

# Locational Marginal Pricing Calculation Methods for Optimal Power Flow under Lossy Conditions

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## Abstract:

This paper deals with concepts of Locational Marginal Pricing (LMP). LMP can be calculated by solving either DC OPF or AC OPF. Firstly, the calculation of LMP is carried out using DC OPF algorithm for both lossy and lossless cases. Secondly, it is solved by AC OPF algorithm. Finally, Artificial Intelligence(AI) – Genetic Algorithm (GA) technique has been introduced for computing LMP. All the above methods have been implemented for IEEE – 30 bus system and the results are compared.

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## NOMENCLATURE

$P_k$	Real power generation of bus k
$D_k$	Demand on the bus of bus k
$R_i$	Line resistance of line i
$X_i$	Line reactance of line i
$G_i$	Line conductance of line i
$B_i$	Line susceptance of line i
$\theta_k$	Voltage angle of bus k
C	Cost function
N	Number of buses
F	Transmission Line limit
$LF_i$	Load factor
$DF_i$	Delivery factor
$P_{loss}$	Power loss
$GSF_i$	Generation shift factor
Offset	System loss linearization offset.
$F_{nd}$	Fictitious nodal demand
$\mu$	shadow price for line flow constraint

clear and accurate signal of the price of electricity at every location on the grid. These prices, in turn, reveal the value of locating new generation, upgrading transmission, or reducing electricity consumption needed in a well-functioning market to increase competition and improve the systems' ability to meet power demand.

LMP calculation (Litvinov et al, 2004) can be formulated with Optimal Power Flow(OPF). The LMP of electricity at a location (bus) is defined as the least cost to service the next increment of demand at that location consistent with all power system operating constraints.

Calculation of LMP in a market environment involves solving an optimization problem. The LMP models existing in literature can be broadly categorized into two classes, namely, the ACOF model and the DCOF model (H.Liu et al,2009). As such, LMP is usually decomposed into three components: marginal energy price (MEP), marginal loss price (MLP), and marginal congestion price (MCP), which is carefully analyzed in (M. Rivier and J. I. Perez-Arriaga,1993). Because of the inherent nonlinearity of transmission losses, there is a great desire to improve the accuracy in loss calculation and pricing (Z.Hu et al, 2010).

Artificial intelligent techniques can also be used to solve OPF problem through which LMP can be calculated. Inspired by the results of Genetic Algorithm (GA) method, LMP calculation is performed using GA in this work. The OPF is modelled as a nonlinear constrained problem with continuous variables in GA. The algorithm uses a local search method for the search of Global optimum solution. Binary coded Genetic

## I. INTRODUCTION

Locational Marginal Price (LMP), also referred to as Nodal Price is a pricing concept used in deregulated electricity markets. This can be used to manage the efficient use of the transmission system when congestion occurs on the bulk power grid. The Federal Energy Regulatory Commission (FERC) has proposed LMP as a way to achieve short-term and long-term efficiency in wholesale electricity markets. In electricity, LMP may vary at different times and locations based on transmission congestion and losses. LMP provides a

algorithm is replaced with continuous genetic algorithm that uses real values of generation instead of binary coded data.

This paper is organized as follows. Section II discusses the concept of loss factor and delivery factor and presents the iterative DCOF model for LMP calculation. Section III presents the ACOF-based approach to calculate LMP. Section IV presents the test system and results of the proposed approaches. Section V concludes the paper.

## II. LMP CALCULATION USING 4 METHODS

This section first presents the common model for DC optimal power flow without losses. Then, the loss factor and delivery factor are discussed. Based on delivery factor, an iterative DCOF model with marginal losses is presented and discussed (F.Li et al, 2006).

### A. Generic DCOF Model Without Losses

The generic DCOF model without the consideration of losses can be easily modelled as the minimization of the total production cost subject to power balance and transmission (including contingency) constraints. This may be written as follows:

$$\min_{p_i} \sum_{i \in I} C_i(p_i) \tag{1}$$

Subject to:

$$\sum_{m=1, m \neq k}^N \left[ \frac{1}{x_{km} (\theta_k - \theta_m)} \right] = P_k - D_k \text{ for } k=1..N \tag{2}$$

N – total number of buses

There are no voltage magnitude constraints because all voltage magnitudes are assumed to be 1.0 p.u. The generator and branch power flow constraint becomes:

$$P_i^{\min} \leq p_i \leq P_i^{\max} \tag{3}$$

$$F_{km}^{\min} \leq P_{km}(x) \leq F_{km}^{\max} \tag{4}$$

Based on the assumptions,

$$P_{km}(x) = \frac{\theta_k - \theta_m}{x_{km}} \tag{5}$$

### B. Loss Factor and Delivery Factor

To consider the transmission loss in DCOF model, the loss factor has to be considered. The loss

factor (LF) at the  $i^{\text{th}}$  bus may be viewed as the change of total system loss with respect to a 1MW increase in injection at that bus. Mathematically, it can be written as:

$$LF_i = \frac{\partial P_{\text{loss}}}{\partial P_i} \tag{6}$$

where

$LF_i$  = loss factor at bus  $i$ .

$P_{\text{loss}}$  = total loss of the system.

The delivery factor (DF) at the  $i^{\text{th}}$  bus represents the effective MW delivered to the customers to serve the load at that bus. It is defined as

$$DF_i = 1 - LF_i \tag{7}$$

The loss factor and delivery factor can be calculated as follows. Based on the definition of loss factor, we have

$$LF_i = \frac{\partial P_{\text{loss}}}{\partial P_i} = \frac{\partial}{\partial P} \left( \sum_{k=1}^M F_k^2 \times R_k \right) \tag{8}$$

Where  $F_k$  – line flow loss of line  $k$

$M$  – total number of lines

So, the DC OPF problem can be formulated with loss as,

$$\min_{p_i} \sum_{i \in I} C_i(p_i) \tag{9}$$

Subject to:

$$\sum_{i=1}^N DF_i \times P_i = \sum_{i=1}^N DF_i \times D_i - P_{\text{loss}} \tag{10}$$

$$P_{\min} \leq P_i \leq P_{\max} \tag{11}$$

$$F_{km}^{\min} \leq P_{km}(x) \leq F_{km}^{\max} \tag{12}$$

The components of LMP can be calculated by

$$LMP_i = LMP_{\text{ref}} + LMP_{\text{con}} + LMP_{\text{loss}} \tag{13}$$

$$LMP_{\text{con}} = \sum_{i=1}^N CSF_i \times \mu_i \tag{14}$$

$$LMP_{\text{loss}} = LMP_{\text{ref}} \times (DF_i - 1) \tag{15}$$

Where,  $LMP_{\text{ref}}$  - Energy cost of reference bus

$\mu_i$  - Shadow price for line flow constraint

After obtaining the optimal solution of generation scheduling, the LMP at any bus  $i$  can be calculated as the sum of the following three LMP components: marginal energy price, marginal congestion price and marginal loss price.

### C. FND based DC OPF formulation

By analogy with the approximation idea of the DC power flow model, the following approximate formula can be derived .

$$P_{ij} \approx 0.5G_{ij} \theta_j^2 - B_{ij} \theta_j \tag{16}$$

For DC power flow,  $P_{ij}$  equals  $-B_{ij}\theta_{ij}$ . So,  $0.5G_{ij}\theta_{ij}^2$  can be considered as the approximate active power loss at one terminal of a branch. The approximate loss at the other terminal also equals  $0.5G_{ij}\theta_{ij}^2$ . The total loss of a branch equals  $I_{ij}^2R_{ij}$ . If the loss  $0.5G_{ij}\theta_{ij}^2$  is represented as a fictitious nodal demand at each end of a branch, branch losses can be approximately considered. The iterative DCOPF formulation to solve LMP problem proposed in (F.Li and R.Bo, 2007) can be formulated here

$$\min_{p_i} \sum_{i \in I} C_i(p_i) \tag{17}$$

$$(1 - LF)^T(P - D) + offset = 0 \tag{18}$$

$$P_{min} \leq P(P - D - P_{fnd}) \leq P_{max} \tag{19}$$

$$P_{min} \leq P_i \leq P_{max} \tag{20}$$

$$P_{fnd} = 0.5 \sum (P_{ij} + P_{ji}) \tag{21}$$

$$P_{ij} = G_{ij}V_i^2 - V_iV_j(G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \tag{22}$$

Where LF is derived from DC power flow model; offset and  $P_{fnd}$  are obtained from the results of DC power flow, which are updated after each iteration. It should be noted that the loss variable is not included here.

D. ACOPF model

In general the ACOPF model can be formulated as

$$\min_{p_i} \sum_{i \in I} C_i(p_i) \tag{23}$$

Subject to:

$$P_{cal} + D_k - \sum p_i = 0 \text{ (Real power balance)} \tag{24}$$

$$Q_{cal} + Q_k - \sum q_i = 0 \text{ (Reactive power balance)} \tag{25}$$

$$p_{imin} \leq p_i \leq p_{imax} \text{ (Gen. real power limits)} \tag{26}$$

$$q_{imin} \leq q_i \leq q_{imax} \text{ (Gen. reactive power limits)} \tag{27}$$

$$P_{cal} = \sum_{i=1}^N V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] \tag{28}$$

$$Q_{cal} = \sum_{i=1}^N V_k V_i [G_{ki} \sin(\theta_k - \theta_i) + B_{ki} \cos(\theta_k - \theta_i)] \tag{29}$$

The LMP from the above formulation is the sensitivity of the Lagrange function with respect to the bus load.

III.GENETIC ALGORITHM

The drawbacks of conventional methods can be summarized as three major problems:

Firstly, they may not be able to provide optimal solution and usually getting stuck at a local optimal.

Secondly, all these methods are based on assumption of continuity and differentiability of objective function which is not actually allowed in a practical system. Finally, all these methods cannot be applied with discrete variables, which are transformer taps.

It is observed that Genetic Algorithm (GA) is an appropriate method to solve this problem, which eliminates the above drawbacks. GAs differs from other optimization and search procedures in four ways:

GAs work with a coding of the parameter set, not the parameters themselves. Therefore GAs can easily handle the integer or discrete variables. GA search within a population of points, not a single point. Therefore GAs can provide a globally optimal solution. GAs use only objective function information, not derivatives or other auxiliary knowledge. Therefore GAs can deal with non-smooth, non-continuous and non-differentiable functions which are actually exist in a practical optimization problem. GAs use probabilistic transition rules, not deterministic rules.

We use GA because the features of GA are different from other search techniques in several aspects, such as: First, the algorithm is a multipath that searches many peaks in parallel and hence reducing the possibility of local minimum trapping. Secondly, GA works with a coding of parameters instead of the parameters themselves. The coding of parameter will help the genetic operator to evolve the current state into the next state with minimum computations. Thirdly, GA evaluates the fitness of each string to guide its search instead of the optimization function.

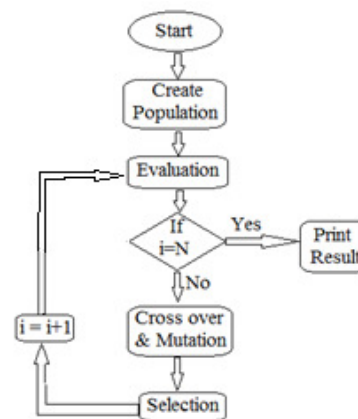


Figure 1: Flow chart

Algorithm  
Step 1: Creating Population for Power generation and LMP

- Step 2: Evaluating all the chromosomes in population
- Step 3: Creating new child chromosomes by performing Cross over
- Step 4: Performing the genetic operation Mutation
- Step 5: Selection of best chromosomes is done after evaluating the mutated parent and on the basis of fitness function.
- Step 6: If the iteration count reaches maximum, Print the results else go to the next step
- Step 7: Increasing the iteration count by 1 and again go to the step 3, cross over

26	3.5032	3.329	3.442
27	3.0318	3.329	3.642
28	3.0353	3.119	2.912
29	3.4086	3.789	3.812
30	3.2212	3.389	3.272

TABLE II  
AC OPF Results

IV. TEST RESULTS ON 30-BUS EXAMPLE SYSTEM

This section presents the test results of above mentioned methodologies for 30 bus system.

TABLE I  
DC OPF Model results

Bus no	LMP(\$/MWh)		
	Conventional	Model 1	Model 2
1	3.0377	3.789	3.662
2	3.5339	3.227	3.432
3	3.283	3.859	3.222
4	3.0899	2.909	2.032
5	4.8981	5.299	5.422
6	2.8297	2.459	2.112
7	4.2418	5.259	4.852
8	3.6504	3.759	3.742
9	4.6397	5.569	5.562
10	5.5315	5.099	5.022
11	2.9483	2.891	3.782
12	3.5347	2.939	2.122
13	2.9747	2.541	3.108
14	3.5455	2.439	3.902
15	2.8597	3.969	2.812
16	3.439	3.689	3.952
17	3.5924	3.729	3.182
18	3.5379	3.609	3.342
19	3.173	3.271	3.358
20	3.3157	3.341	3.878
21	2.4373	1.599	1.882
22	3.9582	3.659	4.012
23	2.2037	1.399	1.402
24	2.2393	2.279	2.932
25	4.8718	5.939	5.902

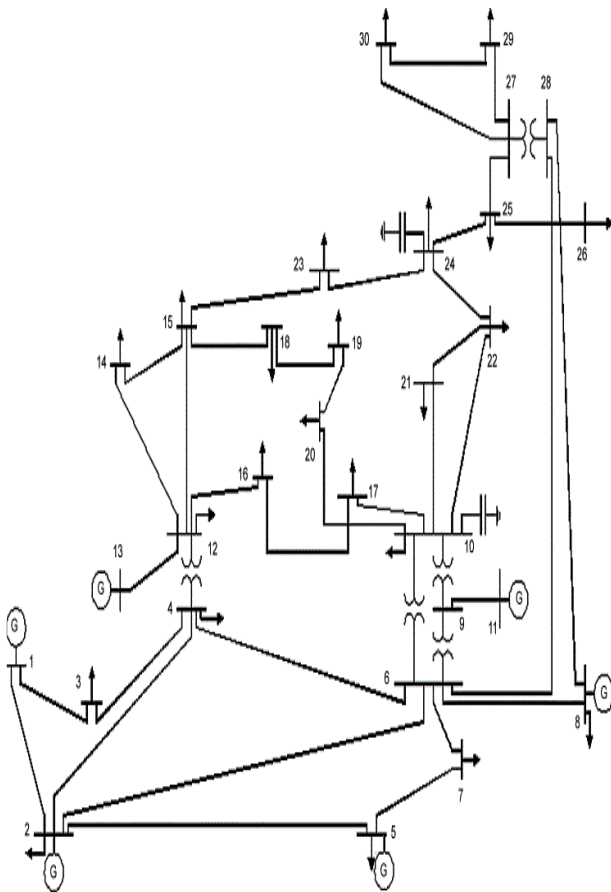
Bus No	LMP (\$/MWh) (Conventional)	LMP (\$/MWh) (GA)
1	3.662	2.9006
2	3.689	2.8772
3	3.754	3.0404
4	3.771	2.8433
5	3.744	2.7083
6	3.779	3.7592
7	3.801	2.5045
8	4.383	3.693
9	3.823	2.5599
10	3.846	3.7481
11	3.823	2.823
12	3.81	3.9815
13	3.81	3.8728
14	3.868	3.7633
15	3.856	2.831
16	3.849	2.9093
17	3.862	3.4813
18	3.911	2.263
19	3.926	2.444
20	3.91	3.4048
21	3.854	2.4975
22	3.843	3.2092
23	3.813	3.4276
24	3.884	2.4119
25	3.932	3.8566
26	3.999	3.2273
27	3.916	2.5147
28	3.106	3.1485
29	3.966	2.8041
30	3.051	2.4242

V.CONCLUSION

Table 1 illustrates DC OPF including losses will give accurate results, either model 1 or model 2, both are similar. From table 2 we came to know that, GA method provides optimized results. In this paper, lossy DC OPF Models are compared with lossless conventional model. AC OPF conventional model is compared with Genetic Algorithm. AC OPF LMP calculation is accurate method compared to DC OPF calculations. But due to some, time consumption difficulties it can't be used in large systems. Since the AI techniques are more efficient, LMP is computed using GA. In spite of its own disadvantages, GA gives proper results for LMP.

APPENDIX

The IEEE 30 bus system taken for the case study is shown below.



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