 256 (mod\ 392)$.

It can be written as $x^4 \equiv 4^4\ (mod\ 8.49)\ \ i.e.\ \ x^4 \equiv 4^4\ (mod\ 8.7^2)$

It is of the type $x^4 \equiv a^4 (mod\ 8.p^n)\ \ with\ a = 4,\ p = 7,\ n = 2$.

It has exactly eight solutions given by

$$x \equiv 2p^n \pm a,\ 4p^n \pm a,\ 6p^n \pm a,\ 8p^n \pm a\ \ (mod\ 8.p^n).$$

$$\equiv 2.7^2 \pm 4; 4.7^2 \pm 4; 6.7^2 \pm 4; 8.7^2 \pm 4\ \ (mod\ 8.7^2)$$

$$\equiv 98 \pm 4; 196 \pm 4;\ \ 294 \pm 4; 392 \pm 4\ \ (mod\ 392)$$

$$\equiv 94, 102;\ \ 192, 200; 290, 298; 388, 396\ (mod\ 392)$$

$$\equiv 94, 102; 192, 200; 290, 298; 388, 4\ (mod\ 392).$$

These are the required eight solutions.

Consider the congruence $x^4 \equiv 625 \ (mod\ 1000)$.

It can be written as $x^4 \equiv 5^4 \ (mod\ 8.5^3)$

It is of the type $x^4 \equiv a^4 \ (mod\ 8.p^n)\ with\ \ a = 5, p = 5, n = 3.$

It has eight solutions given by $x \equiv 2p^n \pm a,\ 4p^n \pm a,\ 6p^n \pm a,\ 8p^n \pm a \ (mod\ 8.p^n).$

$$\equiv 2.5^3 \pm 5; 4.5^3 \pm 5; 6.5^3 \pm 5; 8.5^3 \pm 5 \ (mod\ 8.5^3)$$

$$\equiv 250 \pm 5; 500 \pm 5; 750 \pm 5; 1000 \pm 5 \ (mod\ 1000)$$

$$\equiv 245, 155; 495.505; 745,755; 995, 5 \ (mod\ 1000).$$

These are the required solutions.

Conclusion:

Thus, it can be concluded that the solvable standard bi-quadratic congruence

$x^4 \equiv a^4 \ (mod\ 4.p^n)$ has exactly four incongruent solutions given by:

$x \equiv 2p^n \pm a, 4p^n \pm a,\ (mod\ 4.p^n)$. Also, the congruence

$x^4 \equiv a^4 \ (mod\ 8.p^n)$ has exactly eight incongruent solutions given by:

$x \equiv 2p^n \pm a, 4p^n \pm a, 6p^n \pm a, 8p^n \pm a \ (mod\ 8.p^n)$.

The established formulae are tested by solving different examples.

MERIT OF THE PAPER

A very little material about the standard bi-quadratic congruence is found in the literature of mathematics. The author established direct formulation of solutions of the said congruence. Formulation makes the problems simple and time-saving. This is the merit of the paper.

Reference:

1. Burton D M, "*Elementary Number Theory*", 2/e, 2003, Universal Book Stall.
2. Roy B M, "*Discrete Mathematics & Number Theory*", 1/e, Jan. 2016, Das Ganu Prakashan, Nagpur.
3. Roy, B. M., "Formulation of some classes of standard bi-quadratic congruence of composite modulus," International Journal of Engineering Technology Research and Management, ISSN: Vol-3, Issue-1, feb-2019.

4. Roy, B. M.," Formulation of some classes of standard solvable bi-quadratic congruence of even composite modulus—a generalization" International Journal of Recent Innovation in Academic research, ISSN-:2635-3040.
5. Thomas Koshy, "*Elementary Number Theory with Applications",* 2/e (Indian print, 2009), Academic Press.

