GROUP Q_8 MEAN CORDIAL LABELING

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Abstract. Let G be a (p, q) graph and let Q_8 be a graph. Let f :V(G) → Q_8be a function. For each edge uv assign the label 1 if$\left\lceil \frac{o(f(u)) + o(f(v))}{2} \right\rceil$is even or0 otherwise. f is a group Q_8 mean graph cordial labeling if $|v_f(a) - v_f(b)| \leq 1$and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(a)$, $v_f(b)$ and $e_f(y)$ respectively denote thetotal number of vertices labeled with the elements a, b in Q_8 and the totalnumber of edges labeled with y(y = 0,1). In this paper, we investigate path,comb and ladder related graphs that admit group Q_8 mean cordial labeling.

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1. Introduction

Let G be a simple graph with vertex set V(G) and edge set E(G) respectively.We follow the basic notations and terminologies of graph theory as in Harary [2].A labeling of a graph theory is a map that carries the graph elements to the setof numbers, usually to the set of non-negative or positive integers. For all detailedsurvey of graph labeling we refer Gallian [1]. Lourdusamyet. al introduced theconcept of group S_3 cordial remainder labeling [3]. Somasundaramet. al introduced the concept of mean labeling [6]. Motivated by these concepts, in this paperwe introduced the new labeling called group Q_8 mean cordial labeling. Here wehave proved that path, comb, ladder, slanting ladder, triangular ladder, $L_n \odot K_1$and $TL_n \odot K_1$ admit group Q_8 mean cordial labeling.

Definition 1.1. [4] Ladder graph $L_n (n \geq 2)$ is product graph$P_2 \times P_n$ with 2nvertices and 3n-2 edges.

Definition 1.2. [5] The slanting ladder SL_n is a graph obtained from L_n by addingthe edges $u_i u_{i+1}$, $1 \leq i \leq n-1$, where u_i and v_i, $1 \leq i \leq n$ are the vertices of $L_n$such that $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ are two paths of length n in the graphL_n.

Definition 1.3. [4] The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 andjoining the i^{th} vertices of G_1 with an edge to every vertex in the i^{th} copy of G_2.

2. Main Results

Definition 2.1. Let G be a (p, q) graph and let Q_8 be a quaternion group. Let$f : V(G) \rightarrow Q_8$ be a function. For each edge uv assign the label 1 if $\left\lceil \frac{o(f(u)) + o(f(v))}{2} \right\rceil$is even or 0 otherwise. f is a group Q_8 mean graph cordial labeling if the differencebetween the total number of vertices labeled with a and b is atmost 1, where a, b are1the elements in Q_8 and the difference between the total number of edges labeled with0 and 1 is atmost 1.

Remark 2.2. Consider the quaternion group Q_8. Let the elements of Q_8 be$\{\pm 1, \pm i, \pm j, \pm k\}$. Now $o(1) = 1$, $o(-1) = 2$, $o(\pm i) = o(\pm j) = o(\pm k) = 4$.

Theorem 2.3. The path P_n, $n \geq 1$ are group Q_8 mean cordial graph.

Proof. Let $v_1, v_2, \ldots, v_n$ denote the vertices of P_n. Define a function $f : V(P_n) \rightarrow Q_8$ as follows.

$$f(v_s) = \begin{cases} \text{i if } s \equiv 1 \pmod 8 \\ -\text{i if } s \equiv 2 \pmod 8 \\ 1 \text{ if } s \equiv 3 \pmod 8 \\ \text{j if } s \equiv 4 \pmod 8 \\ -\text{j if } s \equiv 5 \pmod 8 \\ \text{k if } s \equiv 6 \pmod 8 \\ -1 \text{ if } s \equiv 7 \pmod 8 \\ -\text{k if } s \equiv 0 \pmod 8 \end{cases}$$

We observe that $|e_f(0) - e_f(1)| = \begin{cases} 0 & \text{if n is odd} \\ 1 & \text{if n is even} \end{cases}$.

Hence the path P_n is group Q_8 mean cordial graph.

Theorem 2.4. The comb graph $P_n \odot K_1$ is Q_8 mean cordial graph.

Proof. Let $V(P_n \odot K_1) = \{v_s , u_s: 1 \leq s \leq n\}$ and $E(P_n \odot K_1) = \{v_s v_{s+1} : 1 \leq s \leq n-1\} \cup \{v_s u_s : 1 \leq s \leq n\}$. Let $f : V(P_n \odot K_1) \to Q_8$ as follows.

$$f(v_s) = \begin{cases} \text{i if } s \equiv 1 \pmod 4 \\ 1 \text{ if } s \equiv 2 \pmod 4 \\ -\text{j if } s \equiv 3 \pmod 4 \\ -\text{k if } s \equiv 0 \pmod 4 \end{cases}$$

$$f(u_s) = \begin{cases} -\text{i if } s \equiv 1 \pmod 4 \\ \text{j if } s \equiv 2 \pmod 4 \\ \text{k if } s \equiv 3 \pmod 4 \\ -1 \text{ if } s \equiv 0 \pmod 4 \end{cases}$$

In this type of labeling pattern we observe that

When n is even, $e_f(0) = n$ and $e_f(0) = n-1$.

When n is odd,

$$e_f(0) = \begin{cases} n-1 & \text{if } n-1 \text{ is a divisor of } 4 \\ n & \text{if } n-1 \text{ is not a divisor of } 4 \end{cases}$$

$$e_f(1) = \begin{cases} n & \text{if } n-1 \text{ is a divisor of } 4 \\ n-1 & \text{if } n-1 \text{ is not a divisor of } 4 \end{cases}$$

Hence the comb is Q_8 mean cordial graph.

Theorem 2.5. The ladder graph L_n is Q_8 mean cordial graph.

Proof. Let $V(L_n) = \{v_s, u_s : 1 \leq s \leq n\}$ and $E(L_n) = \{v_s v_{s+1}, u_s u_{s+1} : 1 \leq s \leq n-1\} \cup \{v_s u_s : 1 \leq s \leq n\}$. Define $f : V(L_n) \rightarrow Q_8$ as follows:

$$f(v_s) = \begin{cases} i \text{ if } s \equiv 1 \pmod 4 \\ -i \text{ if } s \equiv 2 \pmod 4 \\ j \text{ if } s \equiv 3 \pmod 4 \\ -j \text{ if } s \equiv 0 \pmod 4 \end{cases}$$

$$f(u_s) = \begin{cases} 1 \text{ if } s \equiv 1 \pmod 4 \\ k \text{ if } s \equiv 2 \pmod 4 \\ -1 \text{ if } s \equiv 3 \pmod 4 \\ -k \text{ if } s \equiv 0 \pmod 4 \end{cases}$$

It is easy to verify that

$$e_f(0) = \begin{cases} n + \left\lfloor \frac{n-1}{2} \right\rfloor & \text{if n is even} \\ n + \left\lfloor \frac{n}{2} \right\rfloor & \text{if n is odd} \end{cases}$$

$$e_f(1) = \begin{cases} n + \left\lfloor \frac{n-1}{2} \right\rfloor & \text{if n is even} \\ n + \left\lfloor \frac{n}{2} \right\rfloor - 1 & \text{if n is odd} \end{cases}$$

Thus L_n is group Q_8 mean cordial graph.

Theorem 2.6. The slanting ladder SL_n is Q_8 mean cordial graph.

Proof. Let $V(SL_n) = \{v_s, u_s : 1 \leq s \leq n\}$ and $E(SL_n) = \{v_s v_{s+1}, u_s u_{s+1}, v_s u_{s+1} : 1 \leq s \leq n-1\}$.

Define $f : V(SL_n) \rightarrow Q_8$ as follows:

$$f(v_s) = \begin{cases} i \text{ if } s \equiv 1 \pmod 4 \\ 1 \text{ if } s \equiv 2 \pmod 4 \\ -i \text{ if } s \equiv 3 \pmod 4 \\ -1 \text{ if } s \equiv 0 \pmod 4 \end{cases}$$

$$f(u_s) = \begin{cases} j \text{ if } s \equiv 1 \pmod 4 \\ -j \text{ if } s \equiv 2 \pmod 4 \\ k \text{ if } s \equiv 3 \pmod 4 \\ -k \text{ if } s \equiv 0 \pmod 4 \end{cases}$$

This implies that

$$e_f(0) = \begin{cases} n + \left\lfloor \frac{n}{2} \right\rfloor - 1 \text{ if n is odd} \\ n + \left\lfloor \frac{n-1}{2} \right\rfloor - 1 \text{ if n is even} \end{cases}$$

$$e_f(1) = \begin{cases} n + \left\lfloor \frac{n}{2} \right\rfloor - 1 \text{ if n is odd} \\ n + \left\lfloor \frac{n-1}{2} \right\rfloor \text{ if n is even} \end{cases}$$

Therefore SL_n is group Q_8 mean cordial graph.

A group Q_8 mean cordial labeling of SL_7 is given in Figuer 1.

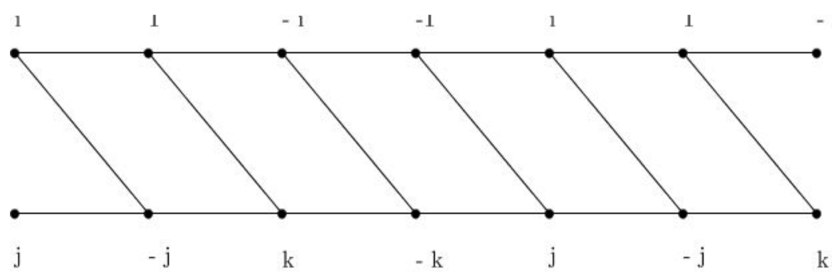

Figure 1

Theorem 2.7. The triangular ladder TL_n is group Q_8 mean cordial graph.

Proof. Let $V(TL_n) = \{v_s, u_s : 1 \leq s \leq n\}$ and $E(TL_n) = \{v_sv_{s+1}, u_su_{s+1}, v_su_{s+1} : 1 \leq s \leq n-1\} \cup \{v_su_s : 1 \leq s \leq n\}$. Define $f : V(TL_n) \to Q_8$ as follows:

$$f(v_s) = \begin{cases} i \text{ if } s \equiv 1 \pmod 4 \\ -1 \text{ if } s \equiv 2 \pmod 4 \\ -i \text{ if } s \equiv 3 \pmod 4 \\ 1 \text{ if } s \equiv 0 \pmod 4 \end{cases}$$

$$f(u_s) = \begin{cases} \text{k if } s \equiv 1 \pmod 4 \\ -\text{k if } s \equiv 2 \pmod 4 \\ \text{j if } s \equiv 3 \pmod 4 \\ -\text{j if } s \equiv 0 \pmod 4 \end{cases}$$

This this type of labeling pattern we observe that$e_f(0) = 2n-2$ and $e_f(1) = 2n-1$. Hence TL_n is group Q_8 mean cordial graph.

A groupQ_8 mean cordial labeling of TL_4 is given in Figure 2.

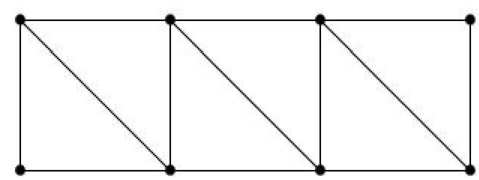

Figure 2

Theorem 2.8. The graph $L_n \odot K_1$ is group Q_8 mean cordial graph.

Proof. Let $V(L_n \odot K_1) = \{v_s^1, v_s^2, v_s^3, v_s^4 : 1 \leq s \leq n\}$ and $E(L_n \odot K_1) = \{v_s^1 v_s^2, v_s^2 v_s^3, v_s^3 v_s^4 : 1 \leq s \leq n\} \cup \{v_s^2 v_{s+1}^2, v_s^3 v_{s+1}^3 : 1 \leq s \leq n-1\}$. Define $f : V(L_n \odot K_1) \rightarrow Q_8$ as follows:

$$f(v_s^1) = \begin{cases} 1 & if\ s \equiv 1 \pmod 2 \\ i & if\ s \equiv 0 \pmod 2 \end{cases}$$

$$f(v_s^2) = \begin{cases} -i & if\ s \equiv 1 \pmod 2 \\ j & if\ s \equiv 0 \pmod 2 \end{cases}$$

$$f(v_s^3) = \begin{cases} -j & if\ s \equiv 1 \pmod 2 \\ -1 & if\ s \equiv 0 \pmod 2 \end{cases}$$

$$f(v_s^4) = \begin{cases} k & if\ s \equiv 1 \pmod 2 \\ -k & if\ s \equiv 0 \pmod 2 \end{cases}$$

This implies that

$$e_f(0) = \begin{cases} 2\text{n} + \left\lfloor \frac{n-1}{2} \right\rfloor & \text{if n is even} \\ 2\text{n} + \left\lfloor \frac{n}{2} \right\rfloor - 1 & \text{if n is odd} \end{cases}$$

$$e_f(1) = \begin{cases} 2\text{n} + \left\lfloor \frac{n-1}{2} \right\rfloor & \text{if n is even} \\ 2\text{n} + \left\lfloor \frac{n}{2} \right\rfloor & \text{if n is odd} \end{cases}$$

Therefore $L_n \odot K_1$ is group Q_8 mean cordial graph.

A group Q_8 mean cordial labeling of $L_3 \odot K_1$ is given in Figure 3.

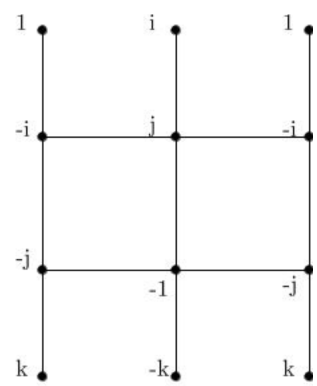

Figure 3

Theorem 2.9. The graph $TL_n \odot K_1$ is group Q_8 mean cordial graph.

Proof. Let $V(TL_n \odot K_1) = \{u_s^1, u_s^2, u_s^3, u_s^4 : 1 \le s \le n\}$ and $E(TL_n \odot K_1) = \{u_s^1 u_s^2, u_s^2 u_s^3, u_s^3 u_s^4 : 1 \le s \le n\} \cup \{u_s^2 u_{s+1}^2, u_s^3 u_{s+1}^3, u_s^2 u_{s+1}^3 : 1 \le s \le n-1\}$.
Define $f : V(TL_n \odot K_1) \to Q_8$ as follows:

$$f(u_s^1) = \begin{cases} 1 & if\ s \equiv 1 \pmod 2 \\ -j & if\ s \equiv 0 \pmod 2 \end{cases}$$

$$f(u_s^2) = \begin{cases} i & if\ s \equiv 1 \pmod 2 \\ -1 & if\ s \equiv 0 \pmod 2 \end{cases}$$

$$f(u_s^3) = \begin{cases} -i & if\ s \equiv 1 \pmod 2 \\ k & if\ s \equiv 0 \pmod 2 \end{cases}$$

$$f(u_s^4) = \begin{cases} j & if\ s \equiv 1 \pmod 2 \\ -k & if\ s \equiv 0 \pmod 2 \end{cases}$$

This implies that$e_f(0) = 3n-2$ and $e_f(1) = 3n-1$.

Thus the graph $TL_n \odot K_1$ is group Q_8 mean cordial graph.

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